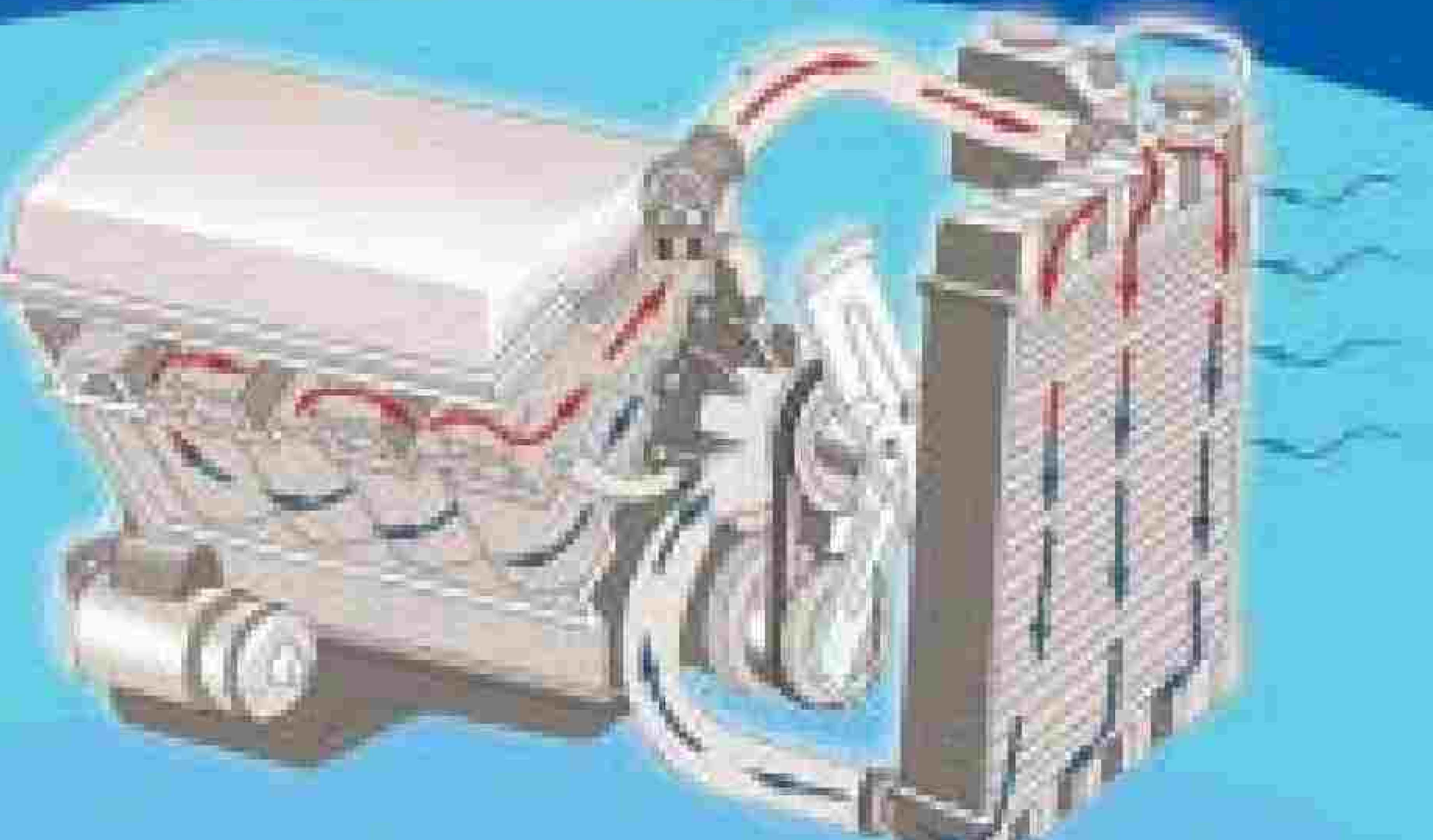
Physics

For Secondary Schools

Student's Book

Form Three



Tanzania Institute of Education

Physics

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Student's Book

Form Three


THE UNITED REPUBLIC OF TANZANIA
MINISTRY OF EDUCATION,
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Table of contents

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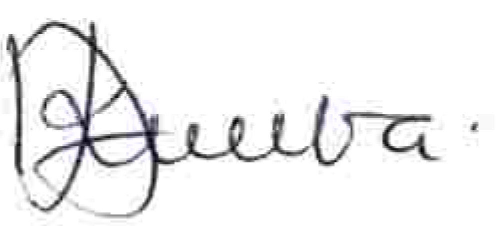

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Preface

This textbook, *Physics for Secondary Schools*, is written specifically for Form Three students in the United Republic of Tanzania. It is written in accordance with the 2007 Physics Syllabus for Ordinary Secondary Education, Form I–IV, issued by the then Ministry of Education and Vocational Training.

The book consists of nine chapters, namely Applications of vectors, Friction, Light, Optical instruments, Thermal expansion, Transfer of thermal energy, Measurement of thermal energy, Vapour and humidity and Current electricity. Each chapter contains illustrations, exercises, activities and answers to numerical questions. You are encouraged to do all the activities, projects, tasks and exercises as well as other assignments that your teacher will provide. Doing so will enable you to develop the intended competencies.

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Chapter One

Applications of vectors

Introduction

Lifting a bucket by hand to a certain height is much more difficult than lifting the same bucket using a single fixed pulley. The fixed pulley does not increase the force applied, so why does the lifting become easier? It is the direction of the force that has made the lifting work easier. To correctly express the effect of force, it is necessary to consider its direction. Physicists classify physical quantities that are described by both magnitude and direction as vector quantities, while those described by magnitude only as scalar quantities. The knowledge of vectors is fundamental in various scientific and engineering works. For example, determining the influence of wind on the motion of an airplane requires the knowledge of vectors. Designing bridges requires knowing not only their length but also the forces acting on them; this shows the importance of using vectors. Vectors are also applied in military and sports activities. In this chapter, you will learn about scalar and vector quantities, relative motion and resolution of vectors. The competencies developed will enable you to determine the shortest path between two points, estimate the influence of wind on the direction of a moving body, and design different support structures at your home. You will also be able to use relative motion in activities such as shooting at a target and kicking a ball when playing football.

Scalar and vector quantities

In the 19th century, scientists developed a suitable way of describing physical quantities. They divided quantities into two categories: scalar quantities and vector quantities. A scalar quantity is specified by its numerical value with an appropriate unit, and it has no direction. The number describing the quantity of a particular scalar is known as magnitude. Examples of scalar quantities are time, mass, volume, density, temperature, energy, distance and speed. For instance, it makes sense to say that an object's length is 3 metres or its mass is 15 kilograms when describing the distance and mass of the object. Depending on their nature, scalar quantities can be added, subtracted, multiplied and divided using the usual arithmetical laws.

On the other hand, a quantity which is described by both magnitude and direction is known as vector quantity or simply a vector. A vector quantity is expressed by its numerical value with the unit in which it is being measured, and its direction. Examples of vectors are force, velocity, acceleration, displacement, momentum, retardation, gravitational force, electric and magnetic intensities. For instance, in describing the velocity of a moving car, one should say, "a car is travelling at a speed of 100 kilometres per hour due North". This implies that, the speed of the car is 100 kilometres per hour and the direction of motion is North. However, if the intention was to describe only the speed of the car and not the direction, the statement could simply be, "the car is travelling at the speed of 100 kilometres per hour".

Therefore, a scalar quantity is defined as any physical quantity that has magnitude only, whereas, a vector quantity is defined as any physical quantity that has both magnitude and direction.

Differences between scalar and vector quantities

A scalar quantity differs from a vector quantity mainly because of direction. That is, a scalar quantity does not have direction while a vector quantity has direction. A vector quantity can therefore be presented in multi-dimensions whereas a scalar quantity can only be presented in one dimension. Table 1.1 shows the differences between scalar and vector quantities.

Table 1.1: Differences between scalar and vector quantities

Category	Scalar quantity	Vector quantity
Definition	A quantity having magnitude only	A quantity having both magnitude and direction
Direction	Has no direction	Has direction
Representation	Represented by the magnitude and unit only	Represented by magnitude, unit and direction
Variation	Varies with changes in magnitude	Varies with changes in magnitude and direction
Operation	Its operations apply normal rules of algebra	Its operations apply a special set of rules known as vector algebra
Symbol	Represented by a quantity symbol. For example, density is presented by ρ	Represented by both quantity and direction. For example, force is denoted as $\vec{F}$ or $\boldsymbol{F}$
Examples	Speed, temperature, energy, mass, volume, density and distance	Velocity, force, displacement, momentum, gravitational force and magnetic intensities

Vector representation

A vector quantity is represented by a straightline segment with an arrow, say $\overrightarrow{PQ}$ or a single bold letter, say ***V*** or non-bold letter accented by a right arrow, for example $\vec{V}$. A vector can also be represented by an arrow drawn to scale and facing a specific direction. The arrow has a head and a tail. The length of the arrow represents the magnitude of the vector. The arrow head indicates the direction of the vector. In Figure 1.1, the vector starts at P and heads towards Q.

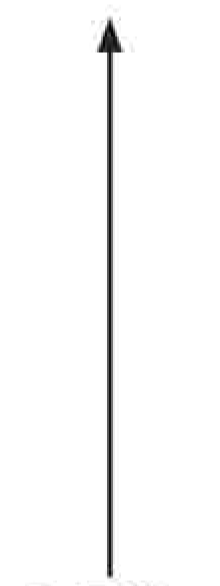


Figure 1.1: *A vector represantation*

Assume the vector in Figure 1.1 has a magnitude of 3 cm representing the distance travelled by a car northwards. If the vector is drawn at a scale of 1cm representing 1 km, the car's actual displacement northwards would be $\frac{3\text{ cm} \times 1\text{ km}}{1\text{ cm}} = 3\text{ km}$. As a result, the entire description of the car's displacement is 3 kilometres due North. Two vectors are equal if their magnitudes and directions are the same, regardless of their point of origin. This means that, if vectors start at different points, they are said to be equal if they are parallel and their magnitudes are the same. For example, in Figure 1.2, the vector **a** is equal to the vector **b**. That is, they have the same magnitude and same direction.

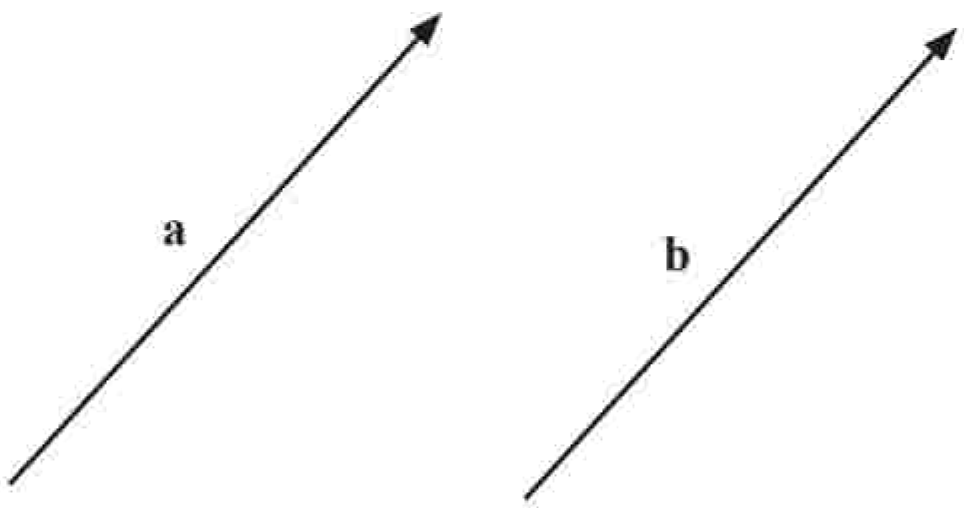


Figure 1.2: *Equal vectors*

If the vector **a** has the same magnitude as the vector **b** but opposite in direction, then the vector **b** takes a negative sign, that is, **a** = −**b**. It is also correct to say that, **b** = −**a**. Figure 1.3 shows two vectors having the same magnitude but pointing in opposite directions.

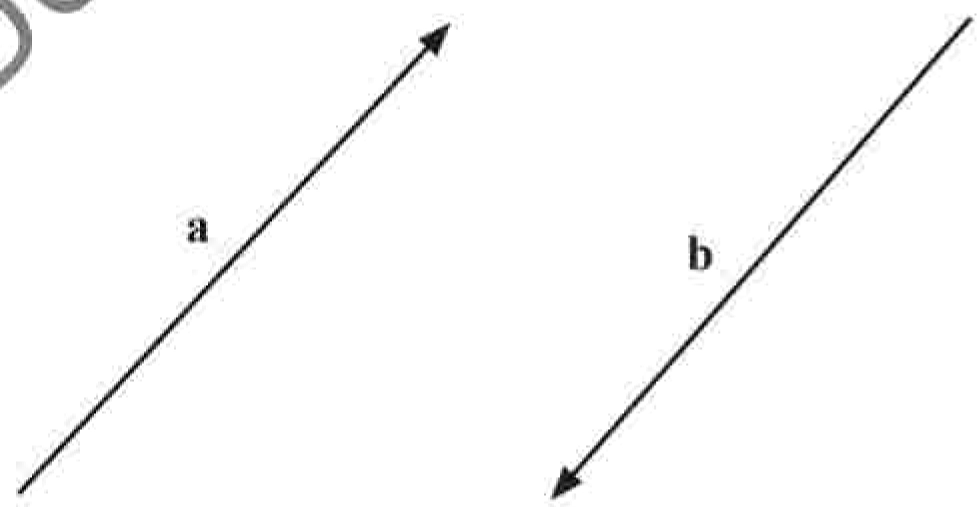


Figure 1.3: *Two vectors with the same magnitude but pointing in opposite direction*

The direction of a vector can be described using compass directions such as North, East, South and West. Vectors can also be described using an angle of rotation from the horizontal axis. For example, one can say, "a force acts at $30°$ to the horizontal plane or a car moves at $45°$ or $60°$ North of East direction". The positive horizontal line or the east direction is therefore considered as 0°. A vector having a direction of 60°, for example, is a vector that has been rotated 60° anticlockwise from the East direction as shown in Figure 1.4 (a). Besides, the vector in Figure 1.4 (b) is rotated 225° anticlockwise from the East direction.

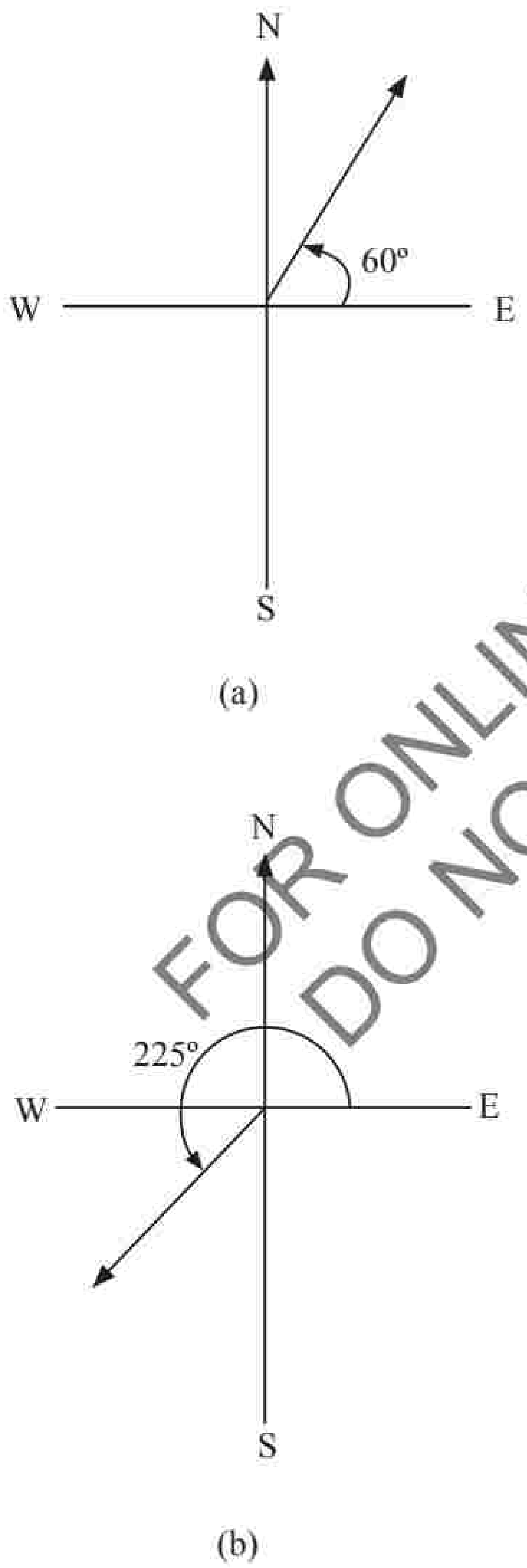


Figure 1.4: *Representation of vector directions using angles*

Addition of vectors by graphical method

The addition of vectors can conveniently be done with the help of vector diagrams. Two or more vectors can be added to obtain a resultant vector. The resultant vector is equal to the net effect of the vectors under consideration. For example, if two forces are acting on a body from different directions, the resultant force is the net effect of the two forces that are acting on the body. Vectors can be added only if they are alike. For example, two displacements can be added, but displacement cannot be added to acceleration. Vector addition can be well understood by performing Activity 1.1.

Activity 1.1

Aim: To add vectors graphically

Materials: a ruler, protractor, pencil, graph paper

Procedure

1. Consider two vectors **A** and **B** as shown in Figure 1.5 (a).

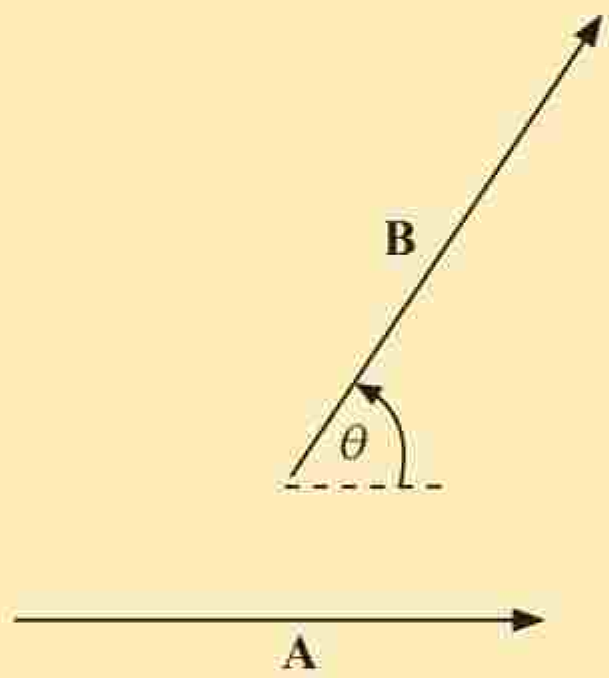


Figure 1.5 (a)

2. Select an appropriate scale. e.g., 1 cm represents 5 units.
3. Draw vector **A** to scale and in the proper direction.
4. Draw vector **B** to the same scale with its tail at the head of **A** and in the proper direction.
5. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of vector **A** to the head of vector **B**. See Figure 1.5 (b).

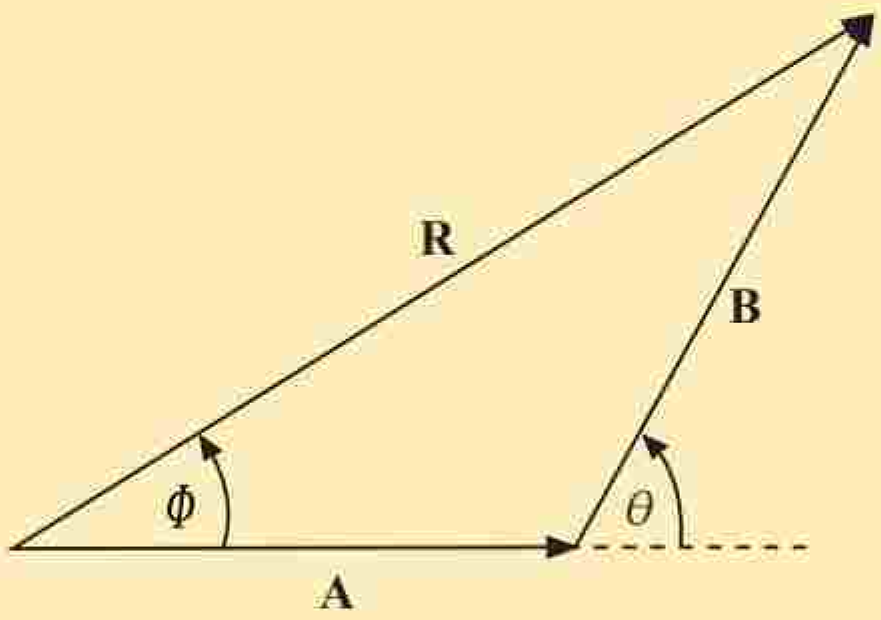


Figure 1.5 (b)

6. Calculate the magnitude of the resultant vector **R** using the selected scale and measure its direction using a protractor.

Note that, the same process applies if you add more than two vectors.

This method of adding vectors graphically is also referred to as the head-to-tail method, analytical method or geometrical method.

Example 1.1

An object is acted upon by two horizontal forces $\mathbf{F_1}$ and $\mathbf{F_2}$ of magnitudes 100 N and 80 N, respectively. Use the graphical method of vector addition to find the resultant force in the following cases:

(a) The forces act in the same direction to the right.

(b) The forces act in opposite directions.

Solution

Use a scale, 1 cm represents 20 N. The magnitudes of the two forces are represented as: 100 N as 5 cm and 80 N as 4 cm.

(a) For the forces acting in the same direction, draw the forces as shown in Figure 1.6 (a).

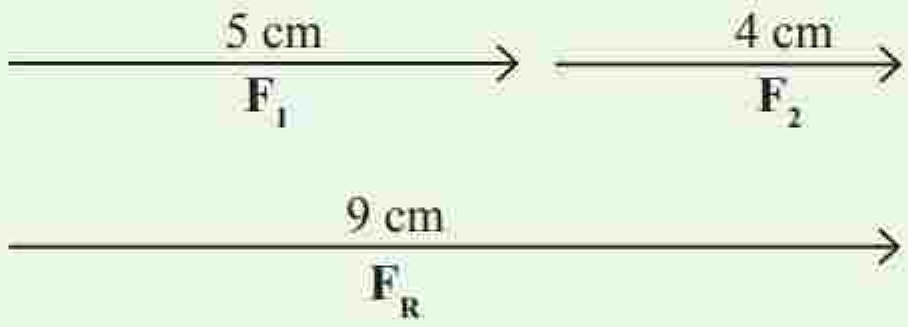


Figure 1.6 (a)

The resultant force is $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$, towards the right. The magnitude of the resultant force is

$$\frac{9\,\text{cm} \times 20\,\text{N}}{1\,\text{cm}} = 180\,\text{N}$$

(b) For the forces acting in opposite directions, draw the forces as shown in Figure 1.6 (b).

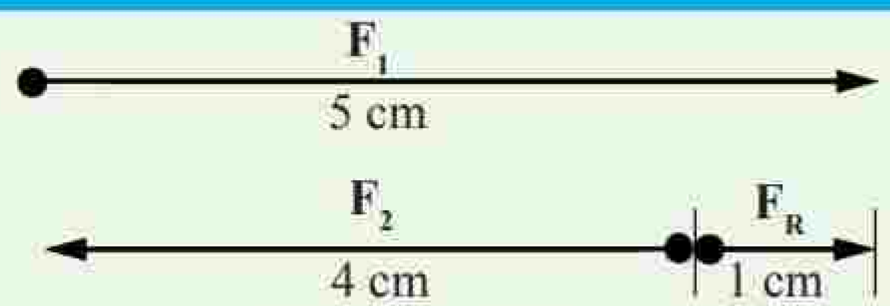


Figure 1.6 (b)

The resultant force is $\mathbf{F_R}$ towards the right. The magnitude of the resultant force is

$$\frac{1\text{cm} \times 20\,\text{N}}{1\,\text{cm}} = 20\,\text{N}$$

Example 1.2

Consider Figure 1.7 with vectors **a**, **b** and **c** having magnitudes of 32 m, 16 m and 24 m, respectively. Use graphical method of vector addition to find resultant vector.

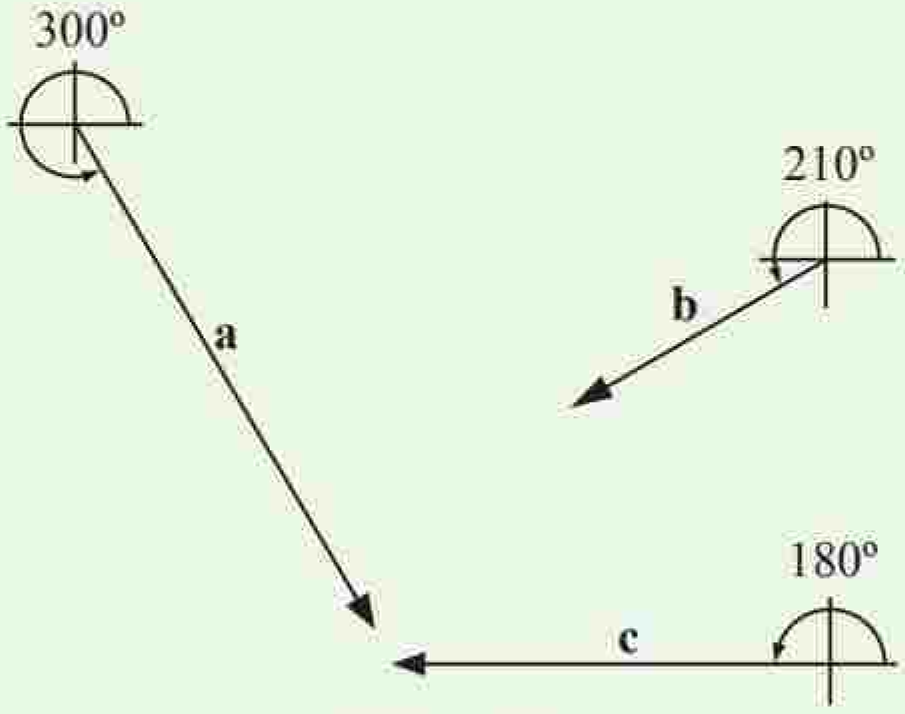


Figure 1.7

To add up the three vectors using the graphical method of vector addition, follow the steps outlined in Activity 1.1.

1. Use a scale, 1 cm represents 8 m. The magnitudes of the three vectors are represented as follows: **a** = 4 cm, **b** = 2 cm, and **c** = 3 cm.

2. Draw vector **a** to scale as shown in Figure 1.8.

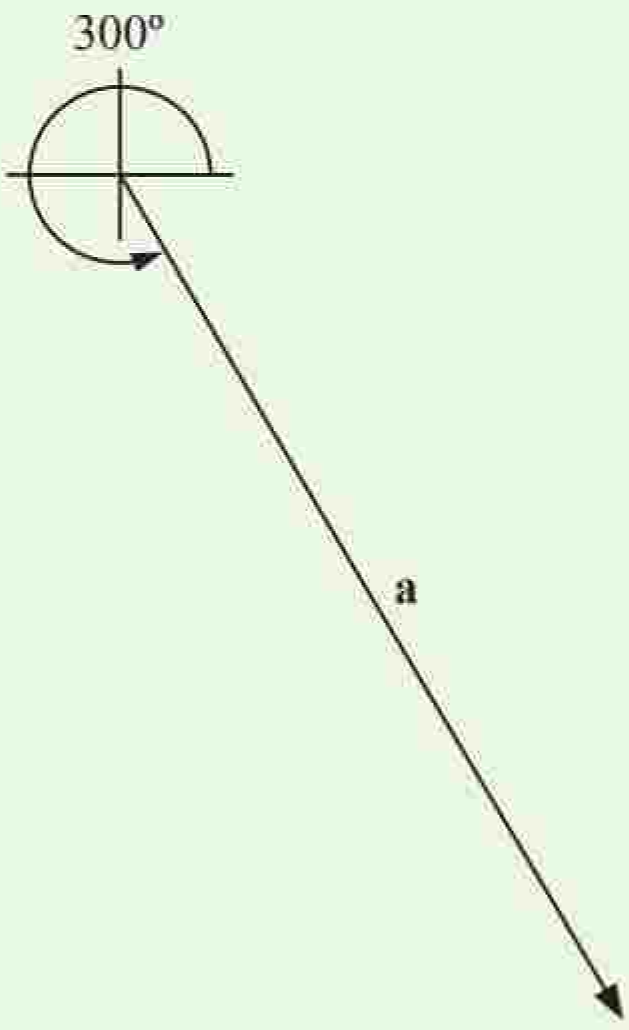


Figure 1.8

3. Starting from the head of vector **a**, draw vector **b** to scale in the direction as shown in Figure 1.9.

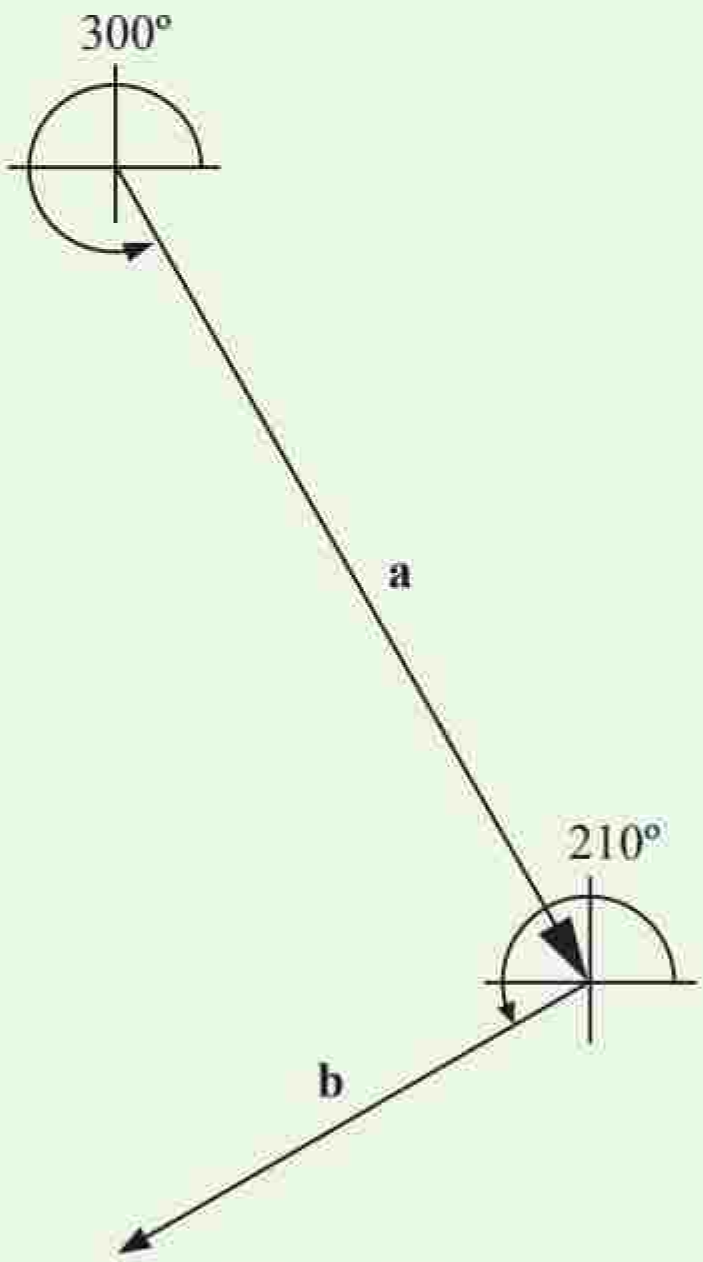


Figure 1.9

4. Repeat step 3 for vector **c** as shown in Figure 1.10.

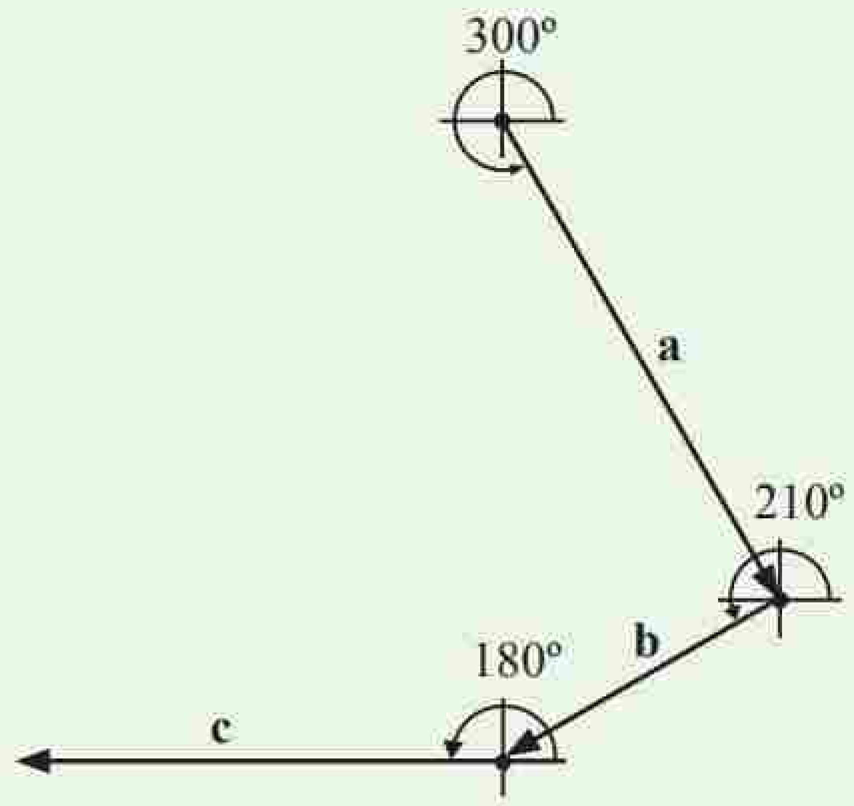


Figure 1.10

5. Draw the resultant vector from the tail of vector **a** to the head of vector **c** as shown in Figure 1.11.

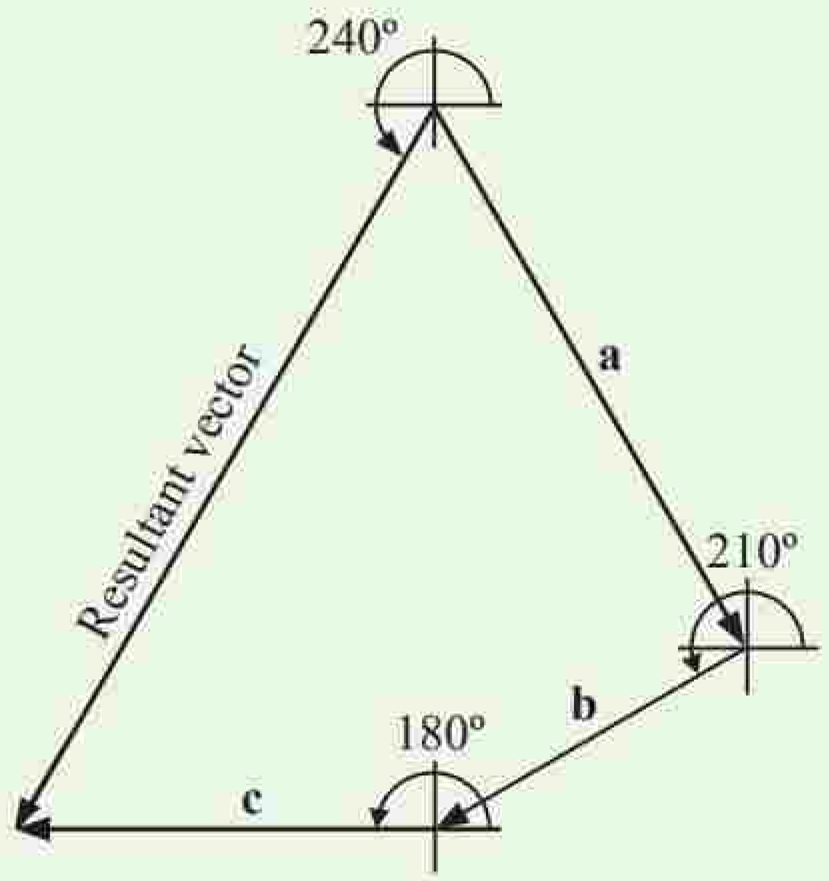


Figure 1.11

6. Measure the length of the resultant vector and convert it to the actual magnitude using the scale chosen in step 1. The resultant vector = 5 cm. Therefore, the actual length is given by

$$\frac{5 \text{ cm} \times 8 \text{ m}}{1 \text{ cm}} = 40 \text{ m}$$

7. Measure the direction of the resultant vector using the anticlockwise rotation from East. The direction of the resultant vector is 240° as shown in Figure 1.11.

   Therefore, the resultant vector is 40 m at an angle of 240°.

Note that, the resultant vector will always have the same magnitude and direction irrespective of the order followed when adding up vectors. Two laws of vector addition can guide the graphical addition of vectors. These laws are the triangle law and parallelogram law of vector addition.

Triangle law of vector addition

Triangle law of vector addition is appropriate when adding two vectors. The law states that, "*If two vectors are represented by the two sides of a triangle in sequence, then the third closing side of the triangle drawn from the tail of the first vector to the head of the second vector represents the resultant of the two vectors in both magnitude and direction*".

Consider two vectors, ***a*** and ***b*** as shown in Figure 1.12.

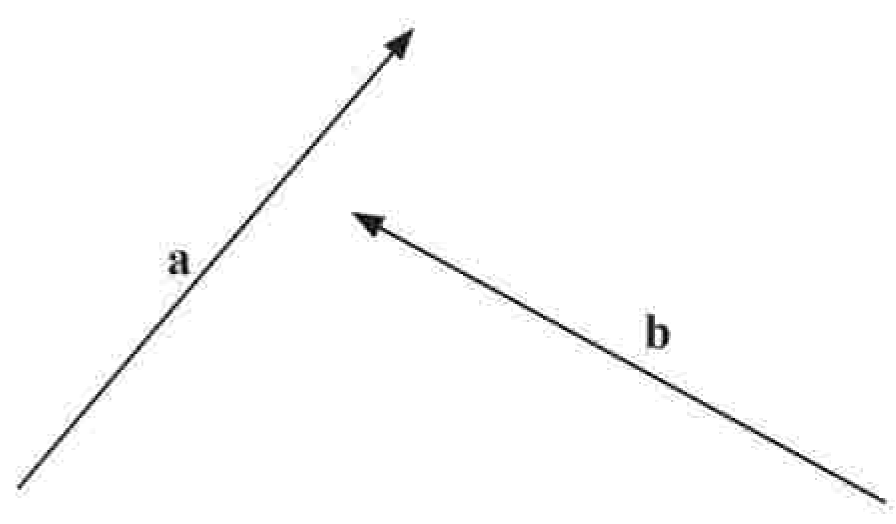


Figure 1.12: *Two vectors*

Using the Tringle law, the two vectors can be added up as shown in Figure 1.13.

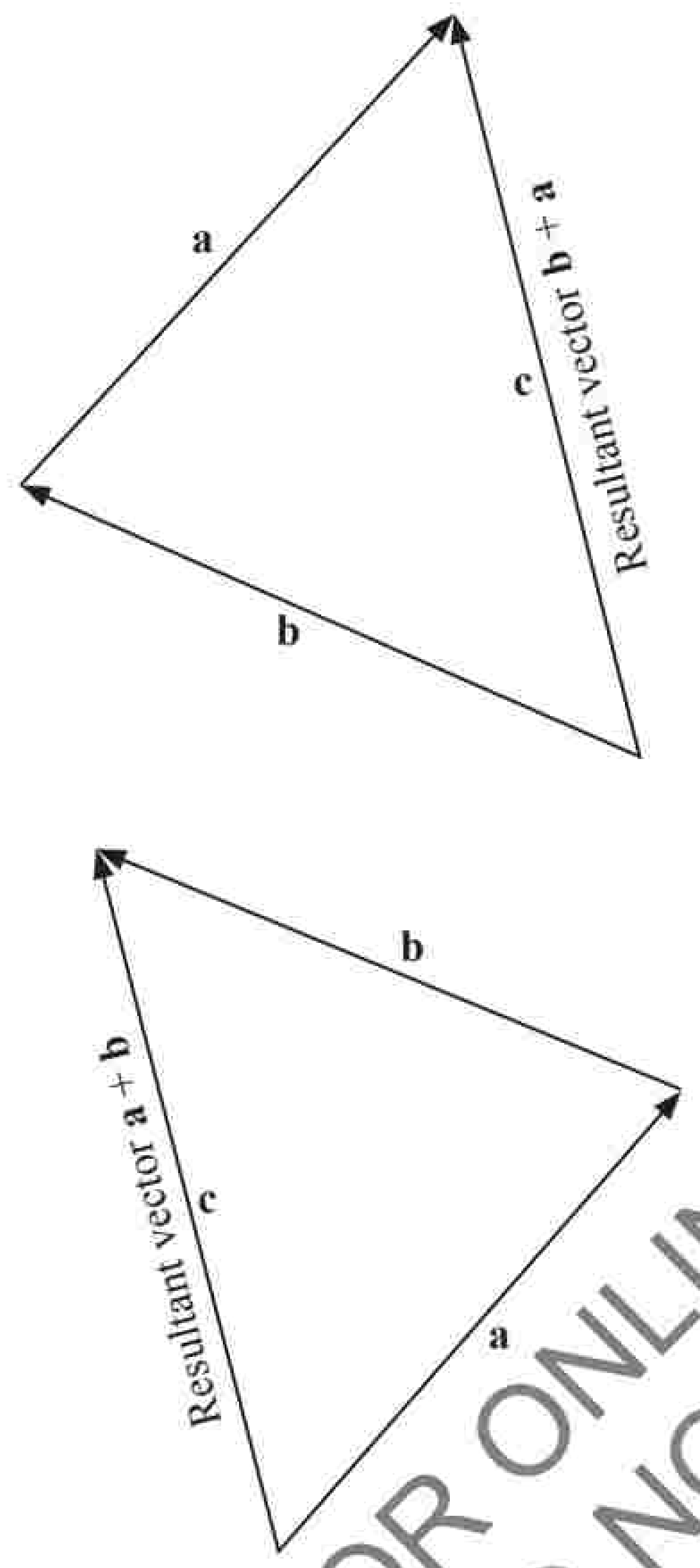


Figure 1.13: *Addition of vectors using Triangle law*

In Figure 1.13, vector **a** is added to vector **b** or vector **b** is added to vector **a**. In both cases, the resultant vector **c** that the same magnitude and direction.

That is, **a** + **b** = **b** + **a** = **c**.

The vectors **a** and **b** are represented by the two sides of a triangle as shown in Figure 1.14. Also, vector **c** is represented by the third side of the triangle in both magnitude and direction.

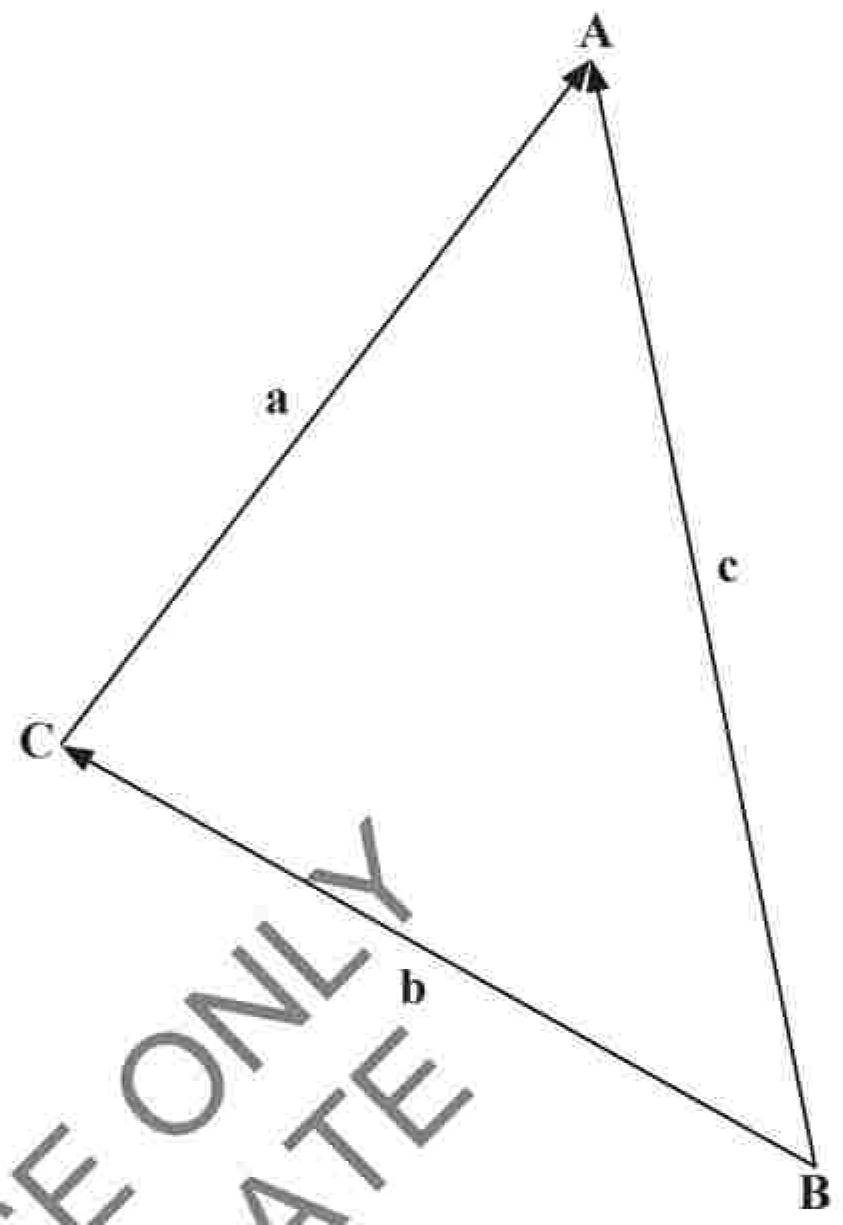


Figure 1.14: *Representation of the Triangle law of vector addition*

Example 1.3

Two forces, one with the magnitude of 8 N and the other 6 N, are acting on a body. Given that the two forces are perpendicular to each other, find the magnitude of a third force, which will just counter the two forces.

Solution

Let the resultant force be **F**.

For **F** to counter the two forces, it must be equal in magnitude to the resultant force of the two forces, but opposite in direction. Therefore, we first find the resultant of the two forces. Take a scale of 1 cm to represent 2 N. Thus, 6 N is represented by 3 cm and 8 N is represented by 4 cm as shown in Figure 1.15.

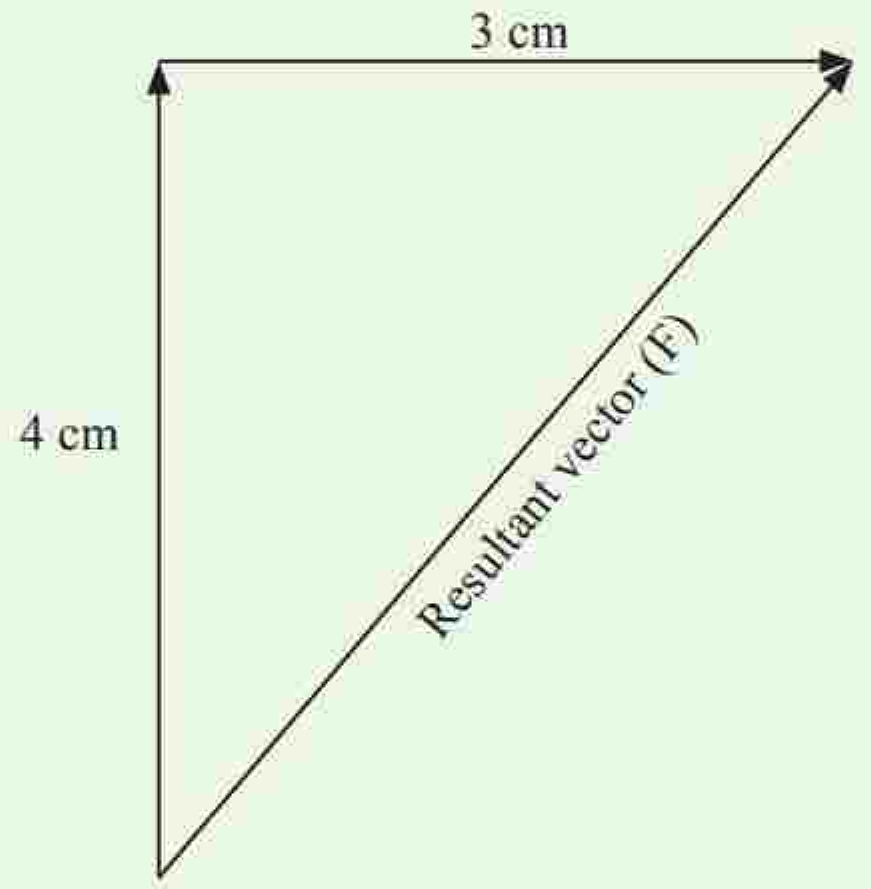


Figure 1.15

The length of **F** in the diagram is 5 cm. The magnitude of the actual force is, therefore, $\frac{5 \text{ cm} \times 2 \text{ N}}{1 \text{ cm}} = 10 \text{ N}$.

Therefore, a force of 10 N acting in the direction opposite to **F** will be needed to counter the two forces.

Parallelogram law of vector addition

Like the Triangle law, Parallelogram law applies when two vectors are to be added. Let vectors **a** and **b** be represented by arrows drawn from a common point O to form two sides of a parallelogram as shown in Figure 1.16. The vector sum $\mathbf{c} = \mathbf{a} + \mathbf{b}$ is then represented by the diagonal of a parallelogram formed by the vectors **a** and **b** as shown in Figure 1.16. Parallelogram law states that, "*If two vectors are represented by two adjacent sides of a parallelogram, then the diagonal of the parallelogram through the common point of the two vectors represents the sum of the two vectors in magnitude and direction*".

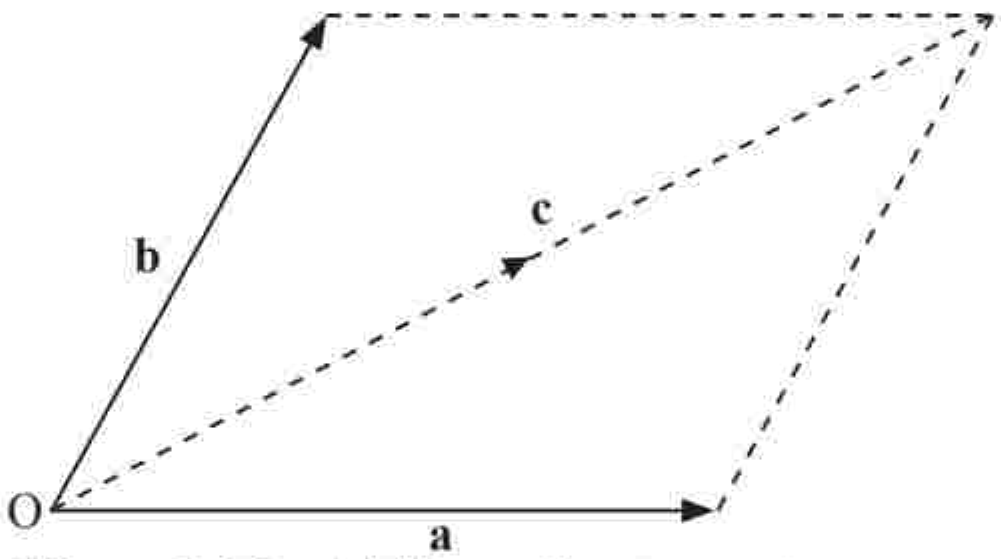


Figure 1.16: *Addition of vectors using Parallelogram law*

Thus, the resultant vector **c** is obtained by adding vectors **a** and **b** which start from the same point, O.

Consider two forces $\mathbf{F_1}$ and $\mathbf{F_2}$ acting at a point O as shown in Figure 1.17.

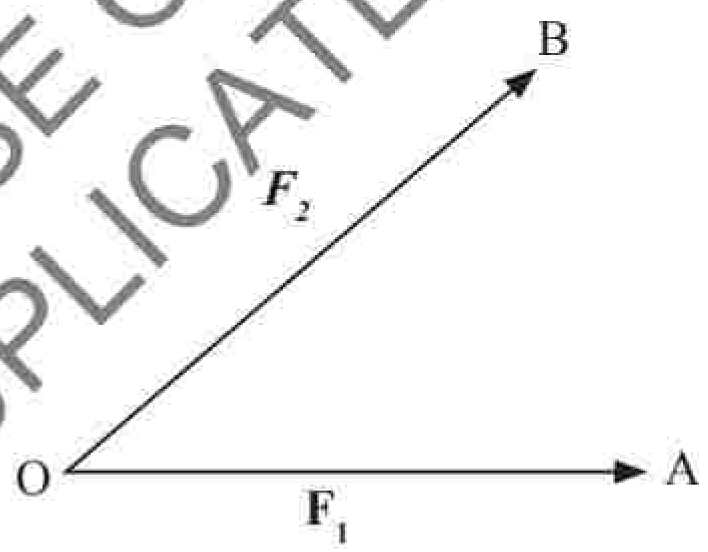


Figure 1.17: *Two forces acting at point O*

Line OA represents force $\mathbf{F_1}$ while OB represents $\mathbf{F_2}$. If a parallelogram is constructed with OA and OB as adjacent sides, then line OC is the diagonal through the common point, O, as shown in Figure 1.18.

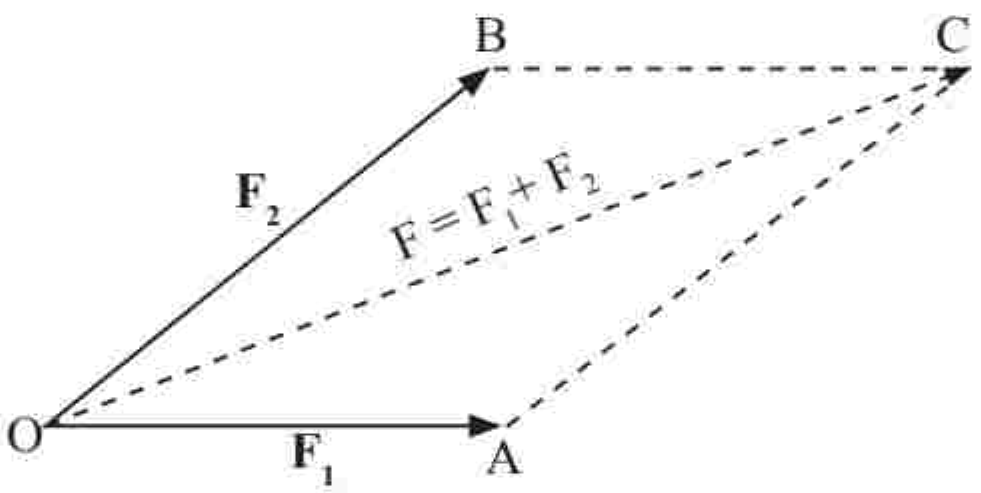


Figure 1.18: *Addition of forces using the Parallelogram law*

Line OC represents the sum of forces $\mathbf{F_1}$ and $\mathbf{F_2}$. It is the resultant force of the two forces in both magnitude and direction. Note that, the direction of the resultant force relative to direction of $\mathbf{F_1}$ is the angle between the resultant and $\mathbf{F_1}$.

Example 1.4

Using the Parallelogram law, find the resultant, $\mathbf{F_R}$ of two forces, $\mathbf{F_1} = 20\,\text{N}$ and $\mathbf{F_2} = 40\,\text{N}$ acting on an object with an angle $45°$ between them.

Solution

Take a scale of 1 cm to represent 10 N.

Using the given scale, 20 N is represented by 2 cm and 40 N is represented by 4 cm.

You can draw the two forces as adjacent sides of a parallelogram as shown in Figure 1.19. Then complete the other sides of the parallelogram using dashed lines. The diagonal of the parallelogram represents the resultant force, $\mathbf{F_R}$.

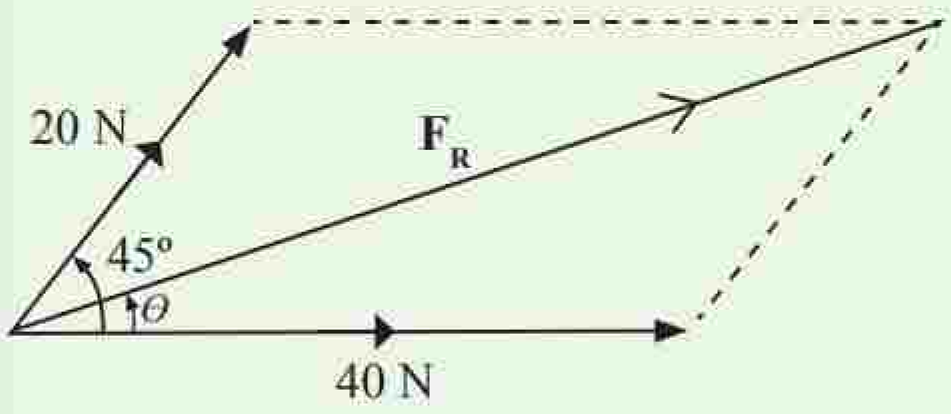


Figure 1.19

The diagonal has a length of 5.6 cm. Using the chosen scale, 1 cm represents 10 N, the magnitude of the resultant force is,

$$F_R = \frac{5.6\text{ cm} \times 10\text{ N}}{1\text{ cm}} = 56\text{ N}$$

The magnitude of the resultant force is 56 N. The angle θ is 14.6°. Therefore, the resultant force is 56 N making an angle of 14.6° with the vector F_2.

Example 1.5

Two forces, $\mathbf{F_1} = 18$ N and $\mathbf{F_2} = 30$ N, are acting on a body that is at rest. If $\mathbf{F_1}$ acts at an angle of 25° and $\mathbf{F_2}$ at 140°, find the resultant force using the Parallelogram law.

Solution

To find the resultant force **F** proceed as follows:

Using a scale 1 cm represents 6 N; draw the two vectors to scale as shown in Figure 1.20.

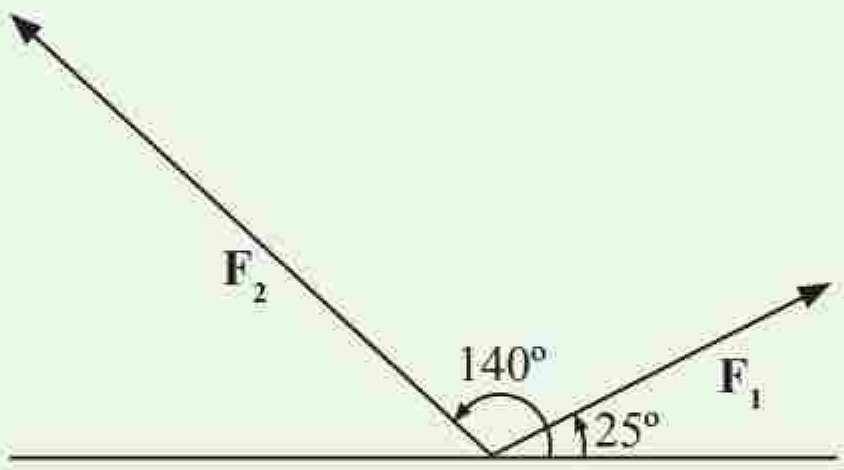


Figure 1.20

Then, complete the parallelogram by drawing the other two sides as shown in Figure 1.21.

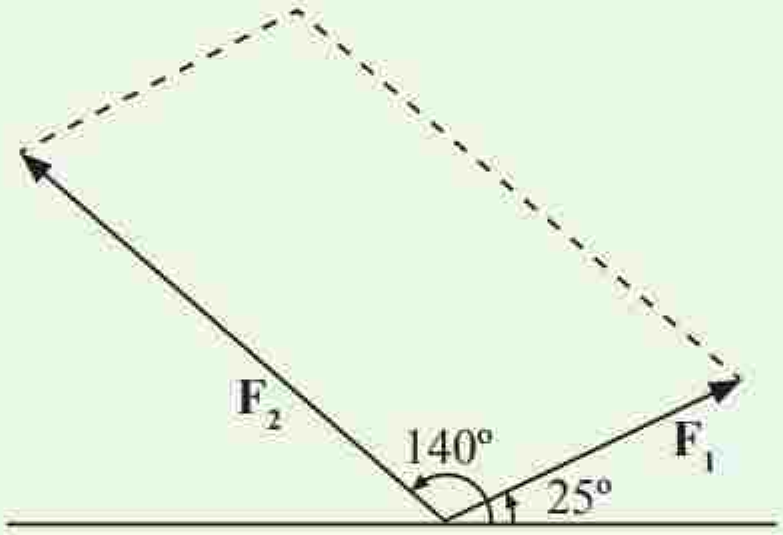


Figure 1.21

Finally, draw the diagonal of the parallelogram starting from the common point where forces $\mathbf{F_1}$ and $\mathbf{F_2}$ meet as shown in Figure 1.22.

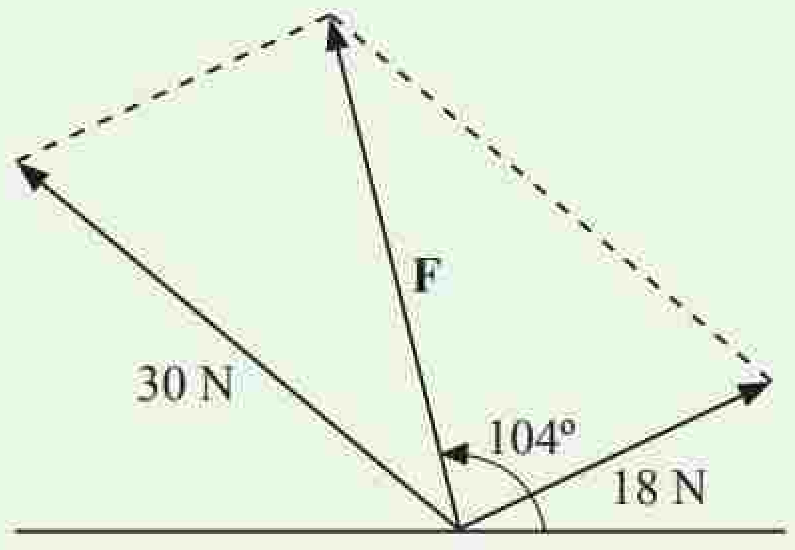


Figure 1.22

The resultant force **F** is represented in both magnitude and direction by the diagonal of the parallelogram with the length of 4.6 cm. Therefore, the magnitude of the resultant force is $\frac{4.6 \text{ cm} \times 6 \text{ N}}{1 \text{ cm}} = 27.6 \text{ N}$.

The direction of **F** is 104° from the horizontal line. Thus, the magnitude and direction of the resultant force are 27.6 N and 104°, respectively.

Exercise 1.1

1. A footballer runs 2.0 km due east, then 1.0 km at 45° north of east, and finally 0.5 km due north. Calculate the displacement of the footballer.

2. A moving car covered 10 km in the northern direction. After making a 30° turn, it covered another 20 km. By drawing an accurate vector diagram, determine the total displacement of the car.

3. Find the resultant force when two forces act as shown in Figure 1.23.

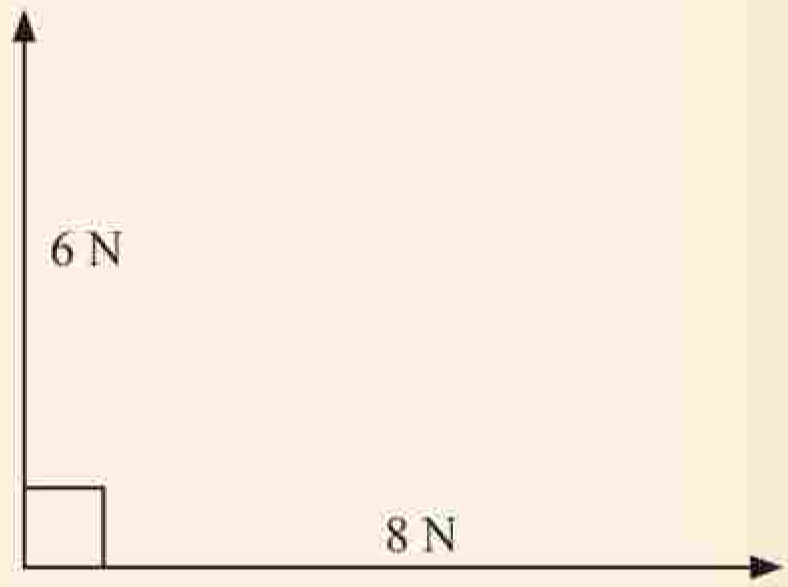


Figure 1.23

4. Two forces of magnitudes 40 N and 60 N are pulling a box on a horizontal table. If the two forces make an angle of 60° between them, find the resultant force on the box using the Parallelogram law.

5. Two cables are used to support a pole as shown in Figure 1.24.

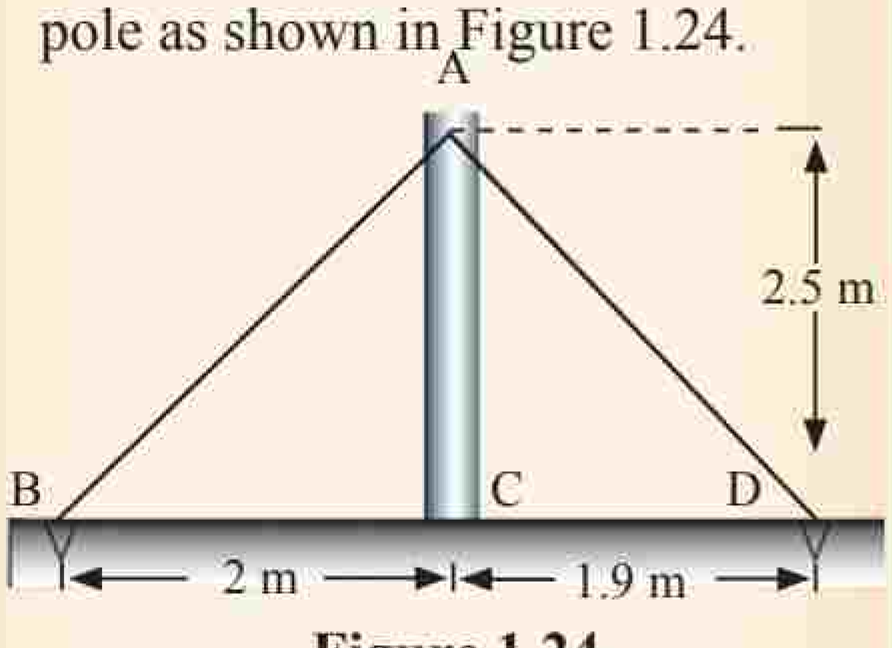


Figure 1.24

If the tension in cables AB and AD is 500 N and 160 N respectively, determine the magnitude and direction of the resultant force exerted by cables AB and AD at point A using the Parallelogram law.

6. Find the resultant force, **F**, when two forces, 9 N and 15 N, act on an object with an angle of 60° between them.
7. Find the resultant force when two forces, 8 N and 16 N, form the following angles:
   (a) 45°
   (b) 60°
   (c) 120°

Relative motion

The same physical principles that apply when you are at rest on the earth, apply also when you are in any reference frame travelling at a constant velocity with respect to the earth. For example, if the motion of a bus follows a straight line at a constant speed, you can toss and catch a coin on such a moving bus. The motion of the coin may differ when viewed by an observer standing on the roadside. The difference in motion of the coin may be explained by including the reference frame in the description of motion.

Relative motion is the movement of an object as seen by an observer on another frame of reference. A passenger in a moving car will see the trees on the roadside moving in a direction opposite to that of the car. However, the same trees appear to be stationary when observed by a person standing at the roadside. Both observers are correct because the motion of an object under observation depends on the frame of reference of the observer relative to the frame of reference of the observed object. A frame of reference is a set of coordinates used to calculate the position and velocity of an object.

Relative velocity

We come into situations when one or more objects move in a non-stationary frame in relation to an observer in a different frame of reference. A boat, for example, may cross a river that is flowing at a certain velocity with respect to an observer on the ground. Similarly, while flying, an airplane may encounter wind which has a certain velocity with respect to the observer on the ground. In both examples, the magnitude of the velocity of a moving object according to an observer on the ground is not the same as that observed by the observer in the object. For example, an observer in the boat may perceive the speed of the boat to be 20 m/s, yet, an observer on the ground sees the same boat travelling at a speed of 25 m/s.

Generally, an observer on a stationary frame of reference observes a speed different from the one observed by an observer in a moving frame of reference. Hence, to describe the motion of an object in a moving medium, one must take into account the effect of the motion of medium on the motion of the object. This effect accounts for the differences in the velocity of the object as observed by the two observers. This means, motion is relative to the observer and therefore, the velocity of the boat must always be described relative to the observer. The velocity of an object that is considered relative to an observer in a different frame of reference is termed as relative velocity.

Relative velocity is calculated by considering the object's velocity as well as the velocity of the medium. For example, airplanes usually encounter winds moving with respect to an observer on the ground. The wind may approach the plane from the front (headwind), behind (tailwind), or sideways (sidewind) as shown in Figure 1.25. A tailwind increases the speed of the plane while a headwind reduces it.

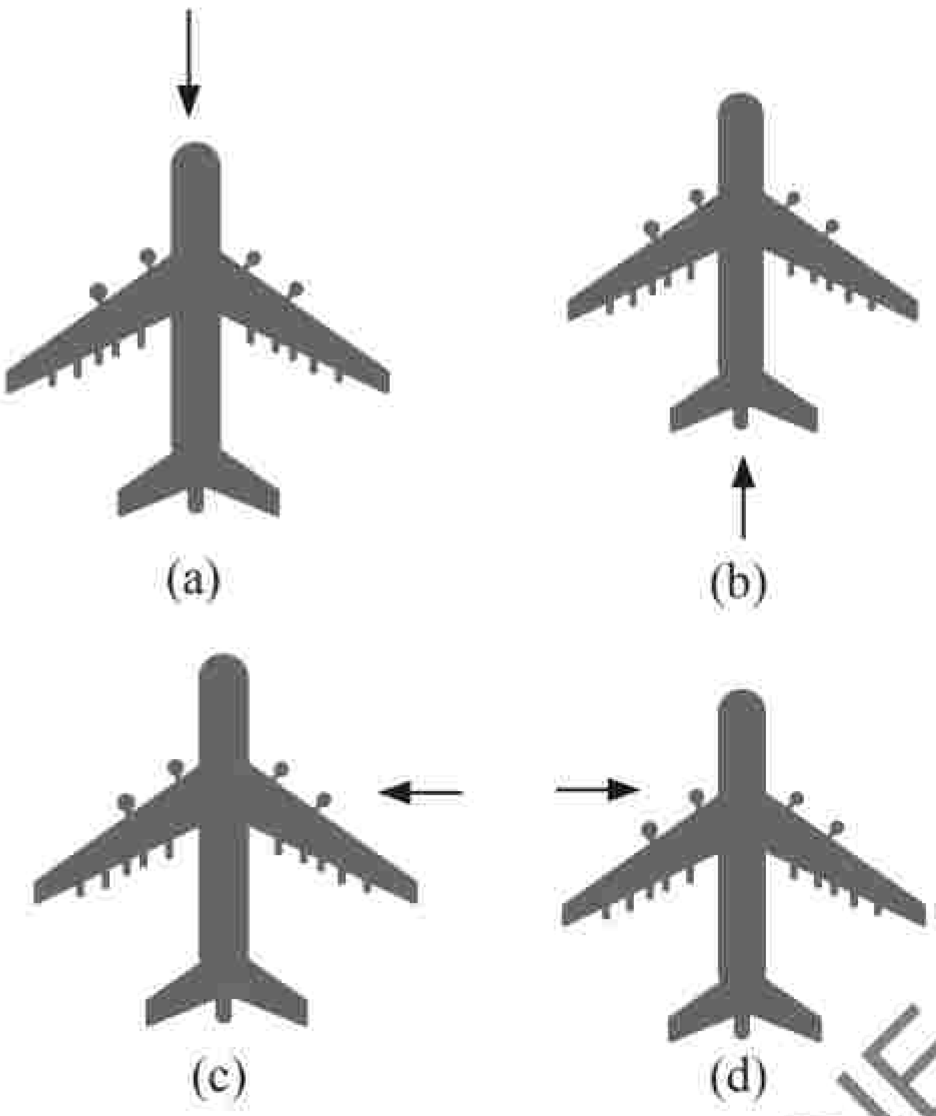


Figure 1.25: *Relative velocity of airplane with respect to wind*

On the other hand, a boat in a river moves along a water current which is moving with respect to an observer on the land. Considering a boat shown in Figure 1.26, the observer on the ground may observe that the speed of the boat is increased if it moves along the water current. Conversely, the speed will decrease if the boat moves against the direction of the water current.

Figure 1.26: *A boat moving amidst a water current*

Example 1.6

If a plane is travelling along with a tailwind at a speed of 100 km/h and the wind speed is 25 km/h, what is the speed of the plane relative to an observer on the ground?

Solution

The resultant speed of the plane is the vector sum of the speed of the plane and the speed of the wind. The resultant of these speeds is illustrated in Figure 1.27.

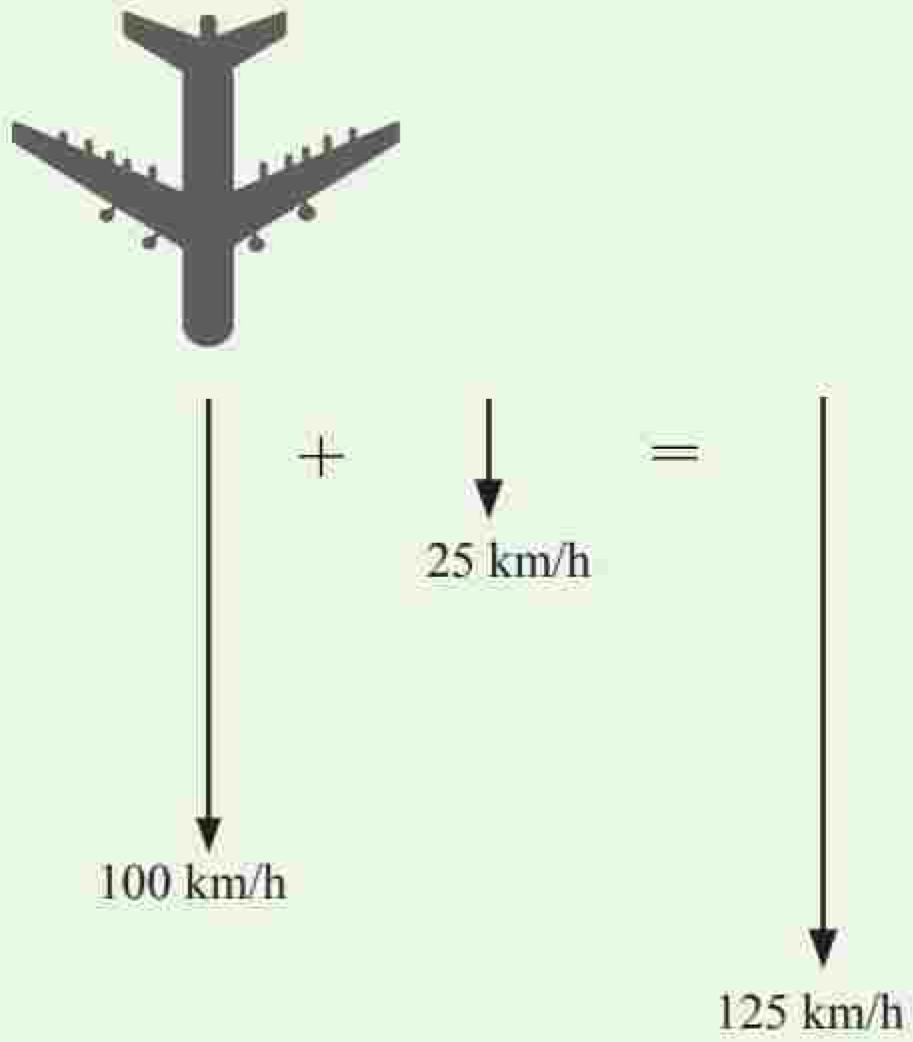


Figure 1.27

The plane travels with a resultant speed of 125 km/h relative to the observer on the ground.

Suppose the same plane is now travelling at 100 km/h and encounters a wind blowing directly against it with a speed of 25 km/h. What will its speed be relative to an observer on the ground?

The resultant speed is the vector sum of the two speeds as shown in Figure 1.28.

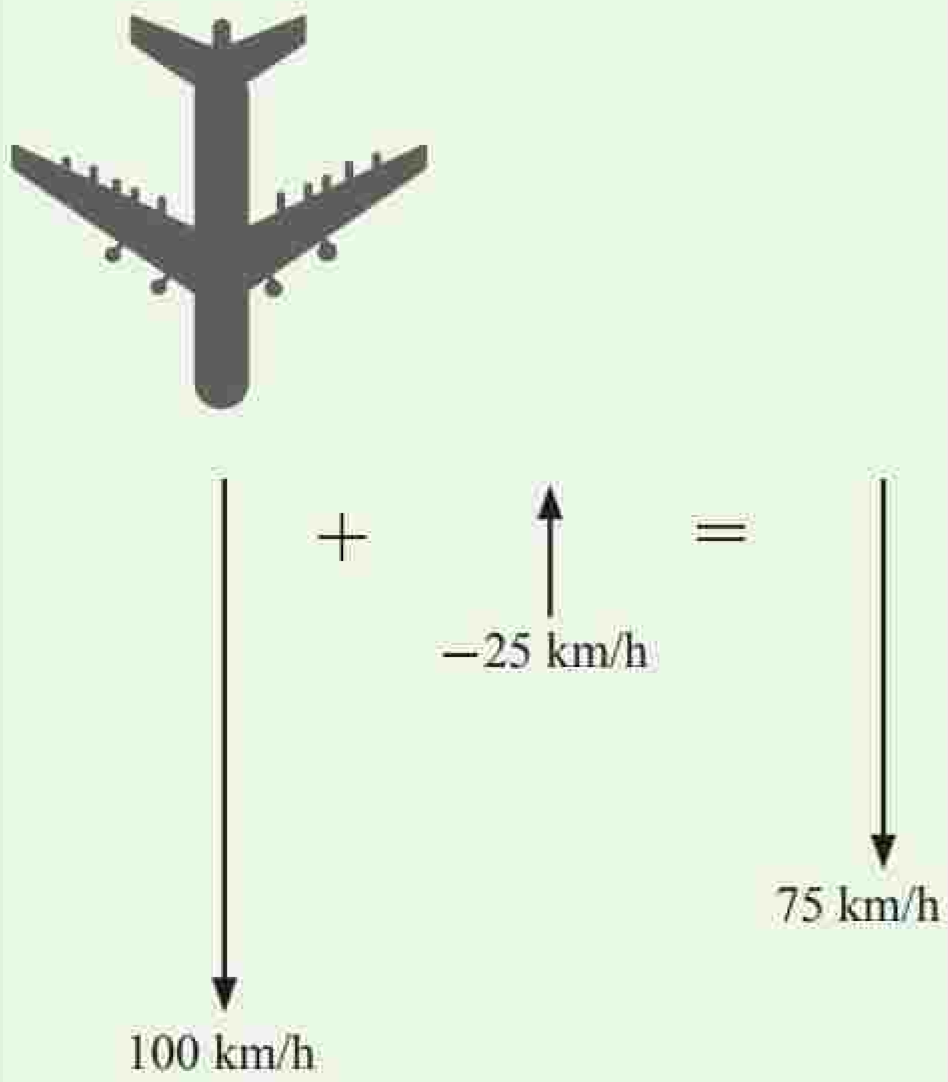


Figure 1.28

In this case, the resultant speed will be 75 km/h. This is the speed of the plane relative to an observer on the ground.

Example 1.7

A plane travelling at a velocity of 100 km/h to the South encounters a side wind blowing at 25 km/h to the West. What is the velocity of the plane as observed by an observer on the ground?

Solution

The resultant velocity of the plane is the resultant vector of the two individual velocities. This is illustrated in Figure 1.29.

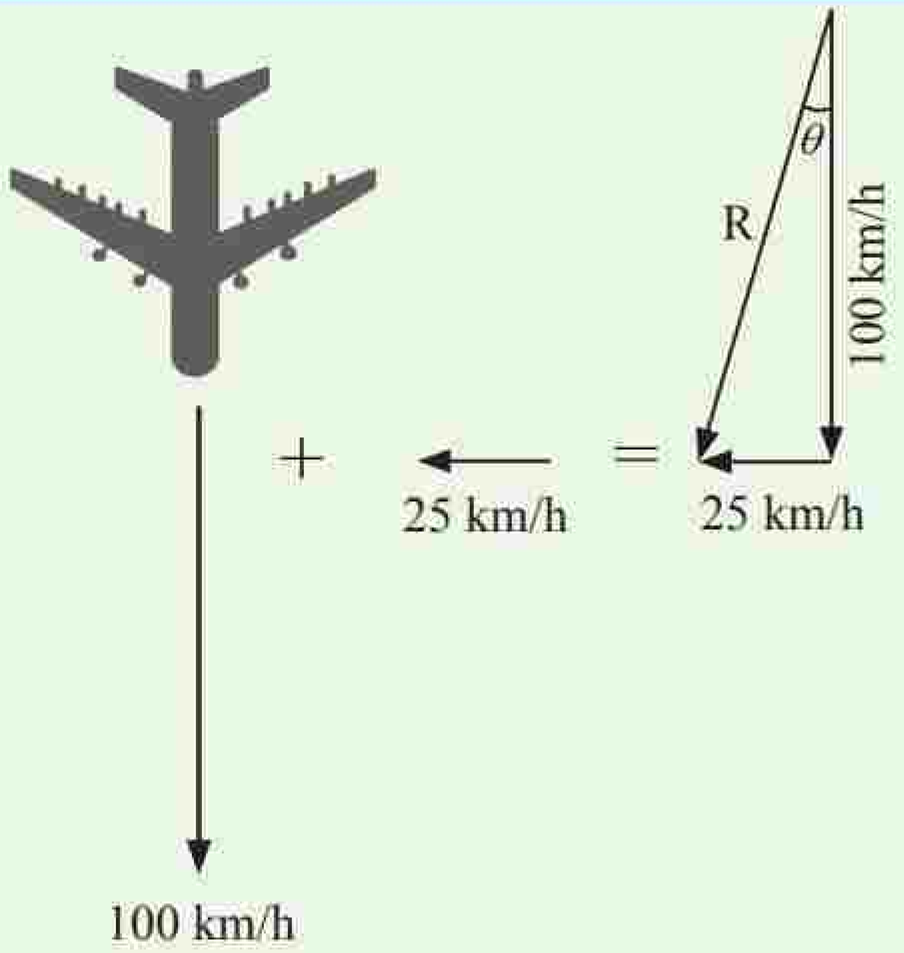


Figure 1.29

Using Pythagoras theorem:

$$R^2 = (100\text{ km/h})^2 + (25\text{ km/h})^2$$

$$R = \sqrt{10625\text{ km}^2/\text{h}^2}$$

$$R = 103.1\text{ km/h}$$

This is the magnitude of the resultant velocity.

The direction of the resulting velocity can be determined using a trigonometrical function (Figure 1.30).

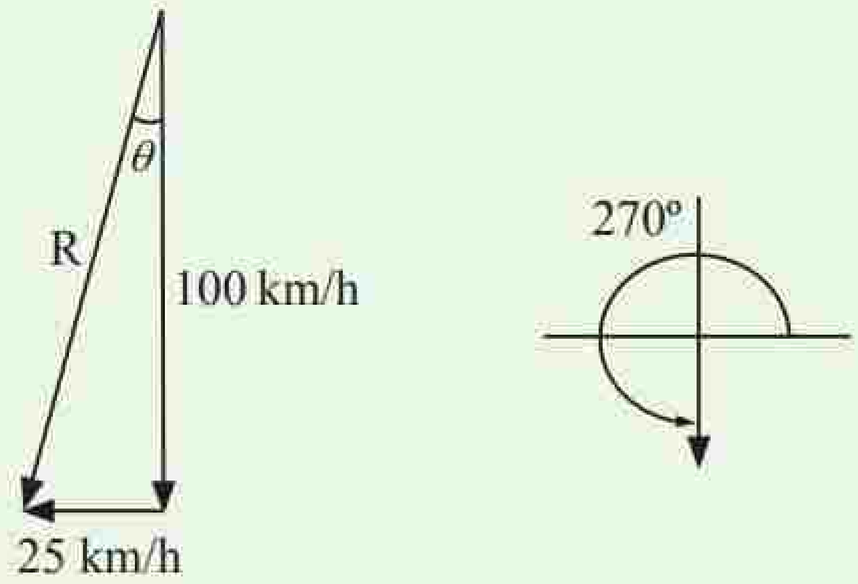


Figure 1.30

It follows that,

$$\tan\theta = \frac{25\text{ km/h}}{100\text{ km/h}} = 0.25$$

$$\theta = \tan^{-1}(0.25) = 14^\circ$$

If the resultant velocity of the plane makes an angle of 14° southward, then the direction of the resultant velocity is 270° − 14° = 256°. This is because the resultant vector's direction is measured as an anticlockwise angle of rotation from due East.

Relative velocity is also a measurement of velocity between two moving objects.

Consider two objects, A and B, moving in the same direction with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively. The velocity of object A relative to B is the difference of the two velocities given as $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$; where $\mathbf{v}_{AB}$ means the velocity of A relative to B. The velocity of B relative to A is therefore,

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

If the objects are moving in opposite directions, then the second velocity will take on a negative value to signify opposite directions, that is,

$$\mathbf{v}_{AB} = \mathbf{v}_A - (-\mathbf{v}_B) = \mathbf{v}_A + \mathbf{v}_B \text{ or}$$

$$\mathbf{v}_{BA} = \mathbf{v}_B - (-\mathbf{v}_A) = \mathbf{v}_B + \mathbf{v}_A$$

Therefore, if the two objects are moving in opposite directions, $\mathbf{v}_{AB} = \mathbf{v}_{BA} = \mathbf{v}_A + \mathbf{v}_B$.

Example 1.8

Car A is moving with a speed of 20 m/s while car B is moving with a speed of 30 m/s. Calculate the speed of car B relative to car A if the cars are moving in, (a) the same direction (b) the opposite directions.

Solution

(a) $\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$

$= 30 \text{ m/s} - 20 \text{ m/s}$

$= 10 \text{ m/s}$

Therefore, the velocity of car B relative to car A, if they are moving in the same direction, is 10 m/s.

(b) $\mathbf{v}_{BA} = \mathbf{v}_B - (-\mathbf{v}_A)$

$= 30 \text{ m/s} - (-20 \text{ m/s})$

$= 30 \text{ m/s} + 20 \text{ m/s}$

$= 50 \text{ m/s}$

Therefore, the speed of car B relative to car A, if they are moving in the opposite direction, is 50 m/s.

Applications of relative motion

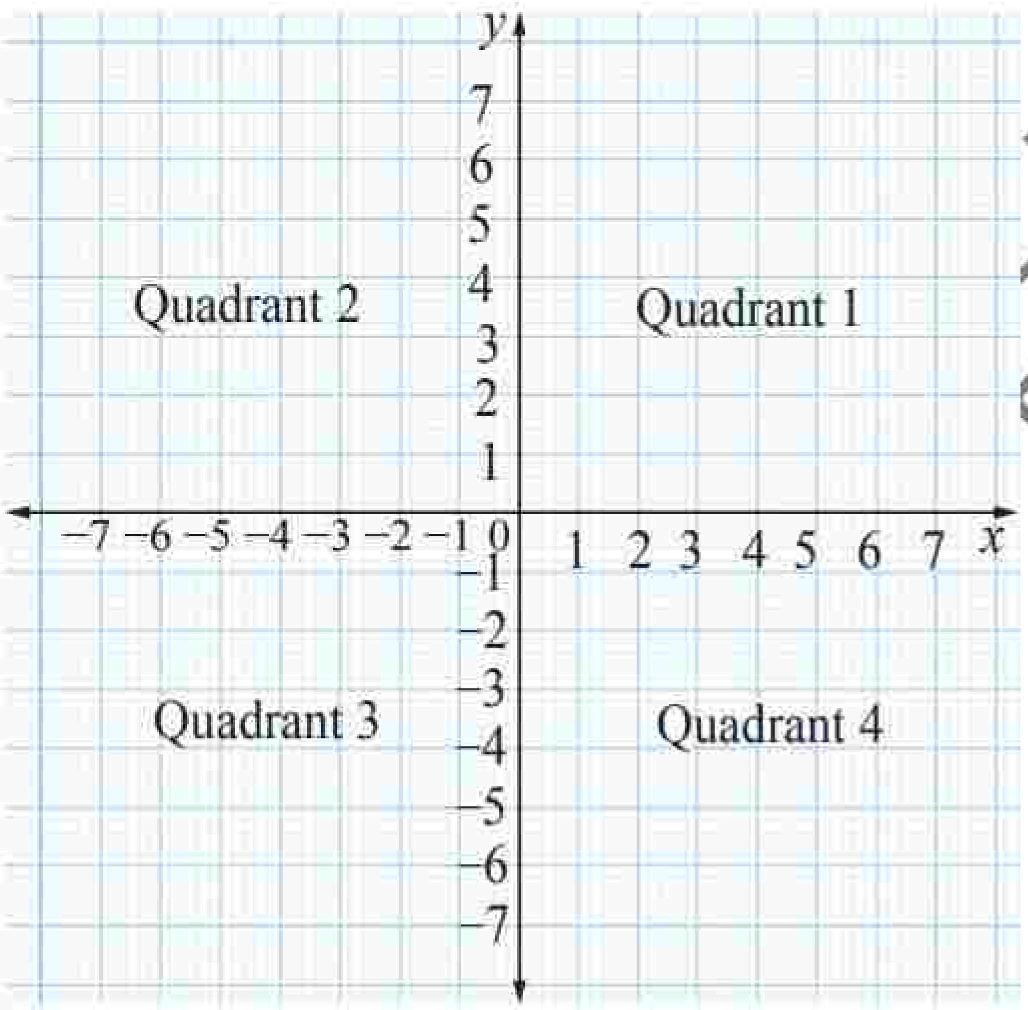
Relative motion is used in navigation to determine the actual velocities of vessels in moving water or air.

Vectors on the Cartesian coordinate system

The direction of a vector can be indicated in several ways. In situations where the vectors are one-dimensional, that is, always horizontal or vertical, the direction can be defined using the positive (+) or negative (−) sign. Normally for horizontal vectors, a (+) sign is assigned to the right and (−) sign to the left motion. For vertical vectors, the (+) sign is assigned for upwards motion and (−) sign for downward motion. Care must be taken not to confuse the (+) and (−) signs indicating the direction of a vector with the signs representing addition and subtraction or giving the magnitude of a number.

For two-dimensional vectors, there are several ways of defining direction. One of these involves representing the vectors on a Cartesian coordinate system. A Cartesian coordinate system consists of two perpendicular number lines called the axes. The point where the axes intersect is called the origin. It is usually represented by zero for each axis. The horizontal axis is usually referred to as the *x*-axis and the vertical axis is the *y*-axis.

In practice, the axes represent measurements. The two axes form four regions called quadrants as shown in Figure 1.31 (a). These regions are numbered in an anticlockwise direction beginning with the upper right quadrant. The position of a point on the coordinate system can be defined by its distance above or below the x-axis and its distance from the y-axis. The position (coordinates) of a point is written in brackets separated by a comma. The *x*-coordinate is written first, followed by the y-coordinate, i.e., (*x*, *y*) as shown in Figure 1.31 (b).



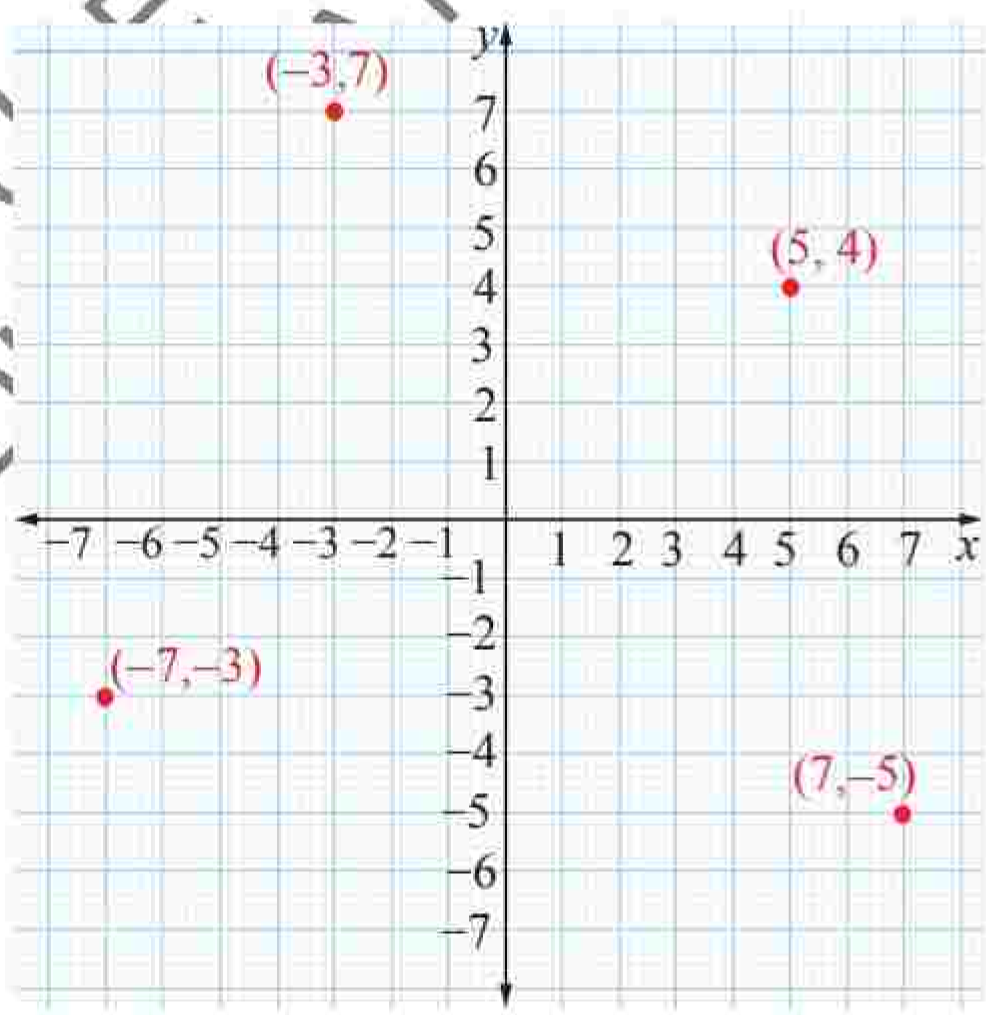


Figure 1.31: *The Cartesian coordinate system*

From Figure 1.31(b) it can be seen that:

(a) In quadrant 1, both the *x*-coordinate and *y*-coordinate are positive.
(b) In quadrant 2, the *x*-coordinate is negative while the *y*-coordinate is positive.
(c) In quadrant 3, the *x*-coordinate and *y*-coordinate are both negative.
(d) In quadrant 4, the *x*-coordinate is positive while the *y*-coordinate is negative.

To represent a vector on a Cartesian coordinate system, draw its tail at the origin. The coordinates of the head of the arrow define the vector (Figure 1.32).

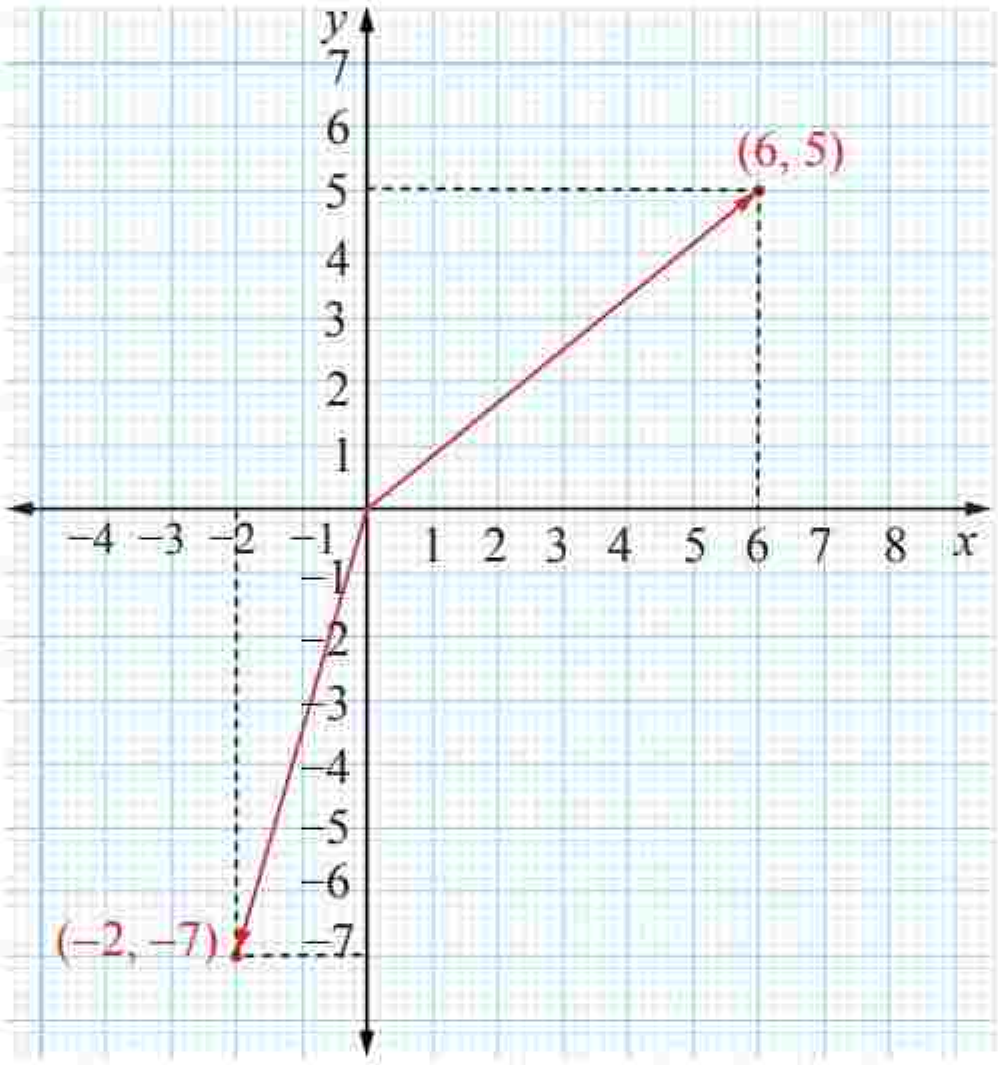


Figure 1.32: *Representation of vectors on a Cartesian coordinate system*

Exercise 1.2

1. A man is walking inside a bus, which is travelling at 56.2 km/h. If the speed of the man relative to the ground is 55.8 km/h, is he walking towards the front or the back? Explain.
2. A boat is moving at 8.9 km/h relative to the water in a river. Initially the boat at the bank aims straight to the opposite bank of the river which is 120.9 m wide. If the speed of the water in the river is 2.9 km/h, how far downstream will the boat be when it reaches the opposite bank?
3. A boat moving at a speed of 9.8 km/h relative to the water in a river wants to get to a fishing camp that is 5.2 km upstream. If the speed of the water in the river is 6.0 km/h, how long will it take the boat to reach the camp?
4. A plane is flying at a velocity of 300 km/h relative to the ground in the direction 300° from East. The plane is flying amidst a wind blowing at 85 km/h relative to the ground in the direction 225° from East. What is the velocity of the plane relative to the air?
5. Two cars, A and B, are moving at speeds of 60 km/h and 80 km/h, respectively. What is the speed of car A relative to car B if the two cars are moving in:
   (a) the same direction?
   (b) the opposite directions?
6. Two motorcars, A and B are approaching each other on a highway (Figure 1.33). At time t = 0, car A and B are 975 m apart at point P and Q, with their speeds being $u_A = 30$ m/s and $u_B = 17$ m/s, respectively. Car A passes point Q after 40 seconds and car B passes point P after 42 seconds.

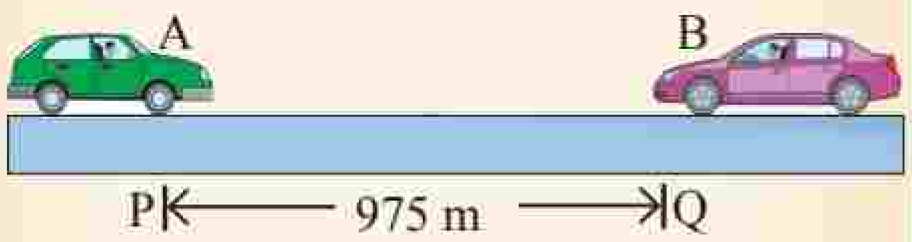


Figure 1.33

   (a) Find their accelerations.
   (b) After how long will the cars pass each other.
   (c) What is the speed of car B relative to car A when they pass each other?

Resolution of vectors

Suppose a ball has been released from its rest position on a perfectly smooth inclined plane in Figure 1.34 (a), what happens to the ball after being released? The ball will roll down the plane, accelerating as it does so. According to Newton's laws of motion, the acceleration of the ball must be caused by a force pointing down the plane. However, it is understood that the only force acting on the ball as it rolls down is its weight, **W** (the force that arises from gravity acting on the mass of the ball and acts vertically downwards). Thus, one might question: where does the force that causes the acceleration come from?

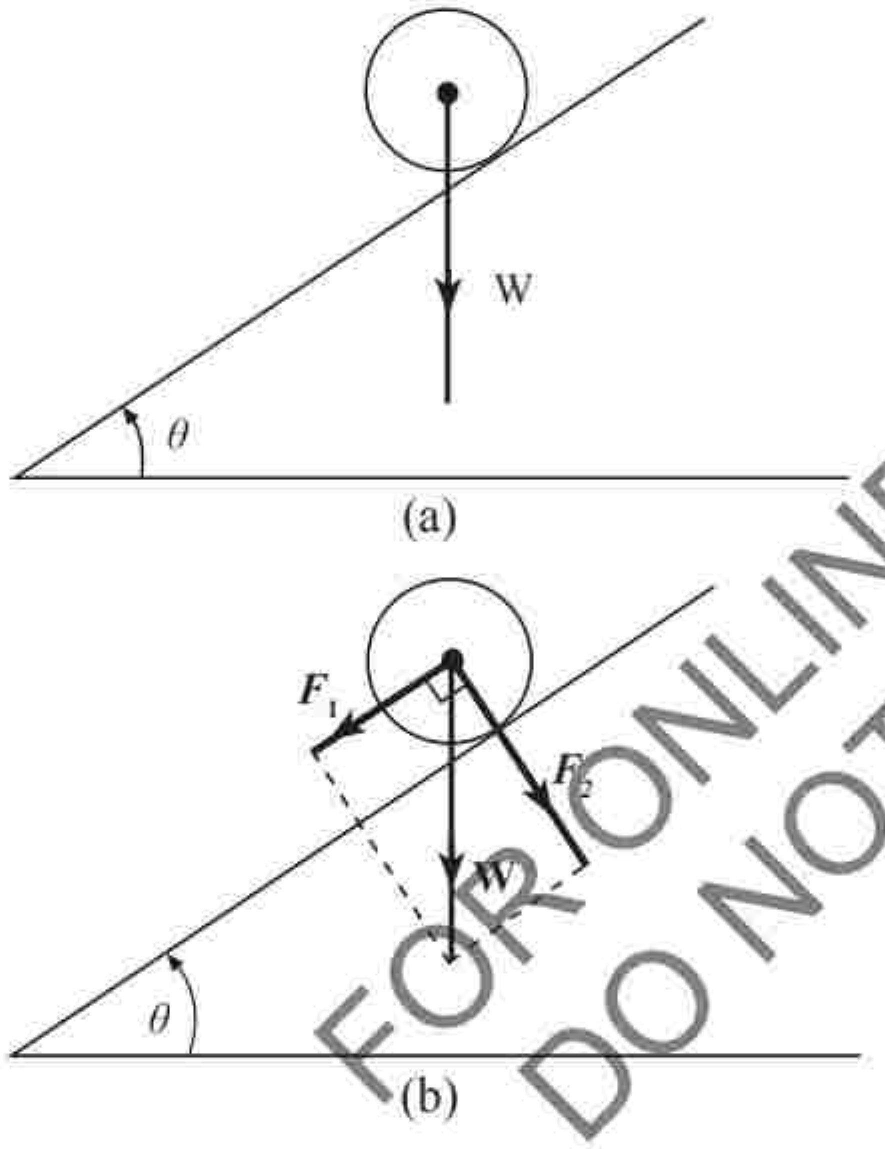


Figure 1.34: *A ball on a smooth inclined plane*

Vector addition gives a simple solution. As illustrated in Figure 1.34 (b), the weight $\mathbf{W}$ of the ball is equal to the sum of two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, expressed as $\mathbf{W} = \mathbf{F_1} + \mathbf{F_2}$. The acceleration is therefore caused by force, $\mathbf{F_1}$, which is parallel to the plane, whereas force $\mathbf{F_2}$, which is normal (i.e., at right angles) to the plane, prevents the ball from leaving the plane and accounts for the difference between $\mathbf{W}$ and $\mathbf{F_1}$.

Components of a vector

In the previous sections, we have dealt with vectors that are directed eastward, westward, northward or southward. For example, when an object had an acceleration, we described its direction in one dimension. Now, in situations where vectors are directed at certain angles to the customary axes of the *x-y* plane, we make use of some mathematical procedure to transform the vector into two parts with each part being directed along the coordinate axis. For example, a vector that is directed northwest can be thought of as having two parts: a northward part and a westward part. If a force pulls an object at an angle of 60^o to the horizontal, the motion of the object will have two parts: an upward motion and the rightward motion.

Generally, any two-dimensional vectors can be considered to have an influence in two different directions. This means, such a vector has two parts and each part is known as a component of a vector. The components of a vector represent the influence of that vector in a given direction. Thus, the combined influence of the two components is equivalent to the influence of the single two-dimensional vector. That is to say, a single two-dimensional vector may be replaced by its two components.

As an example of a two-dimensional vector and its components, consider an airplane in Figure 1.35, which is to fly from Julius Nyerere International Airport, Dar es Salaam to Entebbe in Uganda.

Figure 1.35: *An airplane*

Suppose the plane is to fly in such a way that its resulting displacement vector will be some units towards the northwest. This displacement of the plane has two components: a component in the northward direction and a component in the westward direction as shown in Figure 1.36 (a). The plane would have the same displacement if it was to take the trip to Entebbe in two segments whereby, one segment is directed due north and the other-directed due west as shown in Figure 1.36 (a). The combined influence of the two components would be equivalent to the influence of the two-dimensional displacement in the northwest direction as shown in Figure 1.36 (b).

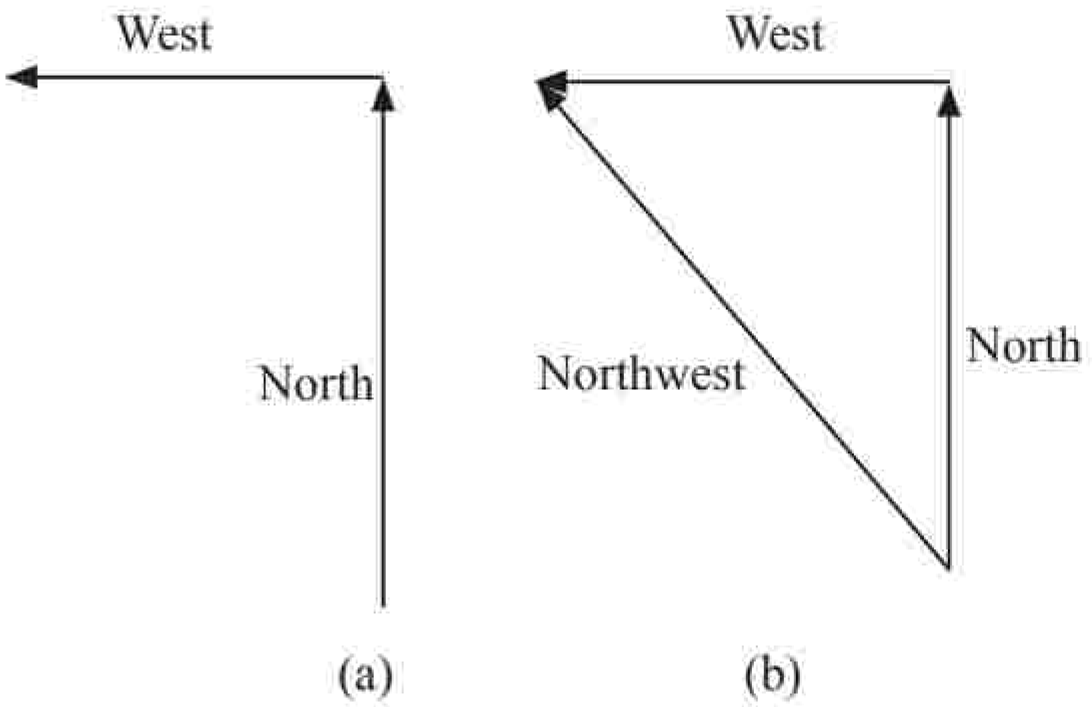


Figure 1.36: *Components of a displacement vector*

Magnitudes of vector components

When a vector is directed at a certain angle to the vertical or horizontal direction, its components are mutually perpendicular to each other. One component is on the *x*-axis (horizontal) and the other on the *y*-axis (vertical) as shown in Figure 1.37. Finding vector components involves the use of trigonometrical functions to relate magnitude of the vector components to the magnitude and direction of a given vector in a two-dimensional coordinate system. This process is known as vector resolution. In other words, vector resolution is the process of splitting a given vector into its constituent parts at right angles to each other.

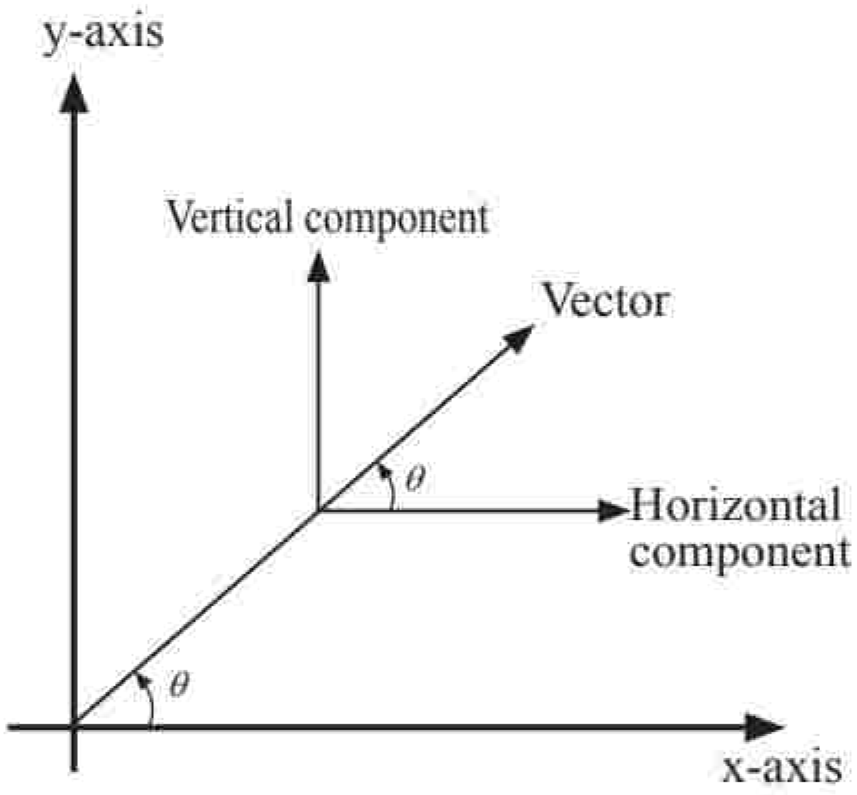


Figure 1.37: *Vertical and horizontal vector components*

In our airplane example, the displacement to the North and West are perpendicular to each other.

Therefore, a northwest displacement of the plane can be resolved into two mutually perpendicular directions as shown in Figure 1.38.

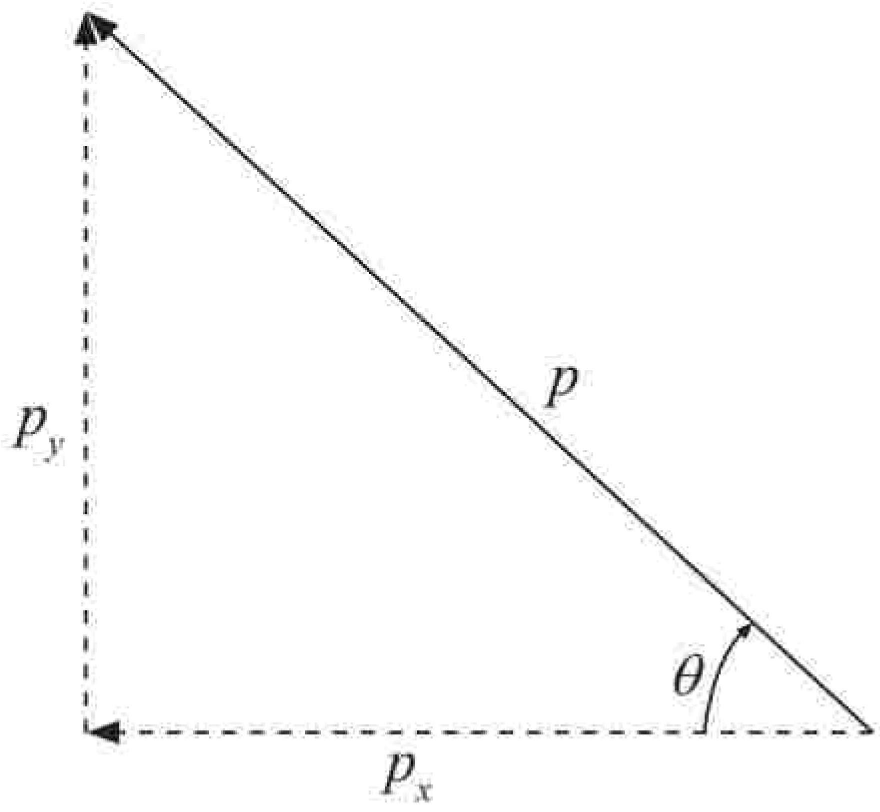


Figure 1.38: *Components of vector* ***p***

p_x and p_y are the components of a vector p. These components form an angle of 90° with each other and their magnitudes can be determined using trigonometric ratios.

Considering the vertical component (y-component);

$$\sin\theta = \frac{p_y}{p}$$

Therefore,

$$p_y = p\sin\theta$$

Similarly, considering the horizontal component (x-component);

$$\cos\theta = \frac{p_x}{p}$$

Hence,

$$p_x = p\cos\theta$$

Therefore, $p = (p\cos\theta, p\sin\theta)$.

Note that, θ is the angle that $\boldsymbol{p}$ makes with the direction of the x-axis and p is the magnitude of the vector $\boldsymbol{p}$.

Example 1.9

A nail drilled in the wall is being pulled out using a string (Figure 1.39). The string forms an angle of 30° with the normal. If the pulling force is 10 N, part of the force will tend to bend the nail while the other part will pull it out from the wall. Determine the magnitudes of these two components of the force.

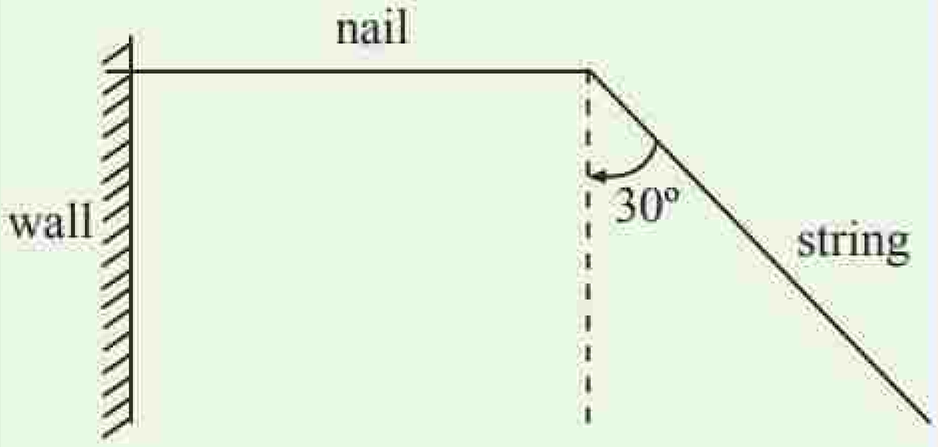


Figure 1.39

Solution

This information can be presented in vector form as shown by Figure 1.40.

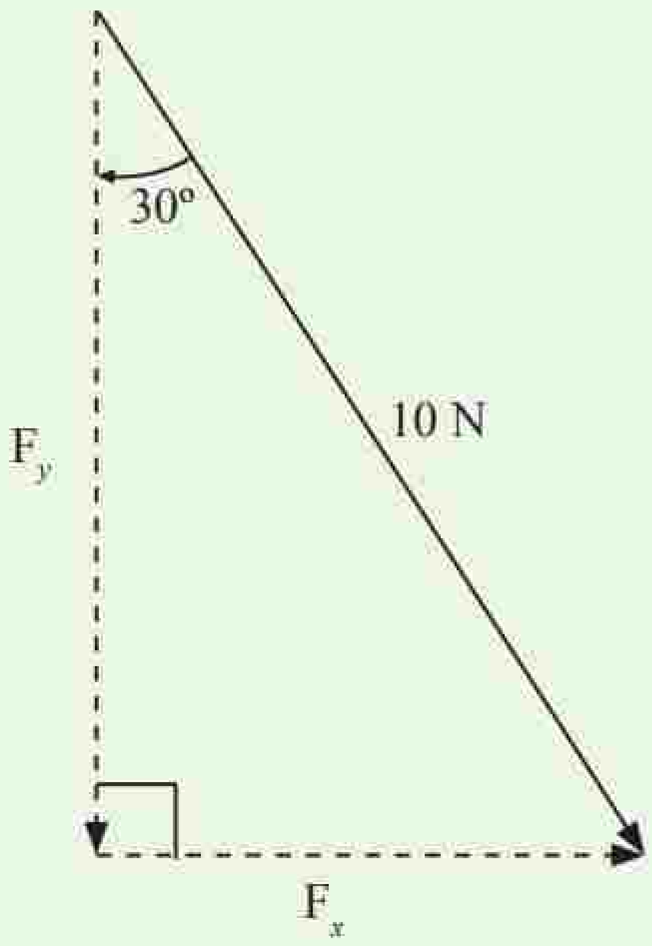


Figure 1.40

Forces F_y and F_x are the components of a 10 N force. F_y tends to bend the nail while F_x tends to pull it out of the wall. Thus,

(a) $\frac{F_y}{10\ \text{N}} = \cos 30°$

$$F_y = \cos 30° \times 10\ \text{N}$$
$$= 0.866 \times 10\ \text{N}$$
$$= 8.66\ \text{N}$$

(b) $F_x = \sin 30° \times 10\ \text{N}$

$$= 0.5 \times 10\ \text{N}$$
$$= 5\ \text{N}$$

Therefore, the pulling force F_x is 5 N while the bending force F_y is 8.66 N. The nail is more likely to bend rather than come out.

Example 1.10

A body is being subjected to two forces: $\mathbf{F_1} = 18\,\text{N}$ acting at an angle of 25° and $\mathbf{F_2} = 30\,\text{N}$ acting at 140° from the East direction. Find the resultant force, **F** of the two forces by separating the forces into x and y components.

Solution

First, resolve the forces into their x and y components, as illustrated by Figures 1.41 and 1.42.

The components for $\mathbf{F_1}$.

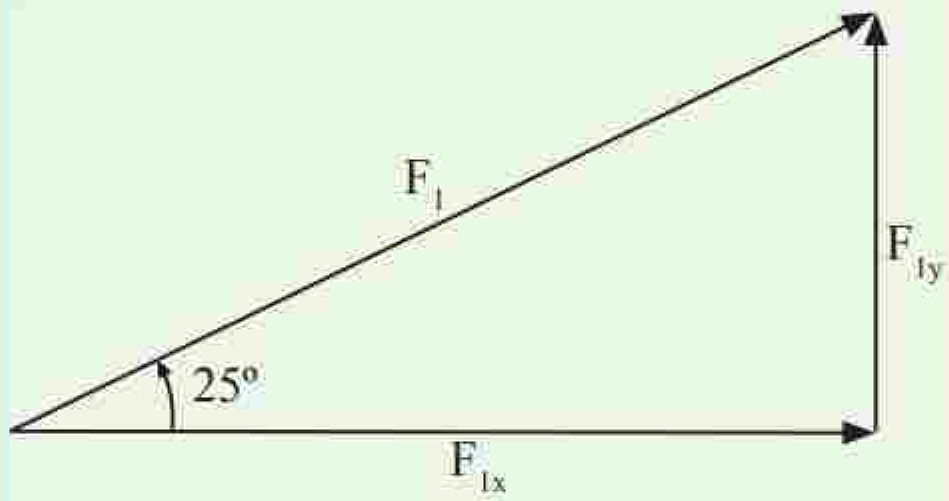


Figure 1.41

$$\sin 25° = \frac{F_{1y}}{\boldsymbol{F_1}}$$
$$F_{1y} = \boldsymbol{F_1} \sin 25°$$
$$= \sin 25° \times 18\ \text{N}$$
$$= 0.4226 \times 18\ \text{N}$$
$$= 7.6\ \text{N}$$

$$\cos 25° = \frac{F_{1x}}{\boldsymbol{F_1}}$$
$$F_{1x} = \boldsymbol{F_1} \cos 25°$$
$$= \cos 25° \times 18\ \text{N}$$
$$= 0.9063 \times 18\ \text{N}$$
$$= 16.31\ \text{N}$$

The components for $\mathbf{F_2}$.

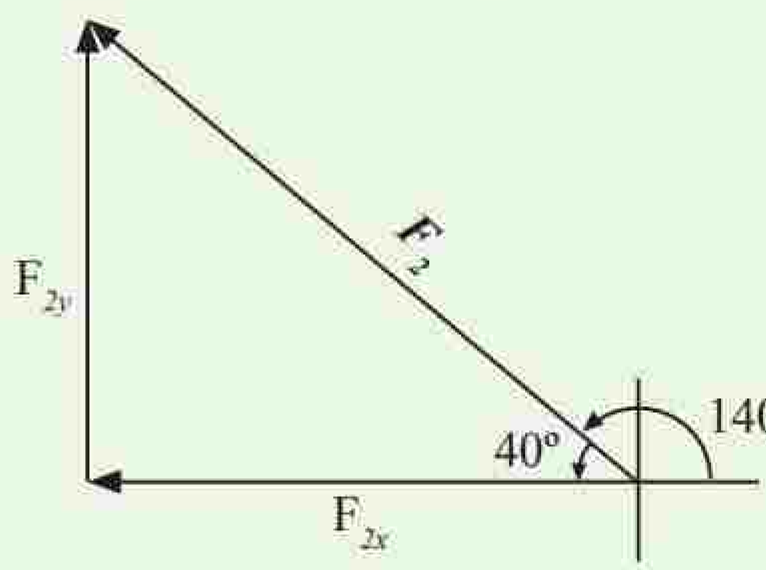


Figure 1.42

$$\cos 40° = \frac{F_{2x}}{\boldsymbol{F_2}}$$
$$F_{2x} = \boldsymbol{F_2} \cos 40°$$

$$= \cos 40° \times 30 \text{ N}$$

$$= 0.7660 \times 30 \text{ N}$$

$$= 22.98 \text{ N}$$

$$\sin 40° = \frac{F_{2y}}{\boldsymbol{F_2}}$$

$$F_{2y} = \boldsymbol{F_2} \sin 40°$$

$$= \sin 40° \times 30 \text{ N}$$

$$= 0.642 \times 830 \text{ N}$$

$$= 19.28 \text{ N}$$

Since the component F_{2x} points to the left, it is negative. Therefore,

$$F_{2x} = -22.98 \text{ N}.$$

In order to get the resultant vector ***F***, calculate the sum of components on the *x*-axis and *y*-axis respectively.

Total component on the *x*-axis:

$$F_x = F_{1x} + F_{2x}$$

$$= 16.31 \text{ N} + (-22.98 \text{ N})$$

$$= 16.31 \text{ N} - 22.98 \text{ N}$$

$$= -6.67 \text{ N}$$

Therefore, $F_x = -6.67$ N.

Total component on the *y*-axis

$$F_y = F_{1y} + F_{2y}$$

$$= 7.6 \text{ N} + 19.28 \text{ N}$$

$$= 26.88 \text{ N}$$

Therefore, $F_y = 26.88$ N.

Applying the Pythagoras theorem, the resultant force is given by:

$$F^2 = F_x^{\ 2} + F_y^{\ 2}$$

$$F^2 = (-6.67)^2 \text{ N}^2 + 26.88^2 \text{ N}^2$$

$$F = \sqrt{44.49 \text{ N}^2 + 722.53 \text{ N}^2}$$

$$= 27.70 \text{ N}$$

If this force was to be presented graphically, it would appear as shown in Figure 1.43.

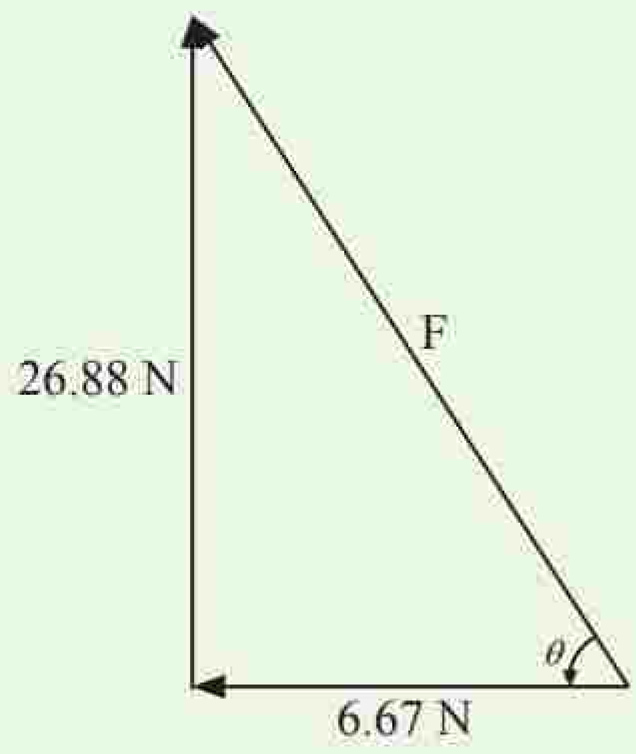


Figure 1.43

To get the angle θ, use the tangent function:

$$\tan\theta = \frac{26.88 \text{ N}}{6.67 \text{ N}}$$

$$\theta = \tan^{-1}(4.03)$$

$$\theta = 76°$$

Therefore, $\theta = 76°$.

The angle θ of the resultant force from East:

$$\theta = 180° - 76°$$

$$= 104°$$

Therefore, the resultant force is 27.70 N at an angle of 104° from East.

Application of vector resolution in solving problems

Vector resolution can be used to solve many problems involving physical quantities such as force, velocity, work and many others. Sometimes one may be required to add up to 10 vectors. To perform the operation using the graphical method described earlier, one adds the first two vectors' head-to-tail, and then adds the third vector to the sum of the first and second vector, then the fourth to the sum and so on. This may be difficult to obtain the vector sum. An easier way of adding vectors is to separate the vectors into their x and y-components, then sum up the respective components.

Example 1.11

An airplane is flying east at 200 km/h, its velocity relative to the air, while a 100 km/h wind is blowing 45° northeast. What is the velocity of the airplane according to an observer on the ground?

Solution

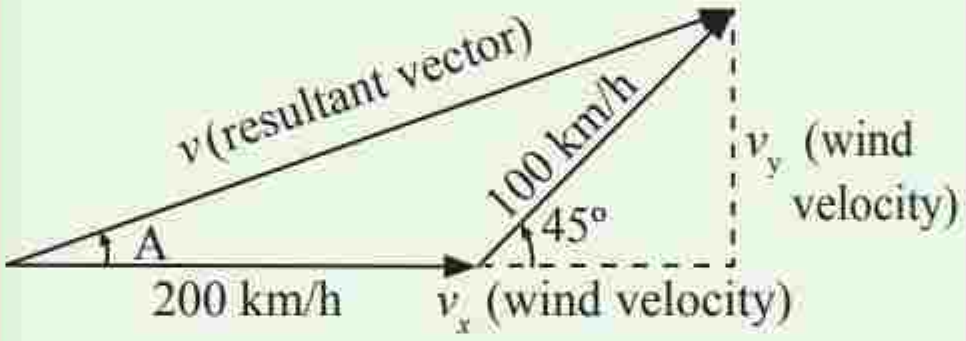


Figure 1.44

The x and y-components of the airplane velocity are 200 km/h and 0 km/h, respectively.

We can use trigonometrical ratios to get the x and y-components of the wind velocity:

$$\cos 45° = \frac{v_x}{100 \text{ km/h}}$$

$$v_x = 100 \text{ km/h} \times \cos 45°$$

$$= 70.71 \text{ km/h}$$

$$\sin 45° = \frac{v_y}{100 \text{ km/h}}$$

$$v_y = 100 \text{ km/h} \times \sin 45°$$

$$v_y = 70.71 \text{ km/h}$$

Thus, the resultant velocity components (v_x, v_y) are given by,

$$v_x = 200 \text{ km/h} + 70.71 \text{ km/h}$$

$$= 270.71 \text{ km/h}$$

$$v_y = 0 + 70.71 \text{ km/h} = 70.71 \text{ km/h}$$

Using the Pythagoras theorem:

$$v^2 = {v_x}^2 + {v_y}^2$$

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

$$= \sqrt{270.71^2 \text{ km}^2/\text{h}^2 + 70.71^2 \text{ km}^2/\text{h}^2}$$

$$= 280 \text{ km/h}$$

The angle A between the resultant velocity and the eastward directed velocity is given by:

$$\sin \text{A} = \frac{70.71 \text{ km/h}}{280 \text{ km/h}}$$

$$= 0.2525$$

$$\text{A} = \sin^{-1}(0.2525) = 14.6°$$

The resultant velocity is 280 km/h at an angle of 14.6° from the east.

Exercise 1.3

1. Given the displacement vectors:

   A = 15 m at 65°

   B = 22 m at 135°

   C = 8 m at 180°

   D = 12 m at 300°

   Find the following vectors using the component method:

   (a) **A** + **C** (b) **B** + **D**

2. Given the components of some vectors:

   (a) $V_x = 12$ m, $V_y = 8$ m

   (b) $R_x = -18$ m, $R_y = 12$ m

   (c) $S_x = 20$ m, $S_y = -14$ m

   Draw the vector represented by each component and determine the magnitude and direction of their resultant.

3. An airplane is moving at 200 km/h at an angle of 60°. The plane experiences a wind blowing at 120 km/h at an angle of 210°. Find the resultant velocity of the plane by resolving the velocities into their components.

Chapter summary

1. Scalars are the physical quantities that have magnitude only while vectors are physical quantities that have both magnitude and direction.
2. Vectors can be added graphically using the parallelogram law or triangle law of vector addition.
3. The Triangle law of vector addition states that: "*If two vectors are represented by the two sides of a triangle in sequence, then the third closing side of the triangle drawn from the tail of the first vector to the head of the second vector represents the resultant of the two vectors in both magnitude and direction*".
4. Parallelogram law of vector addition states that: "*If two vectors are represented by two adjacent sides of a parallelogram, then the diagonal of the parallelogram through the common point of the two vectors represents the sum of the two vectors in magnitude and direction*".
5. A resultant vector is a vector whose magnitude and direction represents the net effect of all vectors starting at a given point.
6. Relative motion is the comparison between the motions of an object as perceived by observers in different frames of reference.
7. A frame of reference is a set of coordinates that can be used to determine the position and velocity of objects.
8. Relative velocity is the velocity of an object that is considered relative to an observer in a different frame of reference.
9. A two-dimensional vector is composed of two parts, which are perpendicular to each other. Each part is called a vector component.

10. Resolving vectors refers to the process of finding the components of a given vector. Trigonometric ratios are useful in vector resolution.

Revision exercise 1

1. Choose the correct answer for each of the following questions.
   (a) Two forces, 4 N and 6 N are acting at an angle of 45° to one another. Which of the following is the resultant force?
      (i) 9.2 N, $\theta = 27.5^{o}$
      (ii) 4.2 N, $\theta = 162.5^{o}$
      (iii) 42 N, $\theta = 45^{o}$
      (iv) 0.42 N, $\theta = 135^{o}$
   (b) Figure 1.45 shows a block being pulled along a track. If a force of 20 N is applied in direction A at an angle of 60°, what is the resolved part of the force in direction B?

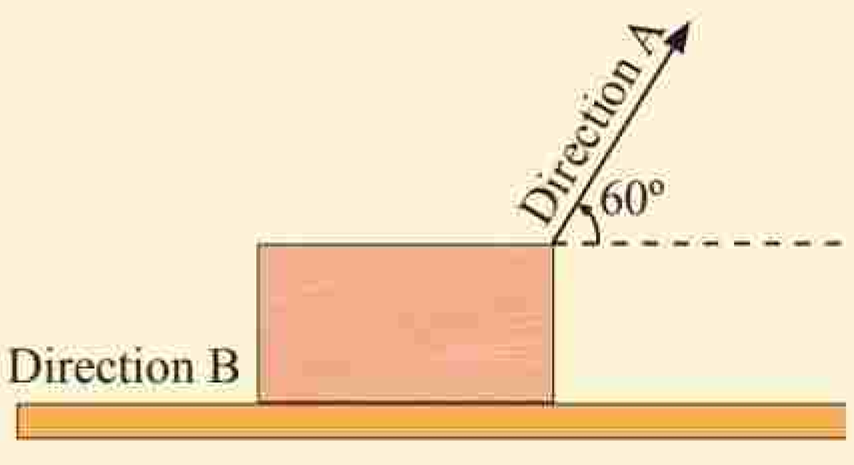


Figure 1.45

      (i) 17 N
      (ii) 20 N
      (iii) 12 N
      (iv) 10 N
2. (a) Give two examples of vector and scalar quantities.
   (b) What is relative velocity?
3. Two forces, 5 N and 12 N are acting on a body. Using the graphical method, find the resultant force of the two forces when the angle between them is:
   (a) 30°
   (b) 45°
   (c) 120°
4. A plane is flying due East with a velocity of 100 m/s when it encounters a wind blowing at a velocity of 20 m/s. Find the resultant velocity of the plane if the direction of the wind is due:
   (a) East
   (b) West
   (c) South
5. A box is being pulled on the floor using a string. The string makes an angle of 30° with the box as shown in Figure 1.46.

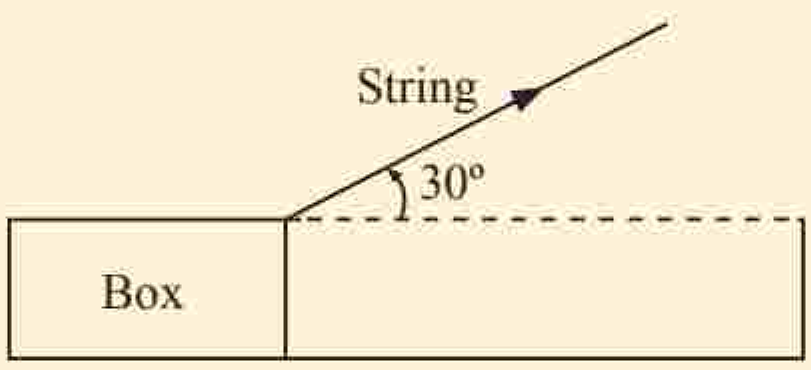


Figure 1.46

   If the force applied at the string is 200 N, find the force which tends to:
   (a) lift the box
   (b) pull the box forward
6. An airplane is taking off at a velocity of 20 m/s. Find the components of the plane's velocity if the take-off angle is,
   (a) 70°
   (b) 45°
   (c) 60°
   (d) 85°

7. The speed of car B relative to car A is 8 m/s when the two cars are moving in the same direction and 28 m/s when the two cars are moving in opposite directions. Determine the speed of each car.

8. A river is flowing at a velocity of 2 m/s due South. A person in a boat wants to move across the river at 10 m/s due East.
   (a) At what angle should the person move?
   (b) At what velocity should the person move the boat?

9. Two forces, **P** and **Q**, are applied on a small boat stuck in a shallow stream as shown in Figure 1.47.

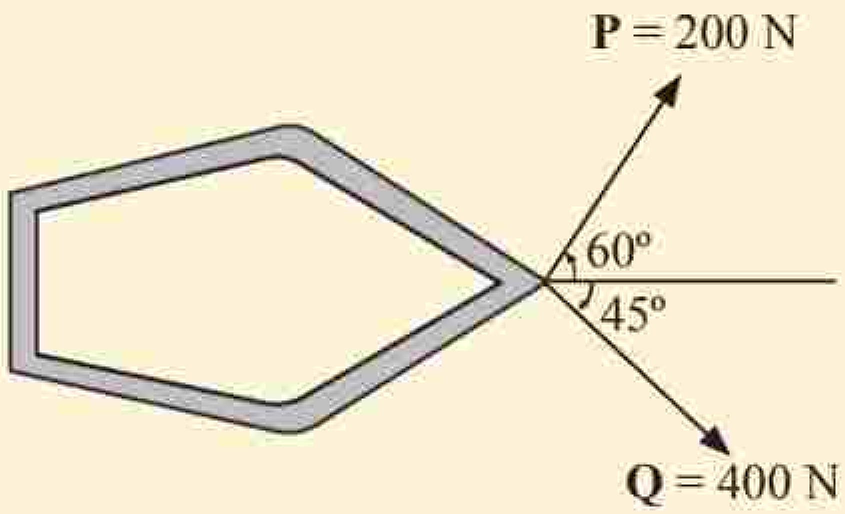


Figure 1.47

Using the Parallelogram law, determine the magnitude and direction of the resultant of the two forces.

10. A mass 3 kg hangs at the end of a string. Find the horizontal force needed to pull the mass sideways until the string is at 30° to the vertical. Find also the tension in the string at the final position.

11. (a) Explain, with examples, the terms relative velocity and resultant velocity.
    (b) Two cars A and B are moving along a straight road in the same direction with speeds of 25 km/h and 40 km/h, respectively. Find the speed of car B relative to car A.

12. How long will a passenger sitting near the window of an SGR train travelling at 360 km/h see a local train passing by in the opposite direction with a speed of 100 km /h? The length of the local train is 315 m.

13. A swimmer's speed in the direction of the flow of a river is 18 km/h. Against the direction of flow of the river the swimmer's speed is 8 km /h. Calculate the swimmer's speed in still water and the speed of the river flow.

Chapter Two

Friction

Introduction

Friction plays an important role in everyday processes. Friction has positive and negative effects depending on the situation. For instance, friction gives shoes and tyres grip on the ground. You can move from one point to another due to friction. On the other hand, the same friction can cause wear and tear to your shoes. In this chapter, you will learn about the concept of friction, types of friction and laws of friction. The competencies developed from this chapter will enable you to use lubricants or ball-bearing between moving parts of machines. You will also be able to apply friction in generating heat by rubbing your hands as well as making fire by rubbing sticks.

Concept of friction

Pulling a block of wood resting on a table or pushing the desk along the floor makes you feel some resistance. That felt resistance is called friction; it is a force in the opposite direction to the applied force. It occurs when a solid object moves over another solid object as well as when an object moves through a fluid (liquid and gas). Friction can also occur within the fluid itself. Friction is caused by the molecular attraction between the parts of two surfaces in contact. The surfaces are microscopically irregular. When two objects are in contact, particles (atoms or molecules) of one object are very close to those of the other object. The particles of the two materials tend to attract each other by molecular and electrostatic forces. These attractive forces existing between different surfaces are known as adhesive forces. Figure 2.1 shows two objects in contact, with the arrows showing adhesive forces acting on them. Any relative motion between the two surfaces must overcome the adhesive forces.

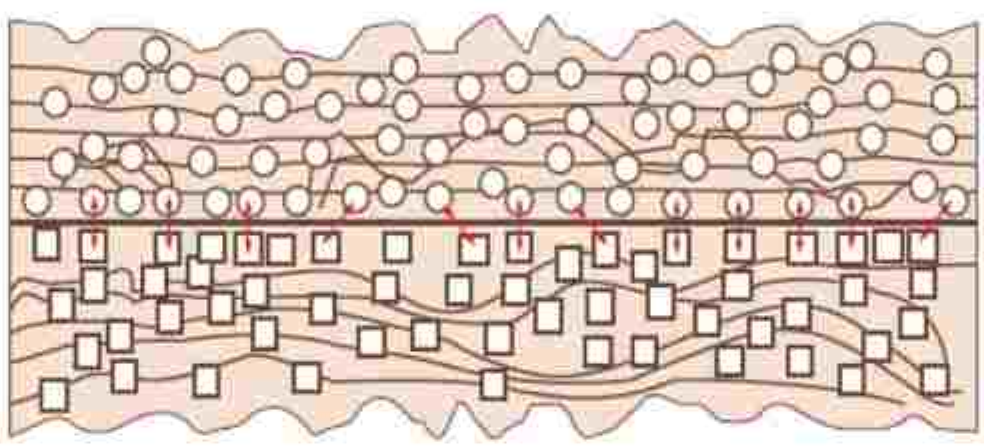


Figure 2.1: *Adhesive forces acting between two surfaces*

All objects that appear very smooth can show some degree of roughness at the microscopic level. There are significant number of ridges and grooves on rough surfaces, as shown in Figure 2.2. Rough elements tend to lock one another as they slide or tend to slide past each other. In moving any surfaces in contact, we must apply a force to overcome the interlocking. The ridges of one surface can lock into the grooves of the other, effectively creating a type of mechanical bond. A large amount of force is therefore needed to slide the objects in contact. The friction between solids and fluids or between different fluids is known as viscosity.

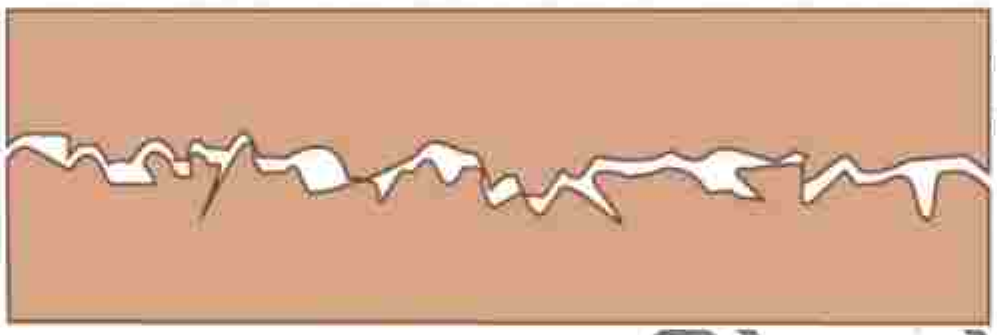


Figure 2.2: *Surface roughness causes friction between surfaces*

Advantages of friction

You have come across different examples all around where frictional force is acting. It is required in accomplishing many daily activities. The following are some advantages of frictional force.

1. Friction aids in walking; we can not walk if there is no friction between the soles of our shoes or feet and the ground. The friction between your feet and the ground makes it possible for you to walk. Without friction, your feet would slip backward as you try to walk forward.
2. Friction helps cars to move on the road. This is possible due to the friction between car tyres and the road's surface. If there is no friction, the tyres will keep on slipping at one point without making the car to move.
3. Moving objects are brought to a stop by the application of friction. Braking systems in moving machines such as vehicles, motorcycles and bicycles make use of friction. For example, a bike slows down when brake pads are applied; the pads clamp onto the bicycle tyre.
4. Friction is used to wear out unnecessary layers during the sharpening and grinding of metal parts. Figure 2.3 shows a knife sharpener.

Figure 2.3: *Sharpening a knife*

5. In lighting matches, friction between the matchstick and matchbox produces heat which ignites the matchstick.
6. Friction enables setting of fire by rubbing sticks. Figure 2.4 shows an example of fire making by rubbing sticks.

Figure 2.4: *Making fire by rubbing sticks*

7. It is friction that enables a nail to remain tightly held in a solid material. You cannot fix a nail in a wood or wall if there is no friction.
8. It is friction that enables us to write on paper or on the board.
9. Friction prevents asteroids or falling satellites from reaching the earth's surface. Asteroids burn in the atmosphere before reaching the earth due to the friction between their surface and air molecules.

Disadvantages of friction

Even though friction has several advantages in our daily activities, it also has some disadvantages as follows:

1. Friction produces unnecessary heat and noise in moving machinery parts leading to the wastage of energy in the form of heat and sound. For instance, the friction between pistons and cylinders of an engine accounts for major energy loss. If the piston-cylinder systems are not properly lubricated and cooled, friction may cause overheating of the engine, and hence lowers the efficiency of the engine. This explains why the efficiency of the actual machines is always less than 100%.
2. Friction slows down the motion of moving parts as it always acts in the opposite direction of the motion, hence more energy is needed to overcome it.
3. Friction causes materials to wear and tear out. Examples of such materials include; soles of shoes, car tyres, and machinery moving parts. As a result of this, the worn out materials must be replaced or properly disposed as unwanted materials.
4. Friction may cause fire in a forest or bush. This is caused by the friction between dry tree branches in high wind velocities.
5. Friction prevents machinery parts from moving freely. If there is no friction, the machinery parts could move freely.

Activity 2.1

Aim: To demonstrate that high friction results in increased wear and tear

Materials: four new identical erasers, A4 white paper, A4 ruled paper, wooden block, a piece of glass

Procedure

1. Rub the erasers on different surfaces of the given material. Each material surface is to be rubbed by a separate eraser, and the number of rubbing striker should be the same, for Example 20.

2. Note the extent of wear and tear on each eraser.

Question

How does the nature of the surface affect the wear and tear of the erasers?

On a rough surface, there is more wear and tear than on a smooth surface. Thus, rough surfaces increase friction resulting in increased wear and tear. A common disadvantage of friction is that it wears out the soles of your shoes. A worn shoe sole is shown in Figure 2.5.

Figure 2.5: *A shoe sole worn out due to friction*

Methods of increasing friction

In instances where friction is an advantage, it is necessary to increase the magnitude of the friction. The following are some ways in which friction force between surfaces can be increased:

1. Pressing together harder the surfaces under consideration. This increases the normal force between surfaces. For most front-wheel-drive cars, the engine is placed over the driven wheels as shown in Figure 2.6 to increase the normal force on the wheel.

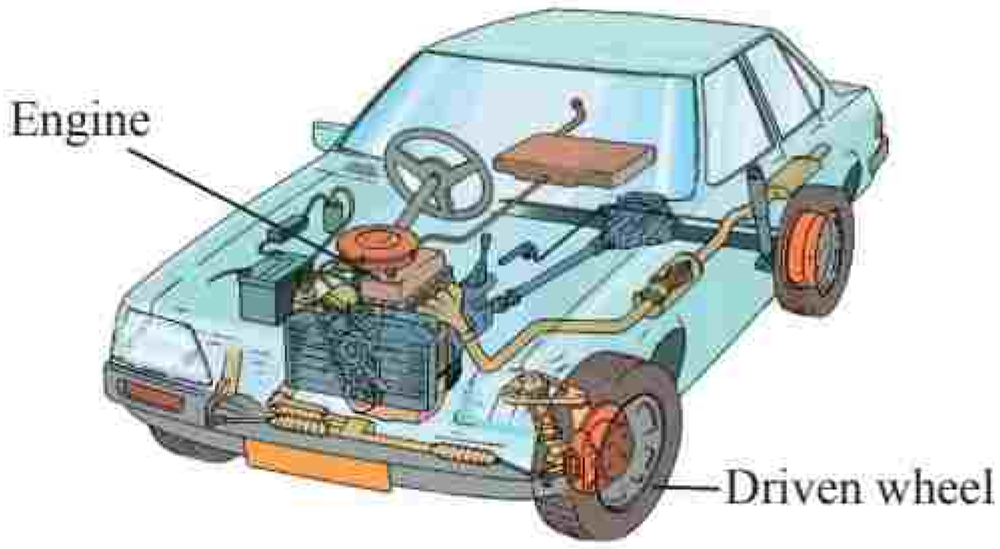


Figure 2.6: *A car engine located over the driven wheels*

2. Changing the roughness of surfaces. For example, increasing treads in tyres increases the friction between the tyre and the road. This is why it is important to change a car or bicycle tyre when its treads are worn off. Spikes on football shoes and groves on normal shoes increase friction between the shoes and the ground. Adding grit to sandpaper improves its scrubbing ability.
3. Use of materials with a high coefficient of friction. Car tyres are made of rubber which has a high coefficient of friction. This increases the frictional force between the tyres and the road. Wrapping sticky tape on the handle of an oar also increases grip on the oar. Sticky materials have a very high coefficient of friction.
4. Scrubbing steel wool is made rough to increase friction. Similarly, a rough cloth will have better cleaning qualities due to large friction than a relatively slippery one. Figure 2.7 shows a scrubbing steel wool in use for cleaning cooking utensil.

Figure 2.7: *Scrubbing a cooking utensil with a steel wool*

Methods of reducing friction

As we have already discussed, friction has a disadvantage in some situations. In such cases, the friction should be minimized. The following are some of the procedures for reducing the frictional force between surfaces:

1. Devices such as ball bearings or rollers can change sliding friction into much smaller rolling friction by reducing the points of contact on the object. Ball bearings or rollers in between two sliding surfaces decrease friction.

2. One technique used by railroad engineers is to create an amount of free movement of one car before it transmits its motion to an adjoining coupled car. This allows the train to pull forward and only take on the static friction of one car at a time, instead of all cars at once, thus spreading the static frictional force out over time.

3. A common way to reduce friction is by using a lubricant, such as oil or water between the two surfaces (Figure 2.8). This often dramatically reduces the coefficient of friction. Acoustic lubrication uses sound as a lubricant.

Figure 2.8: *Lubricant (Engine oil)*

4. The use of special materials, which have a low coefficient of friction such as Teflon, to coat cooking utensils, reduces friction and thus slides easily.

5. Polishing a surface usually reduces roughness, hence friction can be reduced.

6. For a body moving in a fluid, for example airplanes in air and ships in water, the shape is designed so as to reduce friction between the body and fluid.

Activity 2.2

Aim: To investigate the advantages of rollers in moving objects

Materials: seven cylindrical ball pens, 100 g load, wooden block, spring balance, hook screws, a laboratory bench

Procedure

1. Attach a hook screw on a wooden block, fix a spring balance and put it on a bench.

2. Put a 100 g weight on the wooden block as shown in Figure 2.9 and apply a small pull on the spring until the block just starts moving.

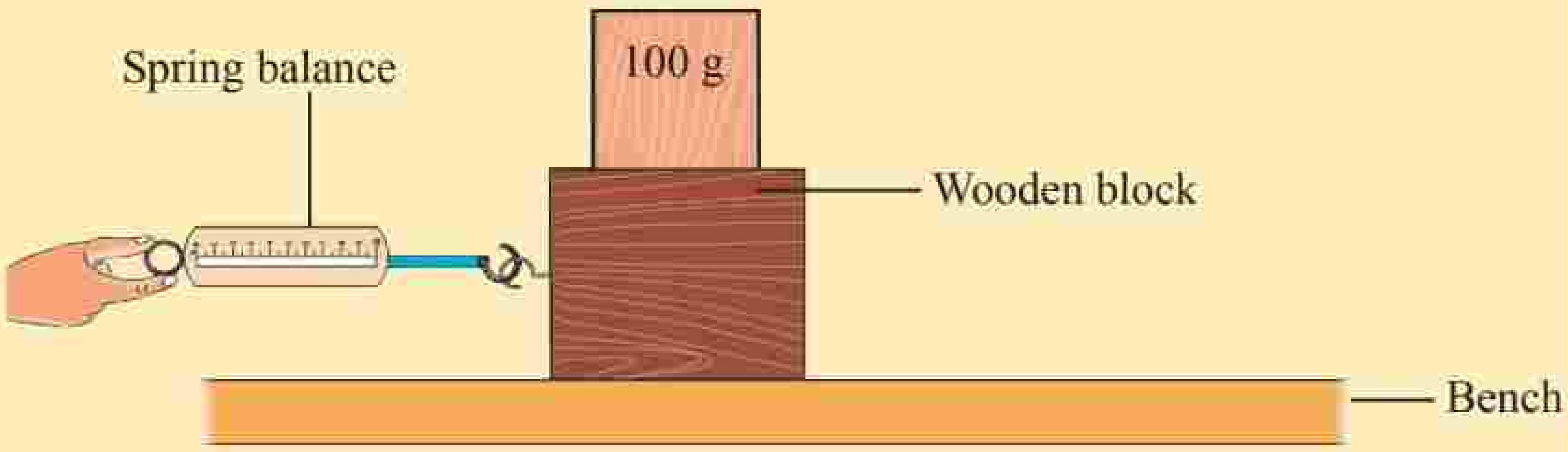


Figure 2.9

3. Record the reading of the force used to pull the wooden block.
4. Take five cylindrical ball pens and place them parallel to each other on the bench.
5. Put the wooden block of the same weight on top of the cylindrical ball pens, and put 100 g weight as shown in Figure 2.10.
6. Apply a small pull on the spring and record the reading of the force used to pull the wooden block as it just starts moving.

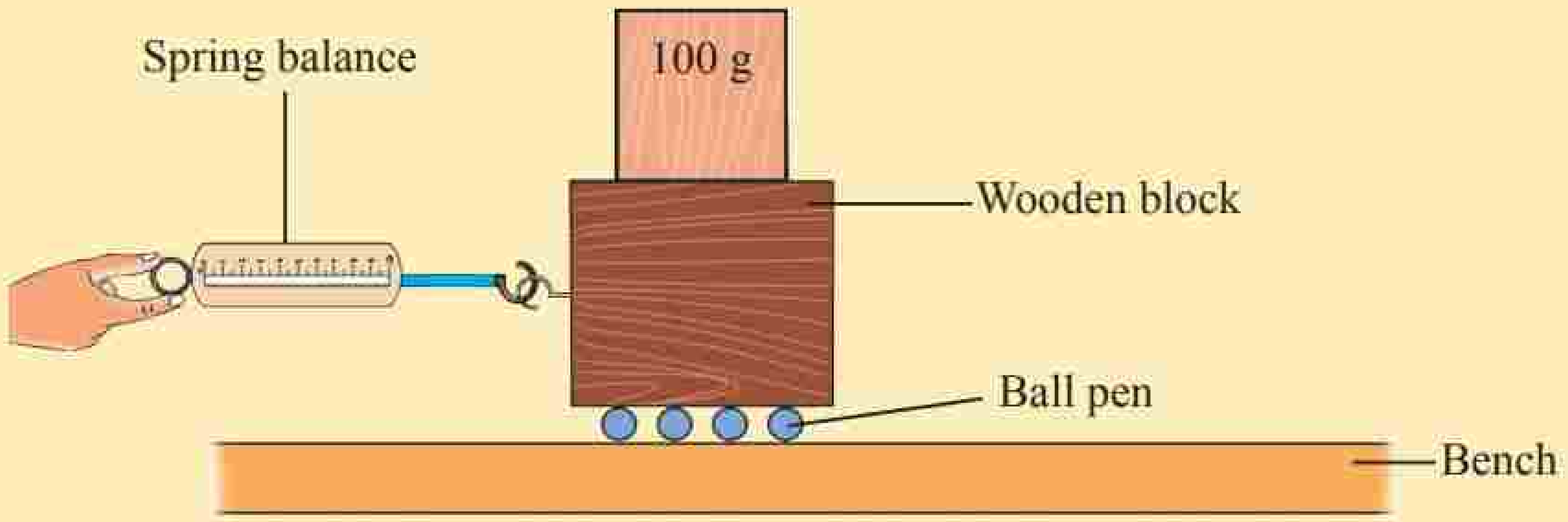


Figure 2.10

Questions

(a) What did you observe when pulling the wooden block when in direct contact with the bench surface?

(b) What did you observe when pulling the block resting on different number of ball pens?

(c) What is the advantage of using ball pens?

With the introduction of several ball pens under the object, moving the object became much easier compared to when the ball pens were absent. Therefore, ball pens reduce friction, such that, object can be easily moved from one position to another using a slightly lower force.

Normal force and friction

Suppose an object is laying on a table as shown in Figure 2.11. The earth pulls the object downwards. According to Newton's third law of motion, the table reacts by exerting an equal and opposite force on the object. This force is perpendicular to the surface of the object, thus, it is called the normal force. Since the object remains in contact with the table, normal force is a contact force. Normal force is, therefore, the net force pressing two parallel surfaces together. Its direction is perpendicular to the surfaces, as shown in Figure 2.11, and is abbreviated as F_N or R, or N. For an object resting on a horizontal surface, the force due to the weight of the object and normal force are the only two forces acting on the object. The two forces are equal but opposite in direction.

Thus, in magnitude,

$$R = W = mg$$

where m is the mass of the object and g is the acceleration due to gravity.

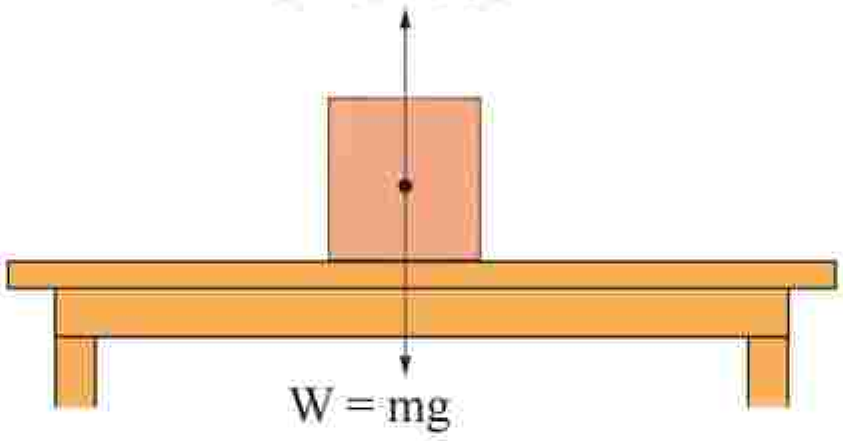


Figure 2.11: *Forces acting on an object resting on a horizontal plane*

From the preceding section, you learnt that friction can be altered by either increasing the contact force or changing the surface roughness. This means that the magnitude of the frictional force depends on the nature of the surface and the magnitude of the normal force. Thus the magnitude of frictional force, F_f is proportional to the magnitude of the normal force. That is,

$$F_f \propto R$$

However, under constant normal force, two different surfaces are expected to give different frictional forces. Thus, the nature of the surface determines the proportionality constant factor, called the coefficient of friction. Thus,

$$F_f = \mu R$$

But, we know that $R = mg$, it follows:

$$F_f = \mu mg$$

It is important to note that the frictional force depends on the materials in which the contact surfaces are made of and not the contact area. Note again that, frictional force acts along the two surfaces in contact, thus, it is a contact force.

Activity 2.3

Aim: To explore the effect of normal force on frictional force

Materials: wooden blocks, assorted masses, string, hook screws, pulley, a light pan, weighing scale

Procedure

1. Measure and record the mass of the wooden block m_1 and that of the pan m_2.
2. Attach the wooden block to the pan using a string passing over a frictionless pulley on the edge of the bench as shown in Figure 2.12.

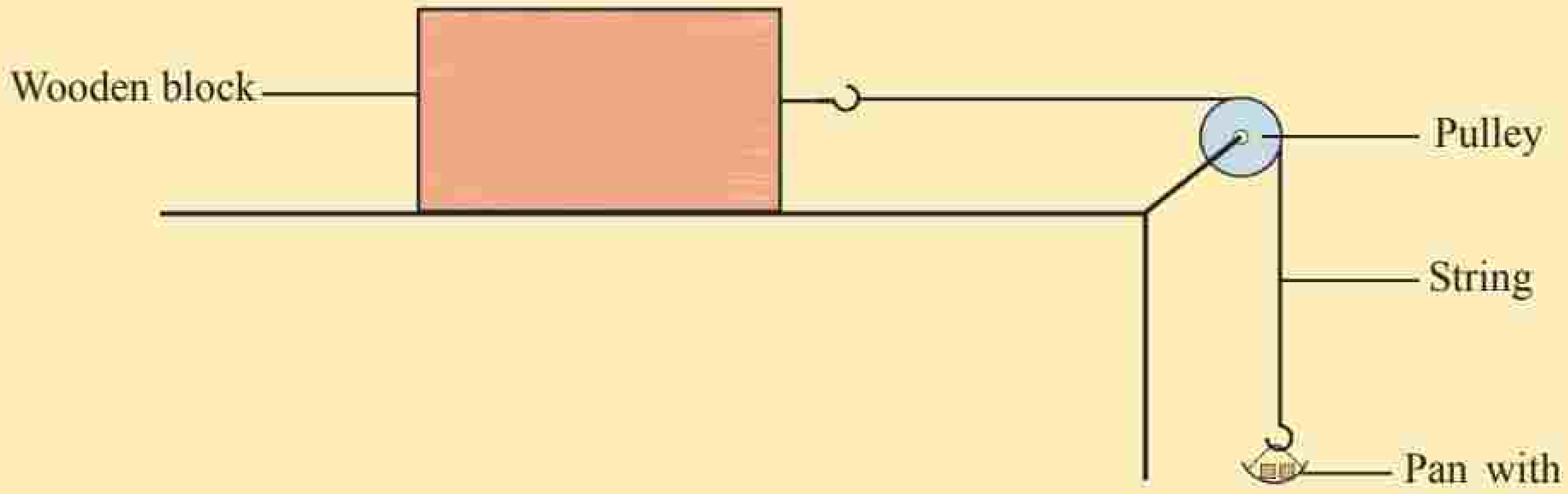


Figure 2.12

3. Place a known mass m_3 on the wooden block and record the total weight of the wooden block and mass m_3 as, $w_1 = (m_1 + m_3) \times g$.
4. Add known masses to the pan until the block starts moving. Ensure that it slides without accelerating.
5. Record the mass m_4 that causes the block to slide without accelerating. Then, record the total weight $w_2 = (m_2 + m_4) \times g$ that causes the block to slide without accelerating.
6. Repeat steps 3, 4 and 5 using three different masses.
7. Record your results in a table similar to the one shown in Table 2.1.

Table 2.1

Total weight at the wooden block (R), $w_1 = (m_1 + m_3) \times g$ (N)	Total weight at the pan (F_f), $w_2 = (m_2 + m_4) \times g$ (N)

Questions

1. Plot the graph of w_2 against w_1.
2. From your graph, explain the relationship between normal force, R, and frictional force, F_f.

The force exerted by weights and pan w_2 will cause the block to slide when it overcomes frictional force. Therefore, w_2 is equal to frictional force whereas w_1 is equal to the total normal reaction on the block. Increasing w_1 results in an increase in w_2.

Exercise 2.1

1. What does it mean by the term friction?
2. Explain any four importance of friction in daily life.
3. In which instances do you think friction is not useful? Give four reasons.
4. Briefly explain any three methods used to increase the frictional effect.
5. Suggest any four ways which can be used to reduce the frictional effect.
6. Why is it easy to slip when there is soap water on the floor?
7. When walking on a low friction surface, one should take short steps. Why?
8. You try to move a table across the floor, but it doesn't move. What is the other force acting on the table?
9. When is the applied force said to be less than frictional force?
10. When a person walks on a surface, the frictional force exerted by the surface on the person is opposite to the direction of motion. Is this true or false? Explain your answer.
11. Large bags are normally fitted with rollers at their bases, explain the importance of the rollers as far as friction is concerned.

Types of friction

Suppose you apply a small force in pushing a cupboard, it will not move. Have you asked yourself why? This is because the applied force to the cupboard is balanced by the frictional force. On the other hand, when more force is applied to push the same cupboard, it will move. However, the cupboard will still experience frictional force while in motion. This means that, there are two types of frictional forces existing in nature. The frictional force that keeps an object stationary in a given position, is called static frictional force. The frictional force experienced by a moving object is called kinetic or dynamic frictional force.

Static friction

From the previous section, you have learnt that, intermolecular force and surface interlocking are the main cause of friction. This means that, the intermolecular force and surface interlocking are also the main cause of static frictional force. Thus, it can be concluded that frictional force opposes the starting of relative motion between two objects in contact with one another.

In static friction, the frictional force resists force applied to an object at rest, and it remains at rest until the force of static friction is overcome. Static friction occurs when two objects are not moving relative to each other. The initial force to get an object moving is applied mainly to overcome this static friction. For instance, if you are walking up the hill, there is static friction between your shoes and the trail each time you step on the ground. Without this static friction, your feet would slip, making it difficult to walk.

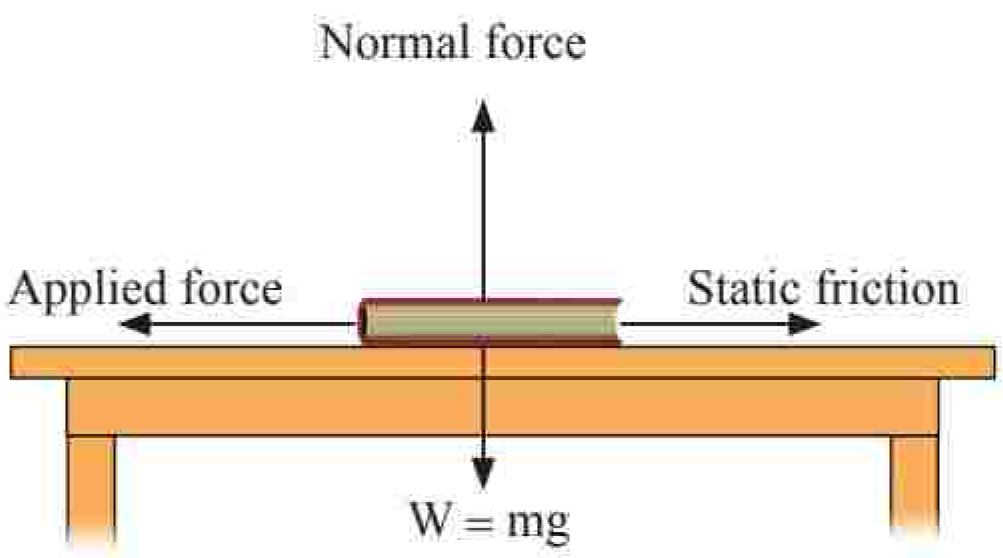


Figure 2.13: *A book resting on a table*

Static frictional force always occurs at the contact region, that is, between the two objects in contact, as shown in Figure 2.13. The coefficient of static friction is denoted as μ_s. The static frictional force is given by,

$$F_s = \mu_s R$$

Dynamic or kinetic friction

Intermolecular forces and surface interlocking are responsible for static friction. This means that for an object to start moving, the applied force must overcome the intermolecular forces and surface interlocking. Once the intermolecular forces and surface interlocking are broken, the surfaces of the two objects can slide against each other. However, they will still experience resistance force, termed as a kinetic or dynamic frictional force. Therefore, dynamic friction is that friction experienced by moving objects. Since friction is continually exerted in a direction that opposes movement, dynamic friction always does negative work. The coefficient of dynamic friction is denoted by μ_k and is usually less than the coefficient of static friction μ_s. The dynamic frictional force F_k of a solid interface is given by,

$$F_k = \mu_k R$$

The coefficient of friction depends on the nature of the surfaces. Generally, it requires less force to keep an object moving at a constant velocity than to start it moving. To set the object in motion, we must overcome the force of static friction. To keep it moving, we must overcome the force of kinetic friction. Once an object is in motion, the kinetic friction remains constant and is independent of the object's velocity as long as it does not become very large.

Limiting friction

Limiting friction is the maximum possible value of static friction. It is the frictional force that object must overcome for it to start moving. If the applied force is more than this limit, a body will move relative to the other. Limiting friction F_L is given by,

$$F_L = \mu_L R$$

where μ_L is the coefficient of limiting friction and R is the normal force.

Activity 2.4

Aim: To determine the limiting friction

Materials: spring balance, wooden block, assorted masses, a piece of plywood, masking tape

Procedure

1. Measure and record the mass of the wooden block.
2. Fix the plywood on the bench using the masking tape.
3. Place the wooden block on the plywood and attach a spring balance, as shown in Figure 2.14.

Note: The spring balance should be parallel to the bench.

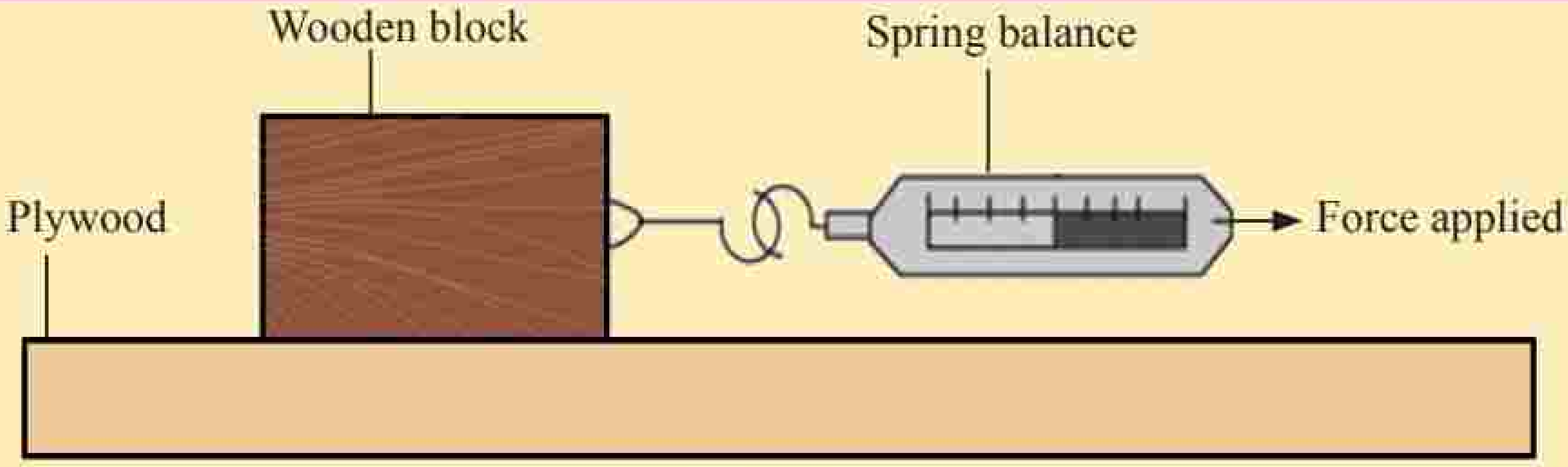


Figure 2.14

4. Pull the wooden block using the spring balance. Increase the pulling force gradually until the block is just about to start moving. Take the reading on the spring balance. This is the value of the force applied on the block.

5. Repeat step 4 with different masses placed on top of the wooden block. Record your results as shown in Table 2.2.

Table 2.2

(Mass of the wooden block + masses) × g (N)	Force required to set the wooden block + masses in motion (N)

Questions

(a) How does the weight W of the wooden block compare with the force required to set it in motion in each case?

(b) Calculate the coefficient of limiting friction in each case.

When the force applied to wooden block is increased, the frictional force also increases. This happens until the frictional force reaches a value equal to the limiting frictional force. When the applied force is increased beyond the limiting frictional force, the block begins to move. The limiting frictional force is independent of the applied force but depends on the nature of surfaces and the normal force. The coefficient of limiting friction, μ_L is given by,

$$\mu_L = \frac{F_L}{R}$$

The coefficient of limiting friction will be the same for all the masses. This is because the coefficient of limiting friction is independent of the normal force.

Example 2.1

An aluminium block of mass 2.1 kg rests on a steel platform. A horizontal force of 15 N is applied to the block.

(a) Given that the coefficient of limiting friction between the two surfaces is 0.61, will the block move?

(b) If the block moves, what will be its acceleration if the coefficient of kinetic friction is 0.47?

Solution

(a) The maximum possible value of the static friction can be determined from,

$$F_{s\,max} = \mu_s R$$
$$= \mu_s mg$$
$$= 0.61 \times 2.1 \text{ kg} \times 10 \text{ N/kg}$$
$$= 12.81 \text{ N}$$

The block will move since the applied force (15 N) exceeds the limiting friction (12.81 N).

(b) Since the block will move, there is a force of kinetic friction opposing the motion, and acceleration can be computed as follows:

$$F_k = \mu_k R$$
$$= \mu_k \times mg$$
$$= 0.47 \times 2.1 \text{ kg} \times 10 \text{ N/kg}$$
$$= 9.87 \text{ N}$$

The net force (F_{net}) is the difference between the applied and kinetic friction because they are in opposite directions.

$$F_{net} = F_{applied} - F_k$$
$$= 15 \text{ N} - 9.87 \text{ N}$$
$$= 5.13 \text{ N}$$

This is the force that will contribute to the acceleration of the block. Using Newton's second law of motion:

$$F_{net} = ma$$
$$a = \frac{F_{net}}{m}$$
$$= \frac{5.13 \text{ N}}{2.1 \text{ kg}}$$
$$= 2.44 \text{ N/kg or } 2.44 \text{ m/s}^2$$

Therefore, the acceleration of the block will be 2.44 m/s^2.

Example 2.2

A box of mass 5 kg is at rest on a wooden floor. If the coefficient of static friction between the box and the floor is 0.6, what minimum external force is required to set the box sliding?

Solution

The maximum possible value of the static friction can be determined using the equation:

$$F_{s\,max} = \mu_s R$$

But, $R = mg$; hence,

$$F_{s\,max} = \mu_s mg$$
$$= 0.6 \times 5 \text{ kg} \times 10 \text{ N/kg}$$
$$= 30 \text{ N}$$

Therefore, a force slightly greater than 30 N will set the box sliding.

Exercise 2.2

1. Define the following terms:
   (a) Limiting friction
   (b) Normal reaction
2. As the normal reaction force increases, what happens to the frictional force?
3. How does friction keep a nail in place in a block of wood? If you try to pull out the nail, in which direction does the frictional force act?
4. From our experience, it is more difficult to push or pull an object to get it moving than it is to keep it moving. Explain why?
5. A box of 12 kg is being pulled across a horizontal floor by a force of 60 N. If the acceleration of the box is 2 m/s^2, what is the force of friction acting between the box and the floor?
6. An object of 0.5 kg is given an initial velocity of 4 m/s, after which it slides a distance of 10 m across a level floor. What is the coefficient of kinetic friction between the object and the floor?
7. A brick of 2 kg is resting on a table. It requires a force of 8 N to set the brick sliding. A 1 kg mass is placed on top of the brick. What force will be required to make the block start sliding?
8. If a body loses half of its velocity on penetrating a 4 cm wooden block, how much more will it penetrate before coming to rest?
9. A refrigeration unit on a work site must be slid into place. If a frictional force of 900 N opposes the motion and two workers apply a total force of 1750 N to the unit, what is the net force on the unit?

Laws of friction

The following are some important laws regarding frictional force.

1. Frictional force F_f between the two surfaces in contact is directly proportional to the normal force, R. That is, friction depends on the normal force, R, and is expressed as:

$$F_f \propto R$$
$$F_f = \mu R$$

where μ is the coefficient of friction which depends on the type of materials in contact.

2. Frictional force is independent of the contact surface area as long as the normal force is the same.
3. Coefficient of static friction is larger than the coefficient of kinetic friction.

4. Frictional force is independent of the velocity once an object has been set in motion.

5. Frictional force depends on the nature of the surfaces in contact. That is, rough surface exerts a large amount of friction compared to a smooth one.

Activity 2.5

Aim: To investigate the effect of surface area on friction force

Materials: block of wood approximately $20\text{ cm} \times 10\text{ cm} \times 5\text{ cm}$, a piece of plywood approximately $100\text{ cm} \times 40\text{ cm}$, spring balance, hooked screws

Procedure

1. Measure and record the weight and dimensions of the wooden block.
2. Place the wooden block such that its broadest face is in contact with the plywood.
3. Insert a hook screw into one side of the block at position, P_1 as shown in Figure 2.15 and attach a spring balance to the hook.
4. Carefully pull the spring balance until the wooden block is just about to start moving. Record the reading of the force, F_1 on the spring balance. Use a table similar to Table 2.3.

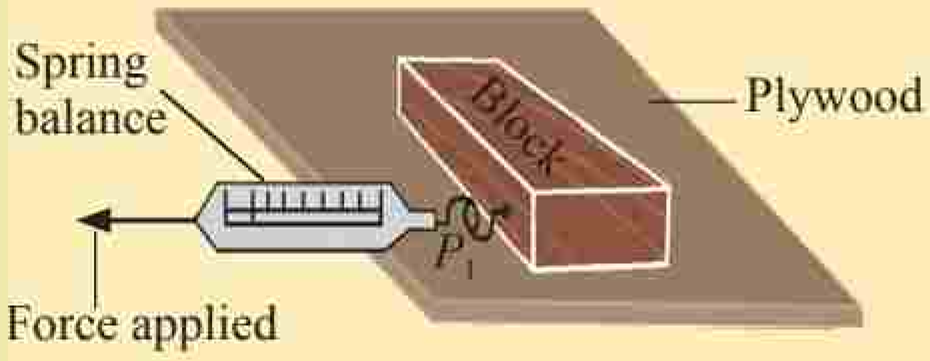


Figure 2.15

5. Repeat steps 3 and 4 for the cases when the hook screw is at positions P_2 and P_3 as illustrated in Figure 2.16. Record the corresponding spring balance readings as F_2 and F_3 for P_2 and P_3 respectively.
6. Repeat the procedure when other faces of the block are in contact with the plywood.

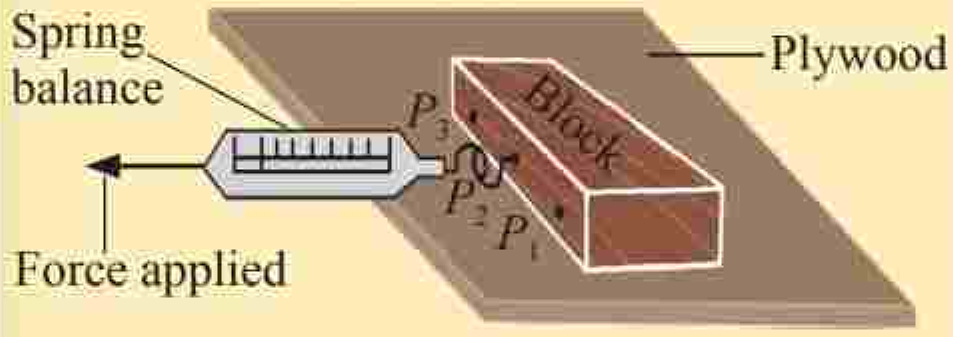


Figure 2.16

Table 2.3

Side in contact	Area, A (cm^2)	P	F (N)
Broadest		P_1	$F_1 =$
		P_2	$F_2 =$
		P_3	$F_3 =$
Narrowest		P_1	$F_1 =$
		P_2	$F_2 =$
		P_3	$F_3 =$
Medium		P_1	$F_1 =$
		P_2	$F_2 =$
		P_3	$F_3 =$

Questions

(a) Do you observe any difference in the readings of the spring balance in the three cases?

(b) What conclusion can you make from the experiment?

From the experimental results, you will note that, frictional force is independent of the contact area. The amount of force required to make the block start moving will be the same regardless of the size of the contact surfaces of the moving object.

Coefficient of friction

The **coefficient of friction**, μ, measures the amount of friction existing between two surfaces in contact. It is a scalar value that describes the ratio of the force of friction between two bodies and the force pressing them together, thus, it does not have any unit. If the value of the coefficient of friction is small, implies that, a minimum force is required to make a body slide. However, if the value of the coefficient of friction is large, a large force is required to make a body slide. The coefficient of static friction μ_s and kinetic friction, μ_k respectively, are given by:

$$\mu_s = \frac{\text{static friction } (F_s)}{\text{normal reaction } (R)}$$

$$\mu_k = \frac{\text{kinetic friction } (F_k)}{\text{normal reaction } (R)}$$

The coefficient of friction depends on the materials used. For example, an ice on a metal has a low coefficient of friction (they slide over each other easily). In contrast, a rubber on the pavement has a high coefficient of friction (they do not slide over each other easily). The coefficient of friction is a function of neither mass nor volume, it only depends on the nature of the material. For example, a large aluminium block will have the same coefficient of friction as a small aluminium block. Rough surfaces tend to have high values of coefficient of friction compared to smooth ones. Most dry materials in combination give coefficient of friction values ranging from 0.3 to 0.6. A value of 0.0 would mean there is no friction at all, which cannot occur in reality. Rubber in contact with other surfaces can yield friction coefficients from 1.0 to 2.0. The coefficient of friction is an empirical measurement, that is, it has to be measured experimentally. The values of coefficient of friction depend on the properties of the surfaces in contact. Table 2.4 shows the coefficient of friction for some materials.

Table 2.4: Values of coefficient of friction for some solid-solid interfaces

Contacting surfaces	Coefficient of static friction, μ_s	Coefficient of kinetic friction, μ_k
Steel on steel	0.74	0.57
Aluminium on steel	0.61	0.47
Rubber on concrete	1.00	0.80
Wood on wood	0.25-0.50	0.20
Glass on glass	0.94	0.40
Ice on ice	0.10	0.03

Activity 2.6

Aim: To determine coefficients of static and kinetic friction

Materials: wooden block (4 cm × 4 cm × 8 cm) with a hook screw, assorted masses, piece of plywood (20 cm × 40 cm), glass plate, spring balance

Procedure

1. Measure the weight of the wooden block using the spring balance.
2. Place the piece of plywood on the bench and the wooden block on it. Attach the spring balance to the hook on the wooden block as shown in Figure 2.17.

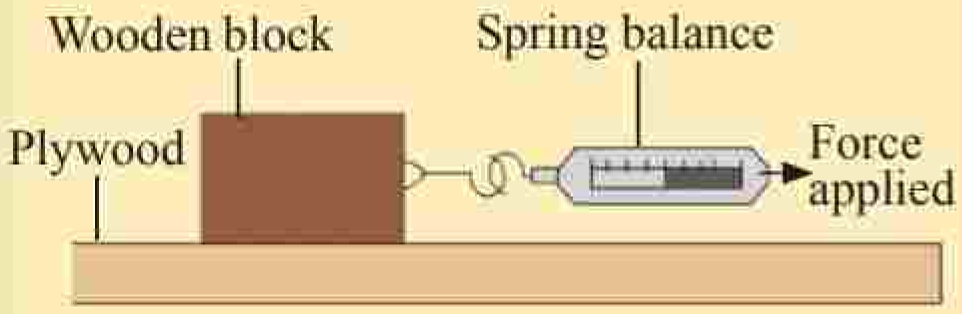


Figure 2.17

3. Gradually pull the spring balance by increasing the force until the block starts to move. Read the maximum force, F_s, measured by the spring balance, just before the block starts moving.
4. Record the force, F_k, measured by the spring balance when the block moves on the surface without accelerating (the reading after the block starts to move).
5. Repeat steps 3 and 4 when various known masses are placed on the wooden block.
6. Record your results as shown in Table 2.5.
7. Repeat the entire experiment with the wooden block placed on the glass plate.

Table 2.5

Weight of the block (N)	Static frictional force, F_s (N)	Kinetic force of friction, F_k (N)	$\mu_s = \frac{F_s}{R}$	$\mu_k = \frac{F_k}{R}$

Questions

(a) Compare the values of μ_s and μ_k for each surface.

(b) Compare the values of μ_s for different surfaces.

(c) Compare the values of μ_k for different surfaces.

(d) What is the relationship between μ_s and μ_k? Is it what you expected? Explain.

From the above experimental results, you will notice that μ_s will be larger than μ_k for both the plywood and glass plates. The coefficients of static and dynamic friction will be different for different surfaces. It is also possible to determine the coefficients by plotting graphs of F_s against R and F_k against R. The gradients of the graphs will give the values of μ_s and μ_k, respectively.

Note: μ_s and μ_k have no units since they are the ratios of the quantities with the same unit.

Consider two blocks of the same mass, one being placed on the horizontal surface while the other on a tilted surface such as an inclined plane. The block in the horizontal plane will experience a higher normal force than that on the inclined plane. This is because a small component of the force of gravity is perpendicular to the face of the plane. From Figure 2.18, the gravitational force or weight is divided into two components: one is the perpendicular component of weight that is equal in magnitude to the normal force. The other component of force is acting parallel to the plane, and it is equal to the frictional force if an object is stationary in the inclined plane. Therefore, the normal force and ultimately the frictional force are determined by vector analysis. Suppose an object with a mass, *m*, rests on an inclined plane as shown in Figure 2.18. Three forces acting on the object are:

(a) the gravitational force *mg* which acts straight down;

(b) the normal force, *R*, exerted on the object by the incline; and

(c) the frictional force, F_f, acting parallel to the incline. Since the object would tend to slide down the incline, the frictional force will be acting up the incline.

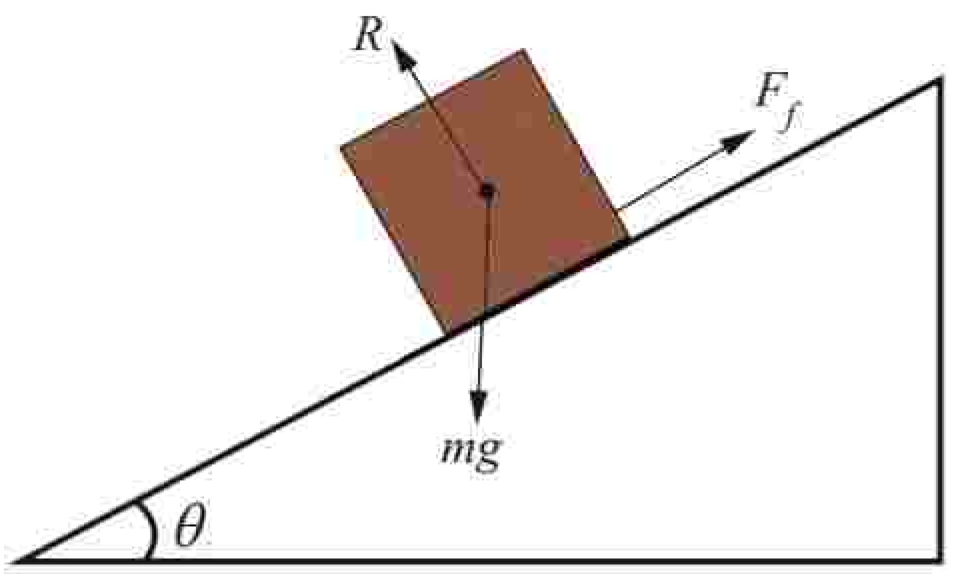


Figure 2.18: *Forces acting on an object at rest on an inclined plane*

From Figure 2.18, the component of the force of gravity acting in opposite direction to the normal force is $mgcos\theta$. The magnitude of frictional force is given as,

$$F_f = \mu R = \mu mg \cos\theta$$

Figure 2.19 shows components of the forces parallel and perpendicular to the plane. The component of the object's weight that is parallel to the incline is given by $mgsin\theta$. This is the component that tends to pull the object down the incline. The component of the object's weight perpendicular to the incline (normal force) is given by $mgcos\theta$.

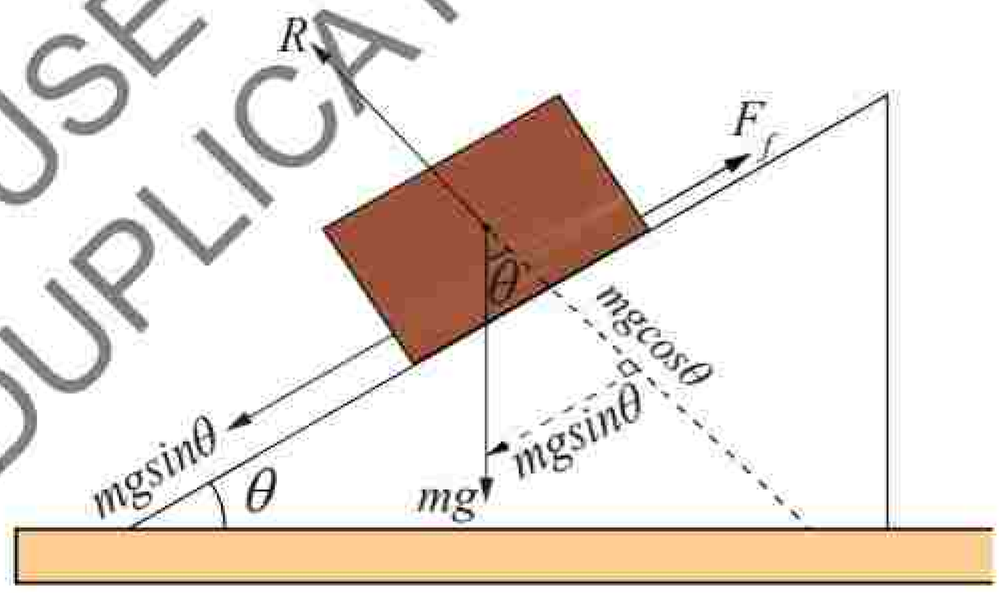


Figure 2.19: *Forces acting on an object at rest on an incline*

When the block is at rest, the static frictional force F_s is given by:

$$F_s = \mu_s R = \mu_s mgcos\theta$$

For the object to begin sliding down the incline, the component of its weight parallel to the incline, $mgsin\theta$, must be larger than or equal to the static friction F_s. That is; $mgsin\theta \geq \mu_s mgcos\theta$.

If the angle of inclination, θ, is slowly increased, $mgsin\theta$ increases while

$\mu_s mgcos\theta$ decreases. But, at certain angle of inclination θ_1;

$$mgsin\theta_1 = \mu_s mgcos\theta_1$$

Beyond this point, the object will begin to slide. This angle can be used to determine the coefficient of static friction as follows:

$$mgsin\theta_1 = \mu_s mgcos\theta_1$$

$$sin\theta_1 = \mu_s cos\theta_1$$

$$\mu_s = \frac{sin\theta_1}{cos\theta_1} = tan\theta_1$$

Once the object begins to slide, there will be kinetic friction between the object and the incline. The net force will be given by;

$$F_{net} = mgsin\theta - \mu_k mgcos\theta$$

Since the coefficient of kinetic friction, μ_k, is usually less than the coefficient of static friction, μ_s, the net force will be larger than zero, and the object will accelerate down the incline.

If the angle of the incline is reduced, the net force will decrease. Eventually, a certain angle, θ_2, will be reached at which the net force will be zero and the object will slide at a constant velocity. This angle can be used to determine the coefficient of kinetic friction as follows,

$$\mu_k = tan\theta_2$$

In summary, if $\theta < \theta_2$, the object will not slide. Suppose $\theta_2 \le \theta < \theta_1$, the object will slide provided that a slight push is given. When $\theta > \theta_2$, the object begins sliding without being pushed.

Example 2.3

A block of mass 270 kg is pulled along a horizontal surface. If the coefficient of kinetic friction between the block and the surface is 0.4, what is the force acting on the block as it slides?

Solution

$$\mu_k = \frac{F_k}{R}$$

$$F_k = \mu_k R$$

$$= \mu_k mg$$

$$= 0.4 \times 270 \text{ kg} \times 10 \text{ N/kg}$$

$$= 1080 \text{ N}$$

Example 2.4

A box of mass 2 kg rests on a horizontal surface. A force of 4.4 N is required just to start the box moving. What is the coefficient of static friction between the block and the surface?

Solution

$$\mu_s = \frac{F_s}{R} = \frac{F_s}{mg}$$

$$= \frac{4.4 \text{ N}}{2 \text{ kg} \times 10 \text{ N/kg}}$$

$$= 0.22$$

The coefficient of static friction between the block and the surface is 0.22.

Exercise 2.3

1. What does the term coefficient of friction mean?
2. Both types of coefficient of friction are dimensionless. Why is this true?
3. Explain the laws of friction.
4. Can the coefficient of friction be greater than 1? Explain.
5. Explain why the coefficient of static friction is larger than the coefficient of kinetic friction?
6. Briefly, explain the factors that determine the frictional force.
7. A box weighing 2 kg is at rest on a wooden floor. The coefficient of static friction is 0.6 and the coefficient of kinetic friction is 0.35.
   (a) What minimum force is required to start the box sliding?
   (b) What minimum force is required to keep it sliding at a constant velocity?
8. A block of mass *m* slides down a plane inclined at an angle of 60° to the horizontal with an acceleration equal to half of the acceleration due to gravity. Show that the coefficient of kinetic friction, $\mu_k = \sqrt{3} - 1$.
9. Starting from rest, the time taken by a body to slide down a 45° inclined plane with friction, is twice the time taken to slide down the same distance in the absence of friction. Determine the coefficient of friction between the body and the inclined plane.
10. A 24 kg box is released on a 30° inclined plane and accelerates at $0.25\ \text{m/s}^2$ down the plane. Find the frictional force impeding its motion.

Chapter summary

1. Frictional force is the force that opposes relative motion or the tendency of motion between two surfaces in contact.
2. Friction is caused by molecular adhesion, surface roughness and deformations.
3. The frictional force is directly proportional to the normal force between two surfaces.
4. The frictional force also depends on the nature of the surfaces in contact.
5. The frictional force is independent of the area of contact and the speed of the object once it is set in motion.
6. There are two main types of frictional force, namely; static friction and dynamic friction.
7. Static friction occurs between two solid objects that are not moving relative to each other.
8. Dynamic friction occurs when two objects are moving relative to each other.
9. Limiting frictional force is the maximum possible value of static friction.

10. The coefficient of friction is a dimensionless quantity that is defined as the ratio between frictional force and normal force.
11. Normally, for a given pair of surfaces, coefficient of static friction, μ_s, is larger than coefficient of kinetic friction, μ_k .
12. Frictional force can be increased by increasing normal force, changing the roughness of surfaces and using materials with a high coefficient of friction.
13. Frictional force can be reduced using special rolling elements such as ball bearings and rollers, lubricants and special materials.
14. Some of the advantages of friction; enables us to walk, slowdown moving cars, enables lighting of a matchstick, used in cleaning cooking utensils and in sharpening different tools.
15. Disadvantages of frictional force include wear and tear in machine parts, loss of energy in the form of sound and heat and the burning of forest or bush.

Revision exercise 2

1. State the laws of friction.
2. Give three examples of static friction.
3. Explain why most vehicles have their engines directly over the drive wheels.
4. Explain four methods of reducing friction.
5. Give five advantages and five disadvantages of friction.
6. Two students are riding their bikes on the same concrete road. One student's bike had new tyres whereas the other had used tyres. Which of them is most likely to skid while moving through a road which has lubricating oil spilled over it? Explain your answer.
7. A horizontal force of 15 N is applied to a 5 kg box placed on a horizontal table. Will the box move if the coefficient of friction is 0.4?
8. Truck A and truck B having a maximum carrying capacity of 1 tone each, were loaded with 300 kg and 600 kg of maize, respectively. After a 3000 km drive, truck B tyres were more worn out than truck A. Explain why truck B tyres were more worn out than A if both trucks had new tyres from the same company?
9. On a wet road, the coefficient of kinetic friction between the road surface and the tyre is 0.1. Two similar cars, A and B, are travelling at speeds of 15 m/s and 30 m/s, respectively. Brakes are suddenly applied to each of the cars. How far will each of the cars travel before coming to a stop?
10. A rectangular box of mass 10 kg rests on an incline. Let the coefficient of static friction be 0.55 and that of kinetic friction be 0.25.

(a) At what angle will the box begin to slide?

(b) If the incline is kept at that angle after the box begins to slide, what will be the box's acceleration?

11. A 10 kg mass is at rest on a horizontal surface with no forces other than gravity and the normal force acting on it.

(a) Given that the coefficients of static and kinetic friction between the surface and the mass are 0.5 and 0.2, respectively:

(i) What is the force of static friction between the object and the surface?

(ii) What is the minimum horizontal force required to start the object sliding?

(b) If the force in (a) (ii) above continues to act on the object after it starts sliding, what will be the object's acceleration?

12. A 6 kg mass is resting on a horizontal surface. It is determined that, a force of 20 N will start the object sliding and keep it sliding with an acceleration of 0.83 m/s^2. What are the coefficients of static and kinetic friction between the mass and the surface?

13. The coefficient of kinetic friction between the tyres of a car and the road is 0.7. The car brakes are applied and it travels a distance of 120 m before stopping. What was the car's velocity just before the brakes were applied?

14. A box of mass 5 kg is at rest on a wooden floor. The coefficient of static friction is 0.42 and the coefficient of kinetic friction is 0.15. Find its acceleration if a force of:

(a) 15 N is applied to the box.

(b) 25 N is applied to the box.

15. A 42 kg refrigerator is placed on the back of a stationary pick-up as shown in Figure 2.20. The coefficient of static friction between the refrigerator and the pick-up bed is 0.44. At what rate can the pick-up accelerate without the refrigerator sliding off back?

Figure 2.20

16. A marble is released on a 30^o inclined plane. Once at the foot of the plane, it moves in turn on three different horizontal surfaces that are covered with:

(a) a silk cloth;

(b) a layer of sand; and

(c) a glass sheet.

On which surface will the marble move the shortest distance? Give reasons for your answer.

Chapter Three

Light

Introduction

Your body seen through a mirror, the view of the moon through a telescope and a car rear seen by the driver on a rear-view mirror, are examples of images. In each case, the object that you are looking appears to be at a position different from its actual position. Your image appears to be on the other side of the mirror, the moon appears to be much closer when seen through a telescope, and a rear car appears further and smaller than it is. All these are examples of applications of mirrors in daily life. In Form One, you learnt about formation of images by light reflected from plane mirrors. In this chapter, you will learn about reflection of light and image formation by curved mirrors, and refraction of light by rectangular and triangular prisms. You will also learn colours of light and refraction of light by lenses. Competencies developed will enable you to make proper use of curved mirrors and lenses. You will also create your own binoculars and use curved reflectors to design a model of a parabolic solar cooker.

Reflection of light from curved mirrors

Have you ever observed that a car side mirror makes objects look smaller than their real size? Why is this happening? A mirror is a thin glass with one side coated with a reflecting material so that it forms images by reflecting light. Mirrors are made by coating a piece of glass by a thin layer of silver nitrate or aluminium. This makes the glass reflective on the uncoated side and non-reflective on the coated side. The piece of glass may be plane or curved. If the piece of glass is flat, the mirror is called a plane mirror, whereas, if the glass is curved, the mirror is called a curved mirror or spherical mirror. Plane mirrors form images whose sizes are the same as those of their respective objects because the reflected rays travel parallel to each other. On the other hand, curved mirrors cause light rays to diverge or converge, thus, a diminished or enlarged image may be formed.

Curved mirrors

A mirror with a curved reflecting surface is called a curved mirror. It is formed by silvering a piece of curved glass. If the glass is thought of as a portion of a hollow sphere, then the resulting mirror is called a spherical mirror. The type of a curved mirror depends on which side of the curved glass is silvered. This means curved mirrors can be classified as concave mirror or convex mirror. If the outer side of the curved glass is silvered such that the inner side becomes reflective, the mirror is called a concave mirror (Figure 3.1(a)). Conversely, if the inner side of the curved glass is silvered so that the outer surface becomes reflective, the mirror is called a convex mirror (Figure 3.1(b)).

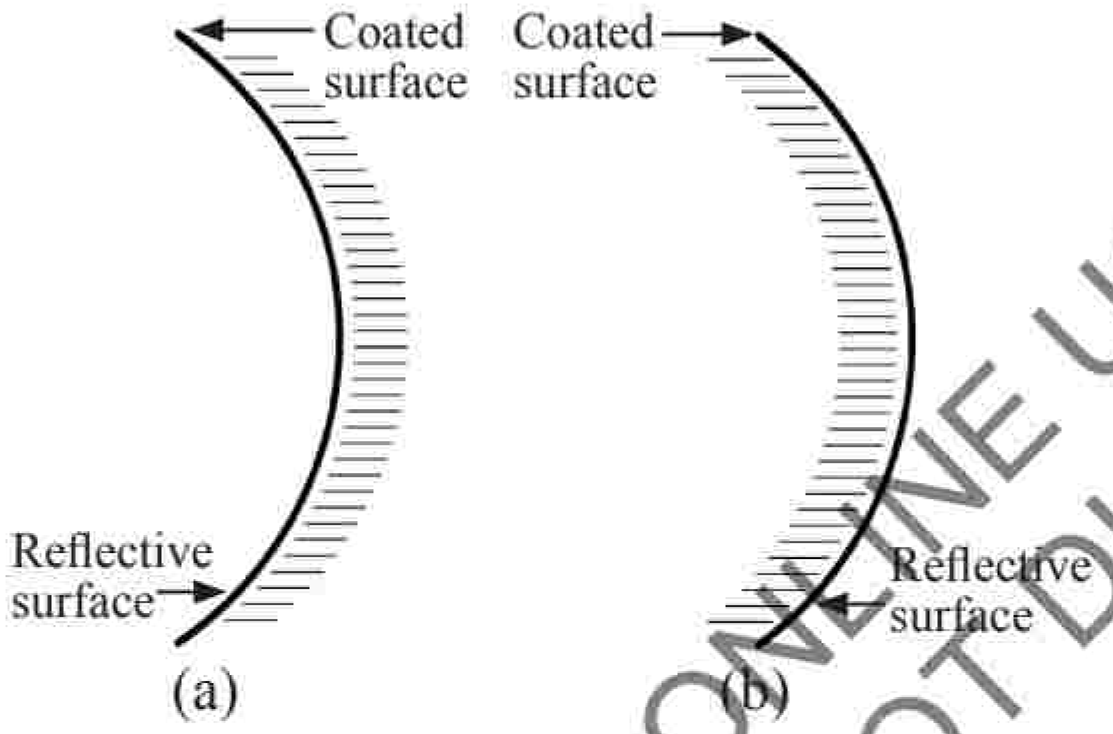


Figure 3.1: *Concave and convex mirrors*

Terminologies related to spherical mirrors

Understanding spherical mirrors demands that, one is knowledgeable with some terminologies that are periodically used. The knowledge on terms such as principal axis, focal point, mirror vertex, focal length and centre of curvature is important for understanding the reflection of light by curved mirrors. These terms are described in Figure 3.2 for both concave and convex mirrors.

If a curved mirror is thought of as being a slice of a sphere, then the centre of the sphere is called the centre of curvature and is normally denoted by the letter C.

The line passing through the centre of the sphere and touching the exact centre of the arc forming the curved mirror is known as the principal axis. The principal axis meets the mirror at a point known as the pole (or vertex) of the mirror and is denoted by the letter P.

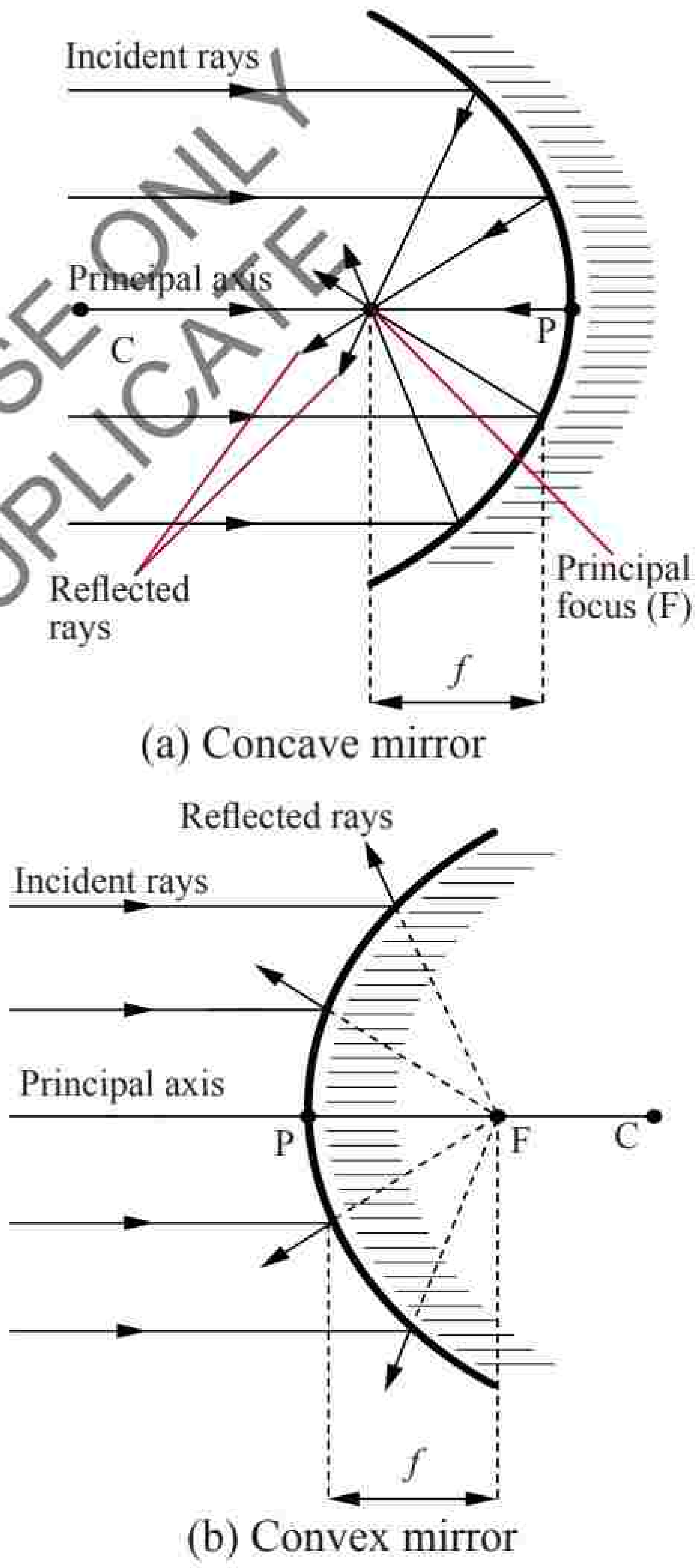


Figure 3.2: *The geometry of curved mirrors*

The pole is the geometrical centre of the mirror. Midway between the pole and the centre of curvature is a point known as the principal focus or focal point. For concave mirrors, focal point is a point in space at which the incident light rays travelling parallel to the principal axis will converge after reflection. For convex mirrors, the principal focus is a point from which light rays appear to diverge after reflection. The focal point is usually denoted by the letter F. The distance from the pole to the centre of curvature is known as the radius of curvature, represented by R. The radius of curvature is the radius of the sphere from which the mirror was made.

In practice, the part of the sphere forming the mirror is smaller compared to its radius of curvature, so that all incident rays parallel to the principal axis will converge at a common point. Finally, the distance from the mirror to the focal point is known as the focal length, represented by f. Since the focal point is the midpoint of the line segment joining the mirror pole and the centre of curvature, the focal length is one-half the radius of curvature. That is, $f = \frac{R}{2}$.

An image formed at a point from where light rays appear to come from is called a virtual image. Virtual images are apparent and cannot be focused onto a screen. On the other hand, an image formed at a point where real light rays intersect is called a real image. Such images can be focused onto a screen.

The distance from the image point to the pole is known as the image distance and is denoted by the letter v. Further, the distance from the object to the pole is called the object distance and is denoted by the letter u. Magnification, m refers to the ratio of the image height, h_i to the object height, h_o. Magnification is also given by the ratio of image distance, v to the object distance, u.

Images formed by curved mirrors

We see an object because light from the object travels to our eyes as we view it along a line to that object. Similarly, we see an image of an object because light from the object is reflected off a mirror and travels to our eyes. Therefore, the image location is the point in space from where light appears to diverge towards our eyes, as seen in Figure 3.3.

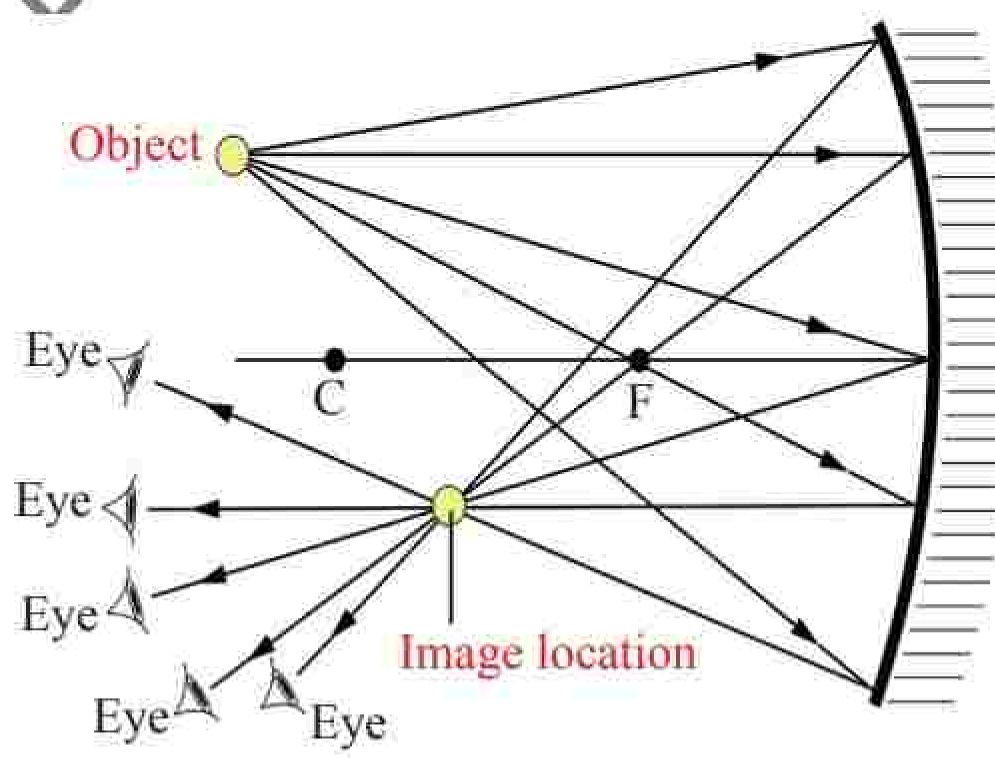


Figure 3.3: *The path of light from an object to a concave mirror to an eye*

Determining the image location for curved mirrors is an important task when using mirrors for various purposes. Method for locating images formed by curved mirrors include the geometrical method and the mirror formula.

The geometrical method involves the use of ray diagrams, which are a valuable tool for determining the path taken by light from the object to the mirror and to our eyes. In this sub-section, the method of drawing ray diagrams for objects placed at various locations in front of a curved mirror are described.

Images formed by concave mirrors

To locate an image formed by a concave mirror, the rules of reflection for concave mirrors are used. These rules are as follows:

1. An incident ray that travels parallel to the principal axis of the mirror is reflected through the focal point.
2. An incident ray that passes through the focal point is reflected parallel to the principal axis.
3. An incident ray that passes through the centre of curvature, is reflected back along its own path.
4. An incident ray not parallel to the principal axis, but strikes the pole of the mirror, is reflected back making the same angle with the principal axis. That is, the angle of incidence equals the angle of reflection.

Considering Figure 3.3, five incident rays are drawn along with their corresponding reflected rays. Each reflected ray converges at the image location and then diverges to the eye of an observer. Every observer can observe the same image location and every light ray follows the laws of reflection. However, only two of these rays are needed to determine the image location since it only requires two rays to find the intersection point. Note that, of the five incident rays in Figure 3.3, two rays correspond to the incident rays described by our rules of reflection for concave mirrors. Since these rays are the easiest pair of rays to draw, they are used to locate the image formed by a concave mirror. However, any two rays out of the four rays described by rules of reflection for concave mirrors can be used to locate the position of the image. Figure 3.4 shows the rays which are used to locate images formed by concave mirrors.

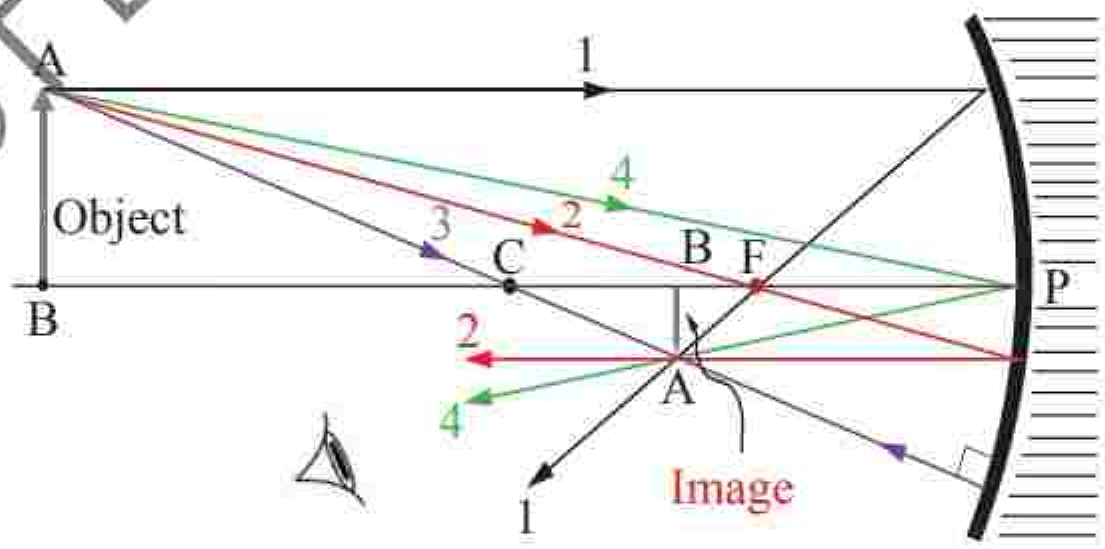
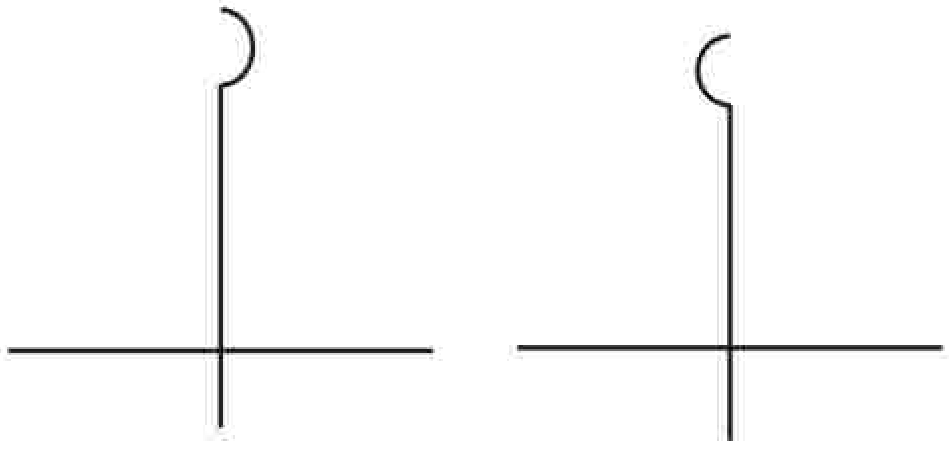


Figure 3.4: *Rays used to locate images formed by concave mirrors*

In drawing ray diagrams, when the light rays diverge, they are extended backwards to locate the image. In such cases, the image formed is virtual because it is formed by virtual rays of light, which are represented by dotted lines. The image is real when it is formed by the intersection of real rays of light.

In a ray diagram, sometimes curved mirrors are represented by straight line segments and identified by using appropriate symbols at the upper end of the line as shown in Figure 3.5.



(a) Concave (b) Convex

Figure 3.5: *Diagrammatic representation of curved mirrors*

Activity 3.1

Aim: To determine the locations and characteristics of images formed by concave mirrors for various object positions

Materials: a graph paper, a sharp pencil, a ruler

Procedure

1. Choose an appropriate scale so that the ray diagram fits on the available space.
2. Draw a concave mirror at the centre of the graph paper.
3. Draw a horizontal line passing through the centre of the concave mirror. This is the principal axis.
4. Locate the centre of curvature and the focal point on the principal axis. Choose the centre of curvature such that there is a considerable distance between the pole of the mirror and the centre of curvature.
5. Using the chosen scale, draw an upright arrow at a point far beyond C. The arrow represents an object.
6. Draw a ray diagram from a tip of the arrow parallel to the principal axis. This ray will be reflected through the focal point.
7. From the same point on the object, draw another ray through the focal point. This ray is reflected parallel to the principal axis. You may also choose to draw a ray that strikes at the pole and reflected with same angles of incidence and reflection.
8. Locate the point at which the reflected rays intersect and draw an arrow from the principal axis to the point of intersection. This represents the image and its location.
9. Measure the object distance, the image distance, the object size and the image size. Calculate the magnification using a formula:

$$Magnification\,(m) = \frac{Image\ size}{Object\ size}$$

Repeat steps 5 to 9 for different positions of the object. That is, object at the centre of curvature, C, object between C and F, object at F, object between F and P.

10. Record your results as shown in Table 3.1.

Table 3.1

u (cm)	v (cm)	h_o (cm)	h_i (cm)	Nature of image	Magnification
Beyond C					
At C					
Between F and C					
At F					
Between F and P					

Questions

(a) What are the positions (image distance) of the images formed for different object positions?

(b) Describe the nature of the images formed. That is, whether the images for various object positions are real or virtual.

(c) Are the images for objects at different positions enlarged, same or diminished?

The activity focused on the reflection of light by a concave mirror and on the formation of images. It is observed that, depending on the object position, an image formed by a concave mirror can be virtual or real, enlarged or diminished, upright or inverted. The amount by which the size of the object is enlarged or diminished is called magnification and is given by:

$$m = \frac{Image\ size}{Object\ size} = \frac{Image\ distance}{Object\ distance}$$

Example 3.1

An object 20 cm high is placed 40 cm in front of a concave mirror of focal length 15 cm. Determine the position, nature, size and magnification of the image formed using a ray diagram.

Solution

1. Choose a scale of 1 cm to represent 5 cm.
2. Draw the principal axis of the mirror and mark the focal point as shown Figure 3.6.

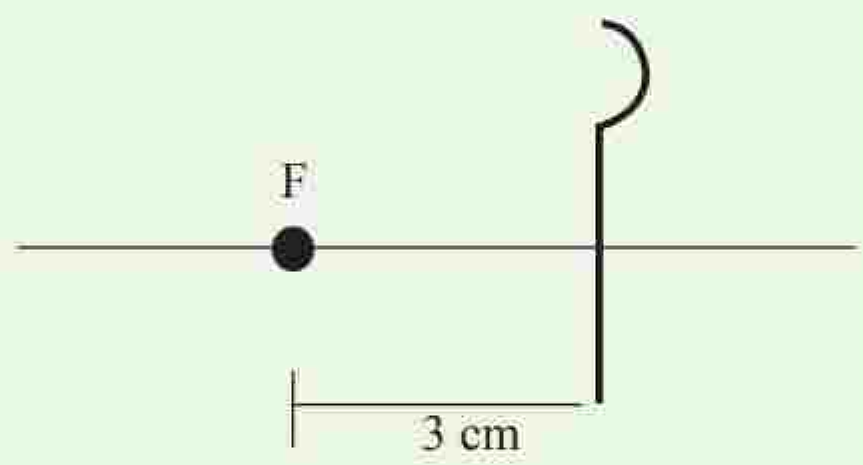


Figure 3.6

3. Draw the object position. Using the chosen scale, the object will be 4 cm high at a distance of 8 cm from the mirror as shown in Figure 3.7.

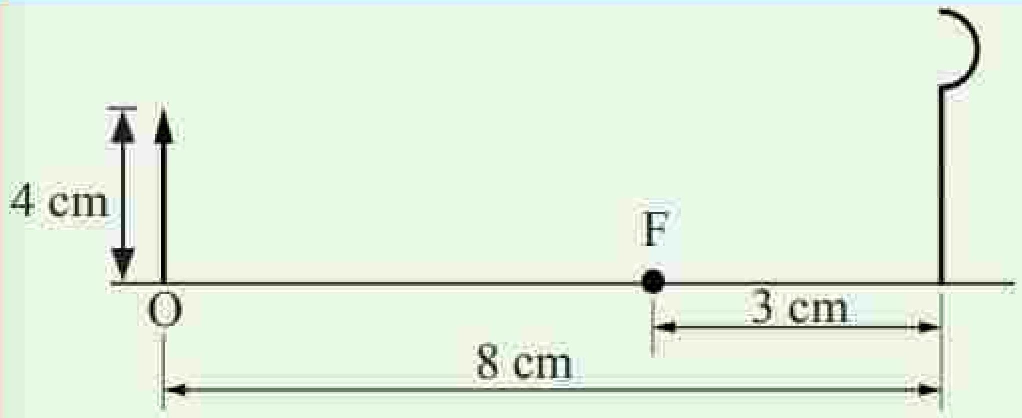


Figure 3.7

4. Locate the position of the image by drawing rays from the object to the mirror and the reflected rays as follows:

(a) From the head of the object, draw ray 1 parallel to the principal axis. This ray will be reflected through the focal point.

(b) From the head of the object, draw ray 2 of light through the focal point. This ray will be reflected parallel to the principal axis.

(c) At the point of intersection of the two reflected rays, construct the image as shown in Figure 3.8.

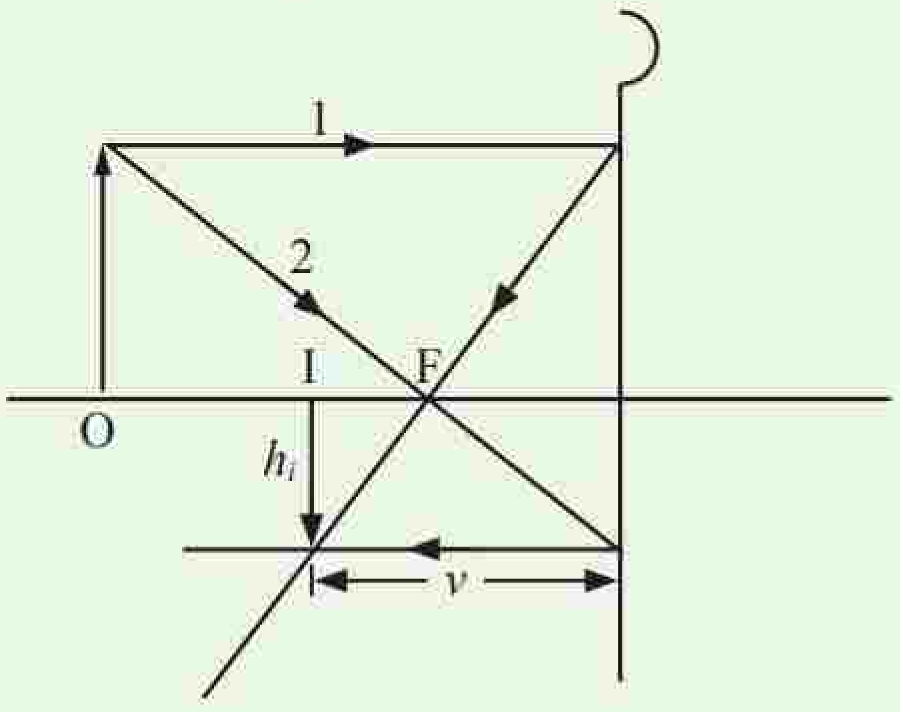


Figure 3.8

5. Using the chosen scale, the nature, size and position of the image can be determined. Thus,

(a) The measured height of the image, h_i = 2.4 cm, which corresponds to the actual height of 12 cm. (1 cm represents 5 cm).

(b) The measured image distance, v = 4.8 cm, which corresponds to the actual image distance of 24 cm from the mirror.

(c) The point of intersection is formed by true rays so the image is real.

(d) Image is inverted.

(e) Magnification, $m = \frac{12 \text{ cm}}{20 \text{ cm}} = 0.6$; the image is diminished.

Task 3.1

Use a ray diagram to observe what happens when the object is located very far beyond the centre of curvature of a concave mirror. Discuss with your classmates about your observations.

Images formed by convex mirrors

A convex mirror is the one in which the outside of the curve reflects light. In this case, the centre of that original sphere (the centre of curvature, C) and the focal point, F are located on the side of the mirror opposite to the reflecting side. They are said to be located behind the mirror. Since the focal point is located behind the mirror, a convex mirror has a negative focal length.

A convex mirror is sometimes referred to as a diverging mirror since incident light rays originating from the same point diverge upon reflection off the mirror surface. Consequently, the reflected light rays will never intersect on the object side (front side) of the mirror, rather, they appear to intersect behind the mirror when extended backwards. For this reason, convex mirrors produce virtual images that are located somewhere behind the mirror. To determine the image location, only a pair of incident and reflected rays needs to be drawn. It is customary to select a pair of rays that is easy to draw. These rays are the same as those used in concave mirror ray diagrams except that, for the current case, the rays appear to diverge from a point behind the convex mirror. This is the point at which the extended reflected rays tend to intersect and form an image. The divergence point can be determined using the laws of reflection for convex mirrors (Figure 3.9), stated as follows:

1. Any incident ray travelling parallel to the principal axis of a convex mirror will be reflected in such a manner that the reflected rays extended backwards passes through the focal point of the mirror.
2. Any incident ray travelling towards a convex mirror such that its extension passes through the focal point, will be reflected and travel parallel to the principal axis.
3. An incident ray, not parallel to the principal axis, that strikes the pole of the convex mirror is reflected such that the angle of incidence equals the angle of reflection.
4. Any incident ray travelling towards a convex mirror such that its extension passes through the centre of curvature will be reflected in its own path.

Note: The image of an object can be located by drawing any two of the stated rays.

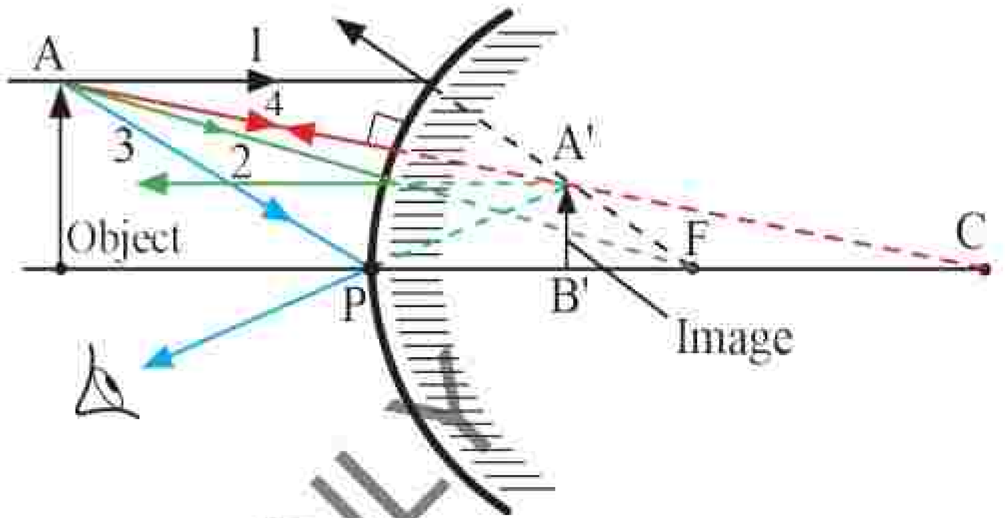


Figure 3.9: *Rays used to locate the image formed by a convex mirror*

Activity 3.2

Aim: To determine the locations and characteristics of images formed by a convex mirror for various object positions

Materials: a graph paper, a sharp pencil, a ruler

Procedure

1. Choose an appropriate scale so that the ray diagram fits on the available space and draw a convex mirror at the centre of the graph paper.
2. Draw a horizontal line passing through the centre of the convex mirror. This is the principal axis. Locate the centre of curvature, C and the focal point, F on the principal axis such that there is a considerable distance between C and P.

3. Using the chosen scale, draw an upright arrow at a point far from the mirror. The arrow represents an object at a position far from the mirror.
4. Draw a ray from a tip point of the arrow towards the focal point which is on the opposite side of the mirror. This ray will strike the mirror before reaching the focal point; stop the ray at the point of incidence with the mirror.
5. Draw the second ray such that it travels exactly parallel to the principal axis. Remember to place arrowheads upon the rays to indicate their direction.
6. Once the incident rays strike the mirror, they are reflected according to the rules of reflection for convex mirrors. For example, the ray that travels towards the focal point will be reflected and travel parallel to the principal axis. The incident ray that travels parallel to the principal axis is reflected and travel in a direction such that the reflected ray extended backwards passes through the focal point. These two rays should be diverging upon reflection.
7. Using dashed lines, extend the reflected rays behind the mirror until they intersect. The point of intersection is the image point of the top of the object.
8. Use an arrow to draw the image by joining the point of intersection of extended rays and the principal axis.
9. Measure the image distance *v*, the object distance *u*, the image height, h_i and the object height, h_o. Note that, distances measured behind the mirror are considered to be negative.
10. Repeat steps 3 to 9 for different positions of the object.
11. Summarize the results in tabular form.

Questions

(a) What are the characteristics of the images formed by a convex mirror?

(b) Do the characteristics of the image formed by a convex mirror change with the change of object position?

From the activity, we observe that, the image of an object in front of a convex mirror will be located at a position behind the convex mirror. Furthermore, the image will be upright, reduced in size (smaller than the object), and virtual.

Example 3.2

An object 5 cm long is placed on and perpendicular to the principal axis of a convex mirror of radius of curvature 20 cm. Using a scaled drawing, find the position, size and nature of the image when the object is 10 cm from the pole of the mirror.

Solution

Height of object is 5 cm, object distance is 10 cm, radius of curvature is 20 cm and focal length is -10 cm.

Scale: 1 cm represents 5 cm.

From the scale:

Height of object $= 1$ cm,

object distance $= 2$ cm and

focal length $= -2$ cm.

1. Draw a convex mirror with the pole P.
2. Locate the focal point and object position by using the provided values.
3. Draw any two of the rays stated in laws of reflection for convex mirror. The sketch is as shown in Figure 3.10.

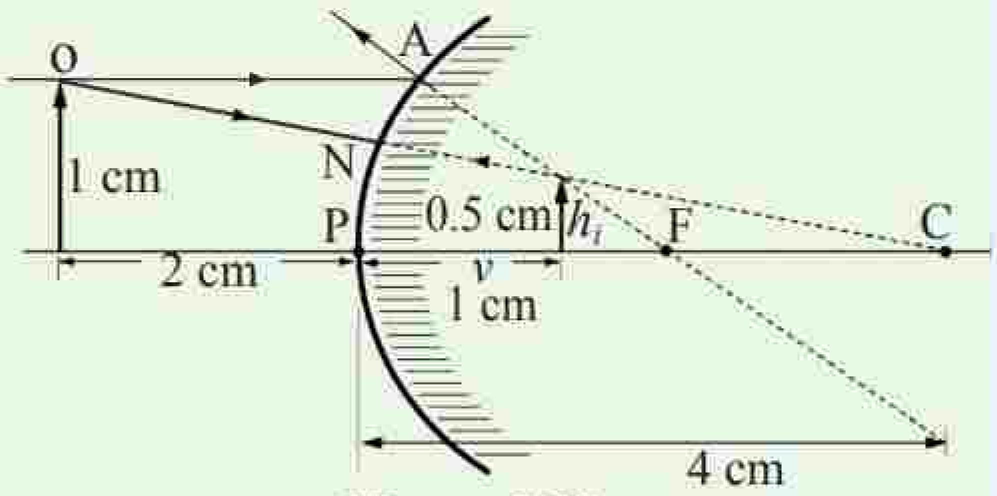


Figure 3.10

4. The image of the object is constructed by considering reflection of the rays OA and ON. The ray OA appears to diverge from the principal focus F after reflections while the ray ON is reflected back along its original path, hence appearing to come from C.
5. The image of the object top is formed at the point of intersection of lines AF and NC.
6. Measure the image distance, v and height of the image, h_i.
7. Since the scale is; 1 cm represents 5 cm, it follows that:

   (a) Image distance,

   $$v = 1\text{ cm} = \frac{1\text{ cm} \times 5\text{ cm}}{1\text{ cm}} = 5\text{ cm}$$

   (b) Height of image, $h_i = 0.5$ cm

   $$= \frac{0.5\text{ cm} \times 5\text{ cm}}{1\text{ cm}} = 2.5\text{ cm}$$

8. The point of intersection is formed by virtual rays so the image is virtual.
9. The image is upright and diminished.

Curved mirror formula

Ray diagrams provide useful information in determining the approximate location and size of the image. However, they do not provide the accurate numerical information about image distance and image size. This is because of several errors that can originate from different sources when drawing the ray diagrams. Therefore, to obtain more accurate numerical information, one ought to use the mirror equation and the magnification equation. The mirror equation expresses the quantitative relationship between the object distance (u), the image distance (v), and the focal length (f). The equation is derived using either the concave or convex mirror geometry.

Consider an object in front of a concave mirror with its image as shown in Figure 3.11.

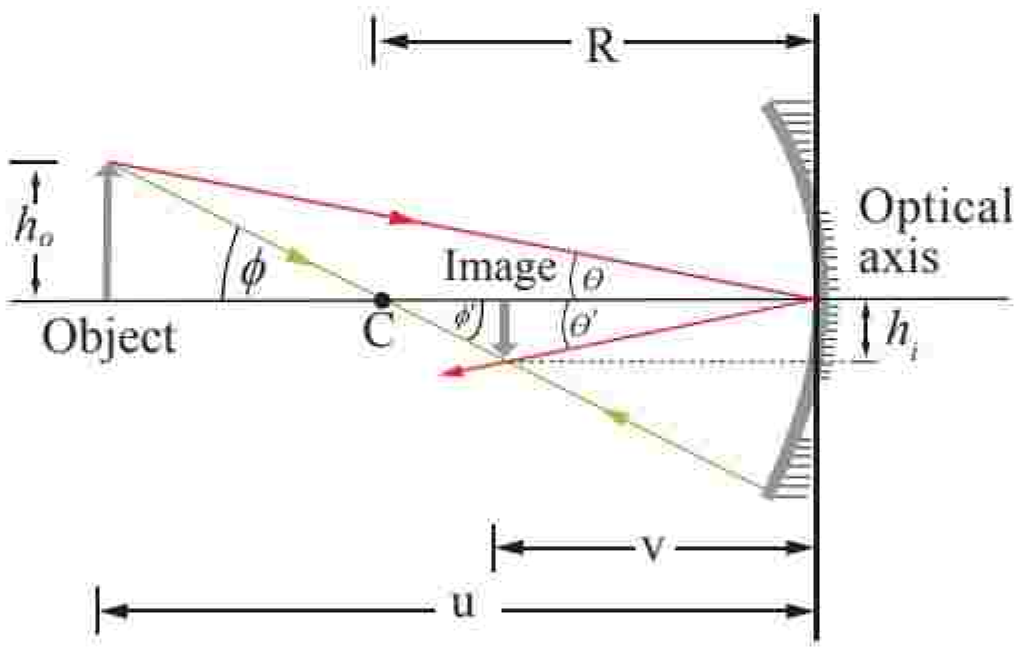


Figure 3.11: *Image formed by a concave mirror*

From Figure 3.11, the angles ϕ and ϕ' are alternate interior angles, thus they have the same magnitude. However, they differ in sign if we measure angles from the optical axis. Therefore, $\phi = -\phi'$. On the other hand, an analogous scenario holds for the angles θ and θ' which have equal magnitudes following the laws of reflection by concave mirrors. However, if measured from the optical axis, $\theta = -\theta'$. Now, considering the trigonometric identity;

$$\tan(-\theta) = -\tan(\theta),$$

we find that;

$$\tan\theta = -\tan(\theta')$$

$$\tan\theta = \frac{h_o}{u} \text{ and } \tan\theta' = \frac{h_i}{v}$$

This implies that, $\frac{h_o}{u} = -\frac{h_i}{v}$, which also can be rearranged to obtain;

$$\frac{h_i}{h_o} = -\frac{v}{u} = m$$

This is the image magnification equation for spherical mirrors. Note that, h_o, h_i, u, v and m are respectively, the object height, the image height, the object distance, the image distance and magnification.

Therefore,

$$m = \frac{\textit{image height } (h_i)}{\textit{object height } (h_o)}$$

$$= -\frac{\textit{distance of image from the mirror } (v)}{\textit{distance of object from the mirror } (u)}$$

Note that, magnification is a ratio and, therefore, has no units. When the value of m is negative, the image is inverted. The image formed by a curved mirror can be larger, smaller or the same size as the object. When the ratio, m is greater than one, the image is enlarged, when it is less than one, the image is diminished, and when it is equal to 1, the image and object sizes are equal. Similarly, from Figure 3.11, we observe that;

$$\tan\phi = \frac{h_o}{u-R} \text{ and}$$

$$\tan\phi' = \frac{h_i}{R-v}$$

Since $\tan(\phi) = -\tan(\phi')$, then,

$$\frac{h_o}{u-R} = -\frac{h_i}{R-v}$$

This implies that;

$$-\frac{h_o}{h_i} = \frac{u}{v} = \frac{u-R}{R-v}$$

Further algebraic manipulation yields;

$$uv - vR = uR - uv$$

$$2uv = uR + vR = R(u+v)$$

$$\frac{2}{R} = \frac{u+v}{uv} = \frac{1}{u} + \frac{1}{v}$$

Since, $R = 2f$, it follows that;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This expression is known as the mirror equation, where u, v and f are respectively object distance, image distance and focal length. This equation is useful in determining the position and nature of the image formed by either concave or convex mirror.

Sign convention for the mirror equation

Recall that, the characteristics of images formed by spherical mirrors depend on whether the mirror is a concave or convex. The size, position and nature of the image also depend on the position of the object in front of the mirror. To determine the image properties, there is a need to indicate the signs of values such as image distance, image height and focal length. Two sign conventions namely: new Cartesian convention and real is positive conventions are commonly used. The two sign conventions are summarized in Table 3.2.

Table 3.2: Sign conventions

New Cartesian convention	Real is positive
All distances are measured from the mirror as origin	All distances are measured from the mirror as origin
Distances measured in the opposite direction of the incident light are negative	Distances of real objects and images are positive
Distances measured along the direction of the incident light are positive	Distances of virtual objects and images are negative

Generally, the sign conventions for the given quantities in the mirror equation and magnification equations are as follows;

(a) f is positive if the mirror is a concave mirror.

(b) f is negative if the mirror is a convex mirror.

(c) v is positive if the image is real and located on the same side of the mirror as the object.

(d) v is negative if the image is virtual and located behind the mirror.

(e) h_i is positive if the image is an upright image and therefore, also virtual.

(f) h_i is negative if the image is inverted, and therefore, real.

Example 3.3

A 4.0 cm tall light bulb is placed at a distance of 8.3 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and the image size.

Solution

Given $h_o = 4.0$ cm

$u = 8.3$ cm

$f = 15.2$ cm

Using the formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v};$$

$$v = \frac{fu}{u - f}$$

$$v = \frac{15.2 \text{ cm} \times 8.3 \text{ cm}}{8.3 \text{ cm} - 15.2 \text{ cm}} = -18.28 \text{ cm}$$

from $m = \frac{h_i}{h_o} = \frac{-v}{u}$, hence

$$h_i = \frac{-vh_o}{u}$$

$$h_i = \frac{-(-18.28 \text{ cm}) \times 4 \text{ cm}}{8.3 \text{ cm}} = 8.81 \text{ cm}$$

Therefore, the image is formed at a distance of 18.28 cm from the mirror with a size of 8.81 cm. The negative sign of the image distance indicates that the image is formed behind the mirror.

Example 3.4

A 4.0 cm tall light bulb is placed at a distance of 35.5 cm from a convex mirror having a focal length of 12.2 cm. Determine the image distance and the image size.

Solution

By new cartesian sign conversion; $u = -35.5$ cm (Measured against incident), $f = +12.2$ cm

from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$;

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}; \quad v = \frac{uf}{u-f}$$

$$v = \frac{-35.5 \text{ cm} \times 12.2 \text{ cm}}{-35.5 \text{ cm} - 12.2 \text{ cm}} = 9.08 \text{ cm}$$

$$m = \frac{-v}{u} = \frac{-9.08 \text{ cm}}{-35.5 \text{ cm}} = 0.26$$

$$h_i = mh_o = 0.26 \times 4 \text{ cm} = 1.04 \text{ cm}$$

Alternatively, by real is positive sign convention; $u = 35.5$ cm, $f = -12.2$ cm (It is behind the mirror)

from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$;

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}; \quad v = \frac{uf}{u-f}$$

$$v = \frac{35.5 \text{ cm} \times (-12.2 \text{ cm})}{35.5 \text{ cm} - (-12.2 \text{ cm})} = -9.08 \text{ cm}$$

It is virtual image.

$$m = \frac{-v}{u} = \frac{-(-9.08 \text{ cm})}{35.5 \text{ cm}} = 0.26$$

$$h_i = mh_o = 0.26 \times 4 \text{ cm} = 1.04 \text{ cm}$$

Therefore, the image size is 1.04 cm and image distance is 9.08 cm behind the mirror.

Example 3.5

An electric room heater uses a concave mirror to reflect infrared (IR) radiation from a hot coil. Note that IR follows the same law of reflection as visible light. Given that, the mirror has a radius of curvature of 50 cm and produces an image of the coil 3 m away from the mirror, where is the coil located?

Solution

Using new Cartesian convention,

$$f = \frac{-R}{2} = -25 \text{ cm}, \; v = -300 \text{ cm}$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$u = \frac{vf}{v-f} = \frac{-300 \text{ cm} \times (-25 \text{ cm})}{-300 \text{ cm} - (-25 \text{ cm})}$$

$$= -27.27 \text{ cm}$$

Therefore, the coil is 27.27 cm in front of the mirror.

Example 3.6

An object 3 cm high is placed at a point 30 cm from the pole of a concave mirror whose focal length is 12 cm. Using the mirror formula, find the position, the height and the nature of the image formed.

Solution

$f = 12$ cm, $u = 30$ cm, $h_o = 3$ cm

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{12 \text{ cm}} - \frac{1}{30 \text{ cm}}$$

$$= \frac{5-2}{60 \text{ cm}} = \frac{3}{60 \text{ cm}}$$

$$v = \frac{60 \text{ cm}}{3} = 20 \text{ cm}$$

Since the image distance is positive, the image is real. To find the height of the image, we apply the magnification formula:

$$m = -\frac{v}{u} = \frac{h_i}{h_o}$$

$$\frac{h_i}{3 \text{ cm}} = -\frac{20 \text{ cm}}{30 \text{ cm}}$$

$$h_i = -2 \text{ cm}$$

The height of the image is less than the height of the object and has a negative sign. This implies that the image is real, diminished and inverted.

Activity 3.3

Aim: To investigate the characteristics of images formed in curved mirrors and verify the mirror equation

Materials: concave and convex mirrors ($f = 15.0$ cm), candle, metre rule, white screen, a V-stand or mirror holder

Procedure

1. Mount the concave mirror on a V-stand or mirror holder
2. Attach the white screen to a movable stand.
3. Light the candle and set up the apparatus as shown in Figure 3.12. Start by placing the lit candle at a point 40 cm from the mirror pole, such that, the object is beyond the centre of curvature, C.

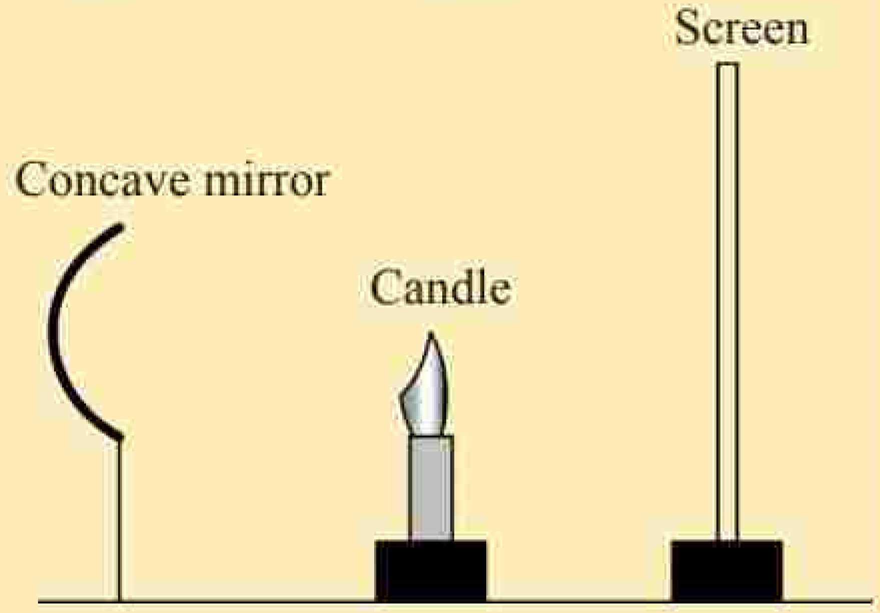


Figure 3.12

Note: The flame of the lit candle should be at the same level as the centre of the mirror and the screen.

4. Adjust the position of the screen until a clear image of the candle flame is formed on the screen. Observe the nature of the image formed. Note whether it is enlarged, diminished, upright or inverted and record your observation.
5. Measure the distance between the screen and the mirror. This corresponds to the image distance, *v.* Verify the measured image distance by calculating *v* using the mirror equation.
6. Move the candle a point 35 cm from the mirror pole and repeat steps 4 and 5.
7. Repeat step 6 for the candle placed at 30 cm, 25 cm, 15 cm, 10 cm and 5 cm from the pole of the mirror. Note that, in some cases, you may have to look on the mirror to see the image. Record your data in the form of Table 3.3.
8. Replace the concave mirror with a convex mirror of focal length 15 cm and repeat steps 4 to 7.

Table 3.3

f(cm)	$\frac{1}{f}$(cm^{-1})	u(cm)	$\frac{1}{u}$(cm^{-1})	v(cm)	$\frac{1}{v}$(cm^{-1})	$\frac{1}{v}+\frac{1}{u}$(cm^{-1})

Questions

(a) Describe the characteristics of the images for all object positions, using:
(i) a concave mirror
(ii) a convex mirror

(b) How do the measured image positions differ from the calculated image positions? Give brief explanation for any discrepancies.

The activity has revealed the characteristics of various images formed by concave and convex mirrors. Although measurements of image and object distances are affected by some errors, the values can be correctly approximated by using the mirror equation. Other image characteristics can also be predicted using the mirror equation. This verifies the mirror equation.

Focal length of a concave mirror

In the previous sub-sections, you have seen that, the focal length of a concave mirror is the distance between the pole of a mirror and a principal focus. The focal length is related to the distance of the object and the distance of the image from the mirror through the mirror equation. Using this concept, it is possible to conduct an experiment to determine the focal length of a given concave mirror. Activity 3.4 is an illustration of such an experiment.

Activity 3.4

Aim: To estimate the focal length of a concave mirror using an illuminated object

Materials: concave mirror, metre rule, cross wires, lamp-box, white screen, a V-stand, bench, a source of light

Procedure

1. Fix the V-stand on a bench and mount the concave mirror on the stand such that the reflecting surface of the mirror faces a distant object. Ensure the image of the distant object is seen.
2. Place the white screen on the bench. Move the screen until a sharp image of the distant object is focused on the screen.
3. Measure the distance between the mirror and the screen. This is the image distance, v. It corresponds approximately to the value of the focal length of the mirror.
4. Make a small round hole on one side of the lamp box and attach the crosswire such that the crossing point coincides with the centre of the hole.
5. Place the box on the bench at a point outside the approximate focal length of the mirror.
6. Switch on the lamp in the box so that the cross wire is illuminated through the hole of the box.
7. Move the screen until a clear inverted image of the crosswire is formed on the screen.
8. Measure the image distance, v from the screen to the mirror and the object distance, u from the crosswire to the mirror.
9. Change the object distance and measure the corresponding image distances. Obtain at least 5 sets of u and v.
10. For each u, calculate the focal length of the mirror using the mirror equation, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$. Determine the average focal length of the mirror.

Question

Why have you got different values of f for different sets of u and v?

This activity has revealed that, the unknown focal length of a concave mirror can be determined by conducting an experiment using an illuminated object. Although the experiment can be affected by measurement errors, the method can successfully estimate the focal length of a given concave mirror.

Activity 3.5

Aim: To determine the focal length of a concave mirror using a pin at the centre of curvature by non-parallax method

Materials: concave mirror, bench, optical pins, screen, metre rule, corks

Procedure:

1. Mount the concave mirror on a bench.
2. Fix an optical pin on a cork, then fix them on a screen.
3. Move the pin (object) until an inverted (real) image of it is seen in front of the mirror as shown in Figure 3.13.

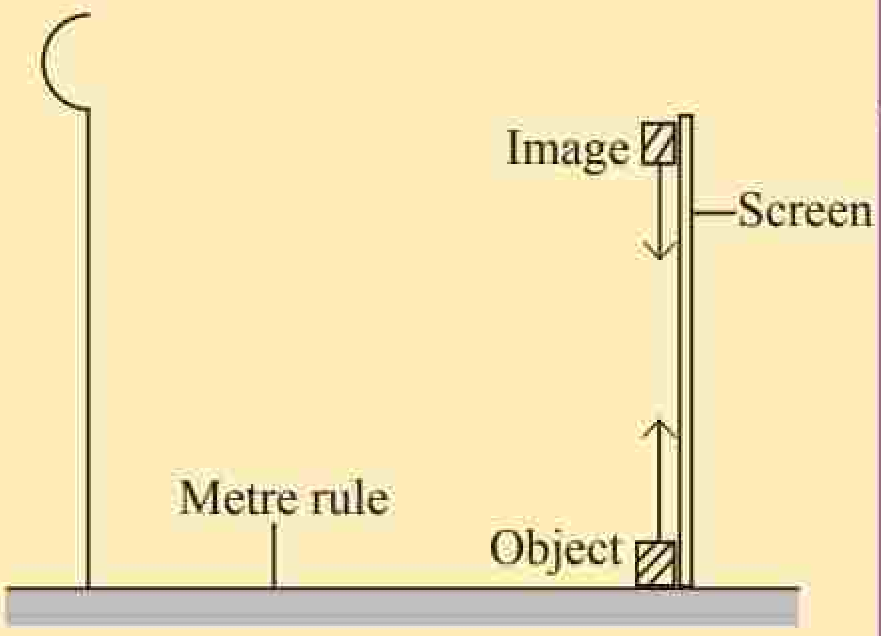


Figure 3.13

4. Adjust the position of the object pin until there is no parallax between it and its image.
5. Measure the distance between the mirror and the image formed at this point.

Questions

What is the significance of the distance between the mirror and the image?

From this activity, the position of the image pin is the centre of curvature. The focal length of the mirror can be obtained from,

$$f = \frac{R}{2}$$

Uses of curved mirrors in daily life

Curved mirrors have various applications in our daily life. Depending on the purpose, some applications make use of concave mirrors while others make use of convex mirrors.

Uses of convex mirrors

Convex mirrors provide a very wide field of view compared to other mirrors, and they form diminished images. Therefore, they are used when a wide field of view is preferred over the size of the image. The following are some applications of convex mirrors;

Convex mirrors are used as side mirrors of cars: This enables the driver to have a wide view of a rear car, as shown in Figure 3.14. However, these mirrors have a disadvantage of making objects appear farther away than they actually are. This is because the image is always formed around the focal point of the mirror, irrespective of the distance of the object.

Figure 3.14: *A side mirror of a car*

Reflecting light in streetlights: A convex mirror is used as a reflector in street lamps. Because of the wide view provided by the diverging mirror, light from the lamp diverges over a larger area. Figure 3.15 shows light diverging from the streetlights.

Figure 3.15: *Divergence of light from a streetlight*

Seeing around corners: Convex mirrors are also placed at road junctions and corners of places like parking lots and supermarkets, allowing people to see around the corners to avoid collision of vehicles or supermarket trolleys. Figure 3.16 shows a convex mirror used as a safety mirror at a road junction.

Figure 3.16: *A safety convex mirror around a corner*

Security and surveillance: Due to their wide field of view, convex mirrors are used for surveillance in business establishments and security installations. An example of a convex mirror used for search security purposes is shown in Figure 3.17.

Figure 3.17: *A security convex mirror in a shop*

Uses of concave mirrors

Concave mirrors are useful because of their ability to produce enlarged images, focus faint light and produce a parallel beam of light. The following are some applications of concave mirrors:

Headlights in a car: A powerful source of light is kept at the focal point of a concave mirror in a smaller space at the back of a headlight. Any light striking the mirror from the focus will get reflected parallel to the axis of the concave mirror forming an intense beam of light directed in front of the car as shown in Figure 3.18.

Figure 3.18: *Car headlights*

Dentist's mirror: A concave mirror helps the dentist to focus light on the tooth to be examined inside the mouth. Figure 3.19 shows a dentist mirror in use to examine a tooth.

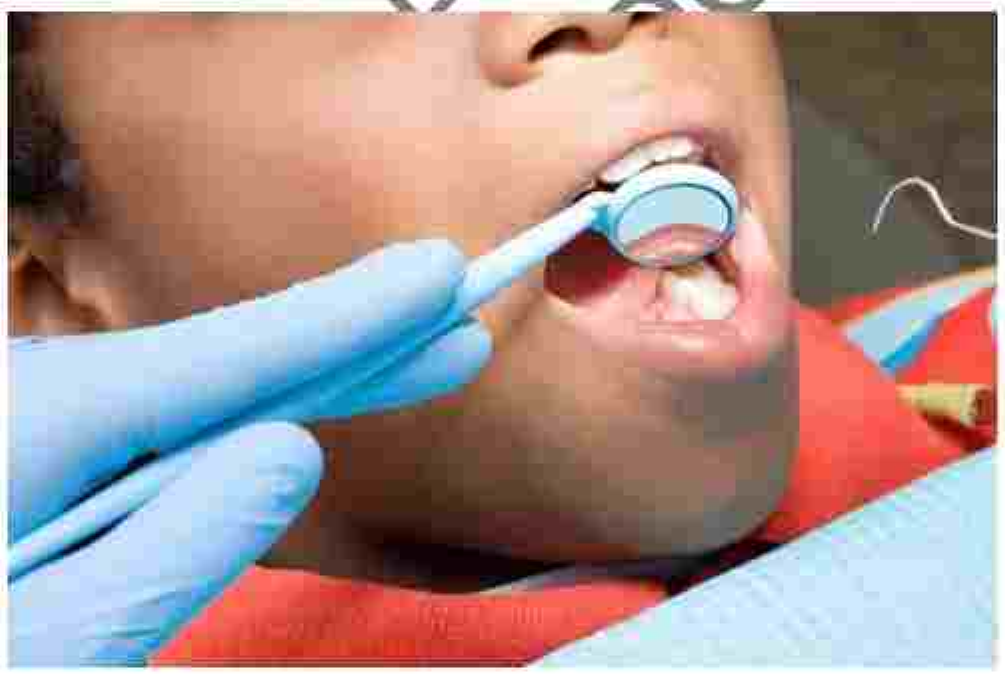


Figure 3.19: *A dentist mirror in use*

Shaving mirrors: A concave mirror produces a magnified and erect image of the object when the object is between the principal focus and the pole. Thus, concave mirrors are used as shaving mirrors, producing an enlarged image of the face when the face is held within the focus of the mirror. A concave mirror shown in Figure 3.20 is an example of a shaving mirror.

Figure 3.20: *A shaving mirror*

Reflecting telescopes: A concave mirror (primary mirror) is used to form images of distant stars at the focal point. A plane mirror (secondary mirror) is then used to project the rays to the side where the image can be viewed through a lens as shown in Figure 3.21.

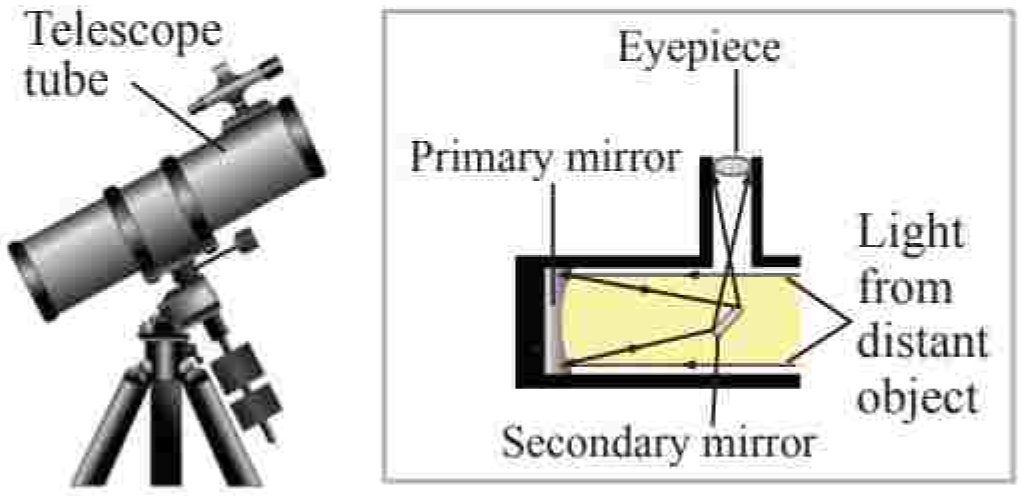


Figure 3.21: *Reflecting telescope*

In some cases, the idea of reflection of light upon falling on concave surfaces is used for other applications apart from light and image formation. Satellite dishes and parabolic solar cookers are the vivid examples of the use of concave shaped reflectors.

Satellite dishes: This is one of the most important uses of concave shaped reflectors. Dishes are used to receive and amplify signals from the communication satellites. The signals strike the concave reflector in parallel rays before being reflected and concentrated at the focus of the reflector. This enables satellite dishes to gather weaker signals coming from large areas and concentrate them at one point. Figure 3.22 shows a satellite dish.

Figure 3.22: *A satellite dish*

Parabolic solar cookers: These are solar cookers that convert sunlight directly into heat by concentrating it through a parabolically arranged set of reflectors. Figure 3.23 shows a parabolic solar cooker.

Figure 3.23: *A parabolic solar cooker*

Exercise 3.1

1. An object 5.0 cm high is placed 35 cm away from a concave mirror of focal length 15 cm. By drawing a ray diagram, determine the position, size and nature of the formed image.

2. An object, 5 cm high, is placed 10 cm away from a convex mirror of focal length 15 cm. By drawing a suitable ray diagram, determine:
   (a) the position, size and nature of the image formed.
   (b) the magnification of the image.

3. An object 25 cm high is placed 60 cm away from a convex mirror of focal length 30 cm.
   (a) Determine the characteristics of the image formed by:
      (i) drawing a suitable ray diagram.
      (ii) using the mirror formula.
   (b) Compare the results obtained using the two methods.

4. An object of 4 cm height is placed at 6 cm in front of a concave mirror of radius of curvature 24 cm. Find the position, height, magnification and nature of the image.

5. An object is placed in front of a concave mirror of focal length 20 cm. The image formed is three times the size of the object. Calculate two possible distances of the object from the mirror.

6. An object is placed at a certain distance from a convex mirror of focal length 20 cm. Find the distance of the object if the image obtained is magnified 4 times.
7. A rod of length 10 cm lies along the principal axis of a concave mirror of focal length 10 cm in such a way that its end closer to the pole is 20 cm away from the mirror. What is the length of the image?

Refraction of light

In the previous sections, it was learnt that, as light travels through a given medium, it travels in a straight line. However, when light passes from one medium into a second medium, the light beam tends to bend. Bending of light is referred as refraction of light. The direction of a ray of light changes as it passes obliquely from one medium into another.

Refraction occurs because different media have different optical densities. If the speed of light in one medium is lower than that in the other medium, the first medium is said to have a higher optical density than the second medium. Thus, the optical density of a medium can be determined by using the speed of light in that medium. Refraction occurs only at a boundary. Once the light has crossed the boundary between the two media, it continues to travel in a straight line.

Activity 3.6

Aim: To investigate the refraction of light rays as they pass from air to water

Materials: coin, bucket, water

Procedure

1. Obtain a wide opaque container, such as a bucket.
2. Place a coin inside but at the bottom of the bucket.
3. In a straight line, slowly move away from the bucket to a point where the coin is just outside your line of view. That is, the coin is not visible. Mark this point as M.
4. Pour gently clear water into the bucket and try to view the coin while standing at point M.
5. Continue adding more water until the coin can be seen when viewed from point M.

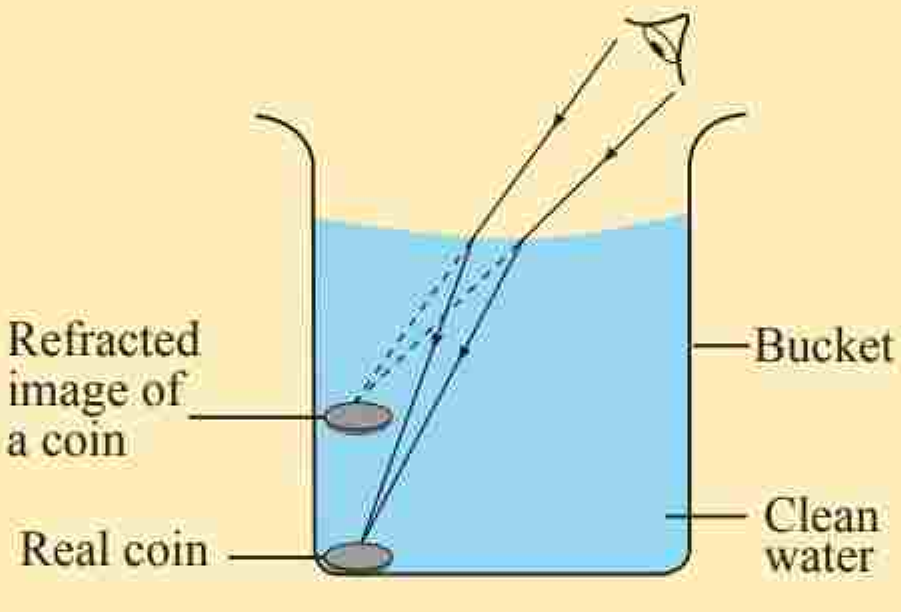


Figure 3.24

Questions

(a) Why does the coin become visible from point M after filling the bucket with water?

(b) Does the coin appear raised? Explain.

The coin appears raised due to the change of direction of rays of light at the boundary between water and air as shown in Figure 3.24.

Activity 3.7

Aim: To investigate the refraction of light as it passes from air to glass

Materials: transparent glass block, white piece of paper, a ray box

Procedure

1. Place a transparent glass block on a white piece of paper. Ensure that the block is fixed so that it does not move.
2. Place the ray box in such a way that, a ray of light falls on the side of the block at a small angle to the normal as shown in Figure 3.25. Draw a line tracing the beam of light and mark the point at which light enters the glass block as M.
3. Mark the point at which the ray of light emerges from the other side of the glass block as N and draw a line tracing the ray of light as it lives the glass block.
4. Remove the glass block and draw a line joining points M and N. Observe the direction of the line MN as compared to the line drawn in step 2.

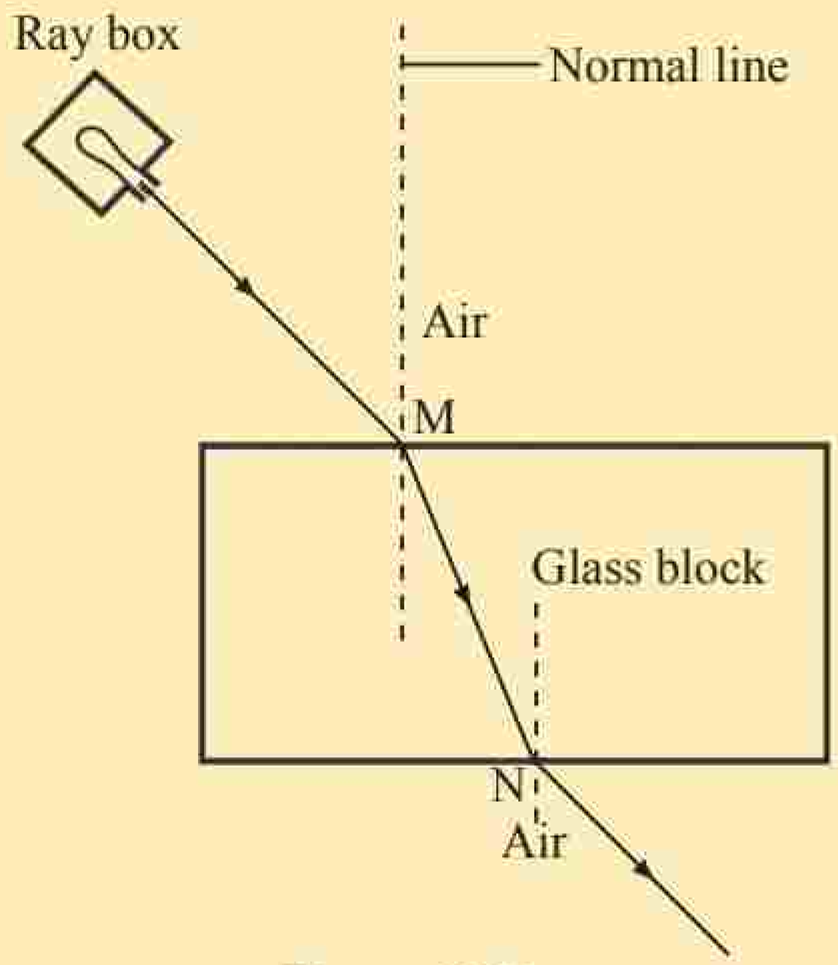


Figure 3.25

Question

Is there any change in the direction of the ray of light as it emerges from the other side of the glass block? Explain.

There is a change in the direction of light when it passes from air to glass (first boundary) and when it passes from glass to air (second boundary). This is because light is passing through different materials with different optical densities.

Angle of incidence and angle of refraction

When light passes from a medium such as air to another medium such as glass, which is optically denser than the first medium, its velocity decreases. Conversely, when light passes from an optically denser medium such as glass to a medium that is optically less dense like air, its velocity increases. The line that is perpendicular to the boundary between the two media through which light travels is called the normal line. The ray of light in the first medium is called the incident ray and the ray of light in the second medium is called the refracted ray. The angle between the incident ray and the normal line is known as the angle of incidence and is denoted by a letter *i*. On the other hand, the angle between the refracted ray and the normal line is called the angle of refraction, denoted by *r*. Figure 3.26 shows the normal line, incident ray, the refracted ray, the angle of incidence (*i*) and the angle of refraction (*r*).

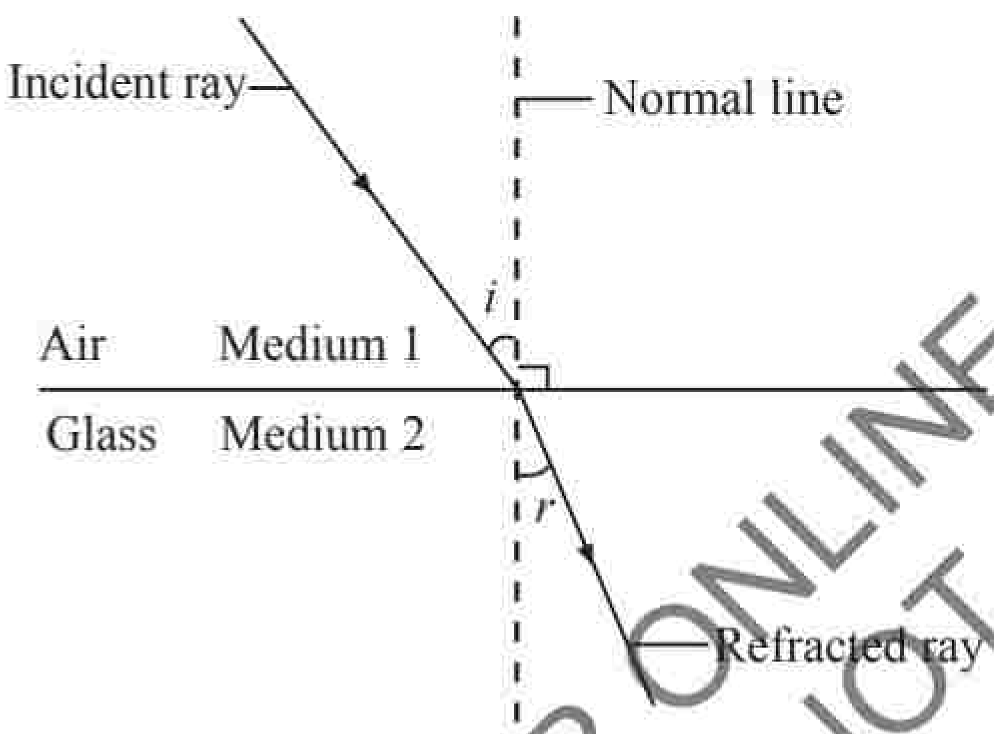
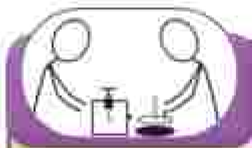


Figure 3.26: *Light travelling from air to glass*



Activity 3.8

Aim: To investigate the path of light from air to glass and measure the angles of incidence and refraction

Materials: plain paper, soft board, drawing pins, transparent glass block, protractor

Procedure

1. Place a plain paper on a soft board. Fix the paper firmly on the board using drawing pins.
2. Place a rectangular glass block on the paper and trace its outline. Draw a line perpendicular to the outline of the glass block. This is the normal line.
3. Draw a light ray that meets the side of a glass block such that it makes a small angle with the normal line. This is the angle of incidence (*i*). Mark the meeting point as M.
4. Stick two pins, P_1 and P_2 along the light ray drawn in step 3.
5. Observe the pins through the block from the opposite side.
6. Stick two other pins, P_3 and P_4, so that they appear in line with P_1 and P_2.
7. Draw a line joining the points of the two pins and the edge of the glass block at point N as shown in Figure 3.27.
8. Remove the glass block and draw a line, MN, joining point M and point N.
9. Draw a line perpendicular to the outline of the glass block at point N. This is also a normal line.
10. Measure the angle between the normal line and the refracted ray. This is the angle of refraction (*r*).

Note: Both the angle of incidence and the angle of refraction are measured from the normal line.

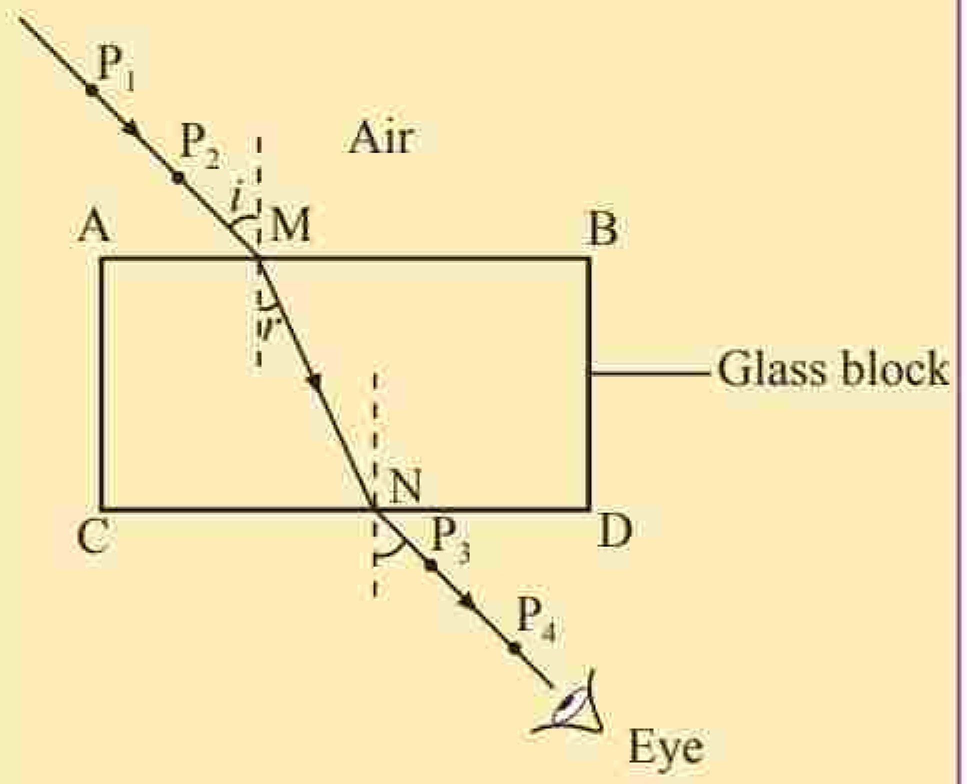


Figure 3.27

Question

Describe the path taken by a light ray travelling from air into the glass block and the air again.

The lines P_1P_2, MN and P_3P_4 show the path that a light ray follows from air, through the glass block and to air again. Light rays are refracted as they pass from air to glass (at the first boundary) and again as they pass from glass to air (at the second boundary).

Laws of refraction

When a beam of light passes through two different media via an interface, it bends. The amount of bending depends on the optical densities of the two media and the angle of incidence of the light beam. Note that, the angle of incidence and the angle of refraction differ in proportion to the difference in the optical densities of the two media. If a light ray passes from the optically less dense medium to the optically denser medium, it is bent towards the normal. However, if the ray travels from a medium of higher optical density to a medium of lower optical density, it bends away from the normal. Following various experimental observations, two laws govern the behaviour of a light ray as it strikes the interface between two media of different optical densities. These laws are known as the laws of refraction of light.

Activity 3.9

Aim: To deduce the laws of refraction and determine the refractive index

Materials: a rectangular glass block, four optical pins, a white sheet of paper, a piece of soft board, pencil, protractor, paper pins

Procedure

1. Place the glass block on the sheet of paper and trace its outline using a pencil.
2. Remove the block.
3. At a point, E, on the glass block outline, draw a normal to the outline. Point E should be near to the centre of the longer edge of the glass block outline as shown in Figure 3.28.

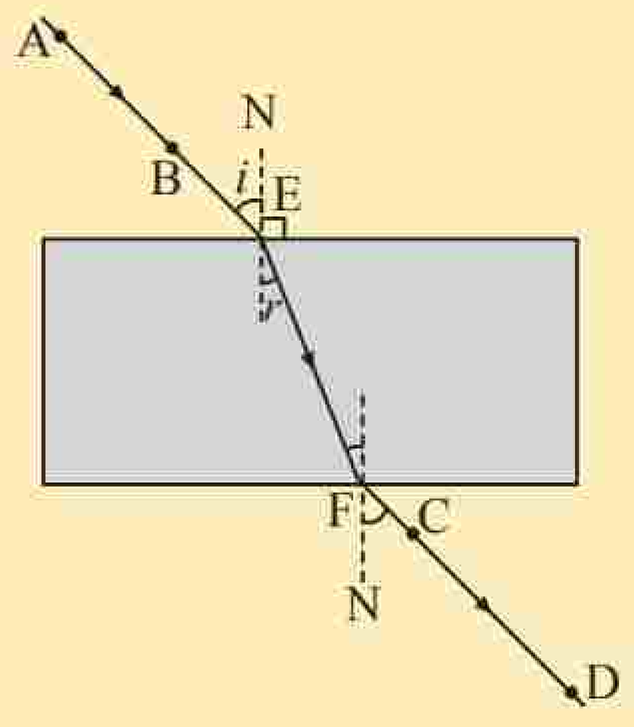


Figure 3.28

4. Draw a line at an angle to the outline, for example 30°, to meet the outline at E.
5. Place the paper on the soft board and hold it in position with the help of paper pins.
6. Stick two optical pins, A and B on the line you drew. Replace the glass block back on the outline.
7. By viewing through the block from the opposite side, set two pins C and D on the opposite side of pins A and B in such a way that all the four pins appear to lie on a straight line. Ensure that the pins appear to lie on a straight line even when viewed from the side with pins A and B.
8. Remove the block and draw a line through the marks made by pins C and D. Mark the point of intersection of the line through C and D and the glass block outline as F.
9. Join points F and E with a straight line and then draw a normal line to the glass block outline at E and F as shown in Figure 3.28.
10. Measure the angle of incidence and the angle of refraction at the first boundary.
11. Repeat the experiment for different values of *i* and record the results in the format shown in Table 3.4.

Table 3.4: Results of observations

$i(°)$	$r(°)$	$sin(i)$	$sin(r)$	$\frac{sin(i)}{sin(r)}$

Questions

(a) Calculate the ratio $\frac{sin(i)}{sin(r)}$ for each pair of angles and record your result in the table.

(b) What conclusion can you draw from the values of $\frac{sin(i)}{sin(r)}$ for the different angles of incidence i?

Several deductions can be made from the results of the above experiment. First, whenever the pins are set in line and viewed from one side, they appear to be in line. This is also the case when pins are viewed from the other side. This illustrates the principle of reversibility of light, which states that; light will follow exactly the same path if its direction of travel is reversed.

The implication of the law of reversibility of light in a rectangular glass prism is that, if *i* is the angle of incidence from air to glass and *r* is the angle of refraction, then, *r* becomes the angle of incidence for glass to air and *i* becomes the angle of refraction.

The second observation to note is that, as a ray of light travels from air to glass (from an optically less dense medium to a denser medium), it is bent towards the normal at the boundary between the two media. Conversely, when a ray of light travels from glass to air (from an optically denser medium to a less dense medium), it is bent away from the normal.

The third observation is that, the ratio $\frac{sin(i)}{sin(r)}$ is the same (constant) for the different angles of incidence.

The fourth observation to note is that at points E and F, the normal line, the incident ray and the refracted ray lie on the same plane. This may not have been obvious. However, you can observe this if you lift one edge of the glass block above the other edge of the block after setting the pins in a line. You will immediately notice that, the linearity of pins A, B, C and D disappears.

These important observations lead into two important conclusions that are essentially the laws of refraction of light. These laws state that;

1. At the point of incidence, the incident ray, the normal line and the refracted ray all lie on the same plane.
2. For a particular material, the ratio $\frac{sin(i)}{sin(r)}$ is constant. This law is also referred to as Snell's law of refraction.

Refractive index of a material

In Activity 3.9, you observed that, for a given material or medium, the ratio $\frac{sin(i)}{sin(r)}$ is constant regardless of the size of the angle of incidence. This constant is known as the refractive index of material (medium). It is a dimensionless (it has no unit) constant denoted by the Greek letter η. The refractive index of a material is a measure of the influence of the material on the speed of light as it passes through it. Therefore, refractive index of a material is also obtained from the ratio of the speed of light in vacuum, c, to the speed of light in the material, v. That is,

$$\eta = \frac{c}{v}$$

To be specific, the ratio $\eta = \frac{c}{v}$ is known as *the absolute refractive index* of a material. If the ratio is obtained from the speed of light v_1 in one material, to the speed of light v_2 in the other material, then $\eta = \frac{v_1}{v_2}$. In this case, the ratio is called the *relative refractive index*.

Normally, the speed of light in air is considered as the speed of light in vacuum, c. Therefore, if light travels from air to water and from air to glass, it will have different speeds in water and in glass because air, water and glass have different optical densities. Letting the speed of light in water be v_1 and its speed in glass be v_2, one can calculate the absolute refractive indices of water and glass as follows;

Absolute refractive index of water; $\eta_1 = \frac{c}{v_1}$. This implies that, $c = \eta_1 v_1$.

Absolute refractive index of glass; $\eta_2 = \frac{c}{v_2}$. This implies that, $c = \eta_2 v_2$.

Therefore,

$$c = \eta_1 v_1 = \eta_2 v_2$$

Hence,

$$\eta_1 v_1 = \eta_2 v_2$$

This expression can be rearranged to read;

$$\frac{\eta_2}{\eta_1} = \frac{v_1}{v_2}$$

The ratio $\frac{\eta_2}{\eta_1}$ is the relative refractive index of glass to water. This ratio can also be related to the ratio of the sines of the angle of incidence and the angle of refraction as follows.

$$\frac{\eta_2}{\eta_1} = \frac{sin(i)}{sin(r)}$$

Therefore,

$$\eta_1 sin(i) = \eta_2 sin(r)$$

Applying the law of reversibility of light shows that ${}_g\eta_a = \frac{1}{{}_a\eta_g}$, where ${}_a\eta_g$ and ${}_g\eta_a$ are the refractive indices for light travelling from air to glass and from glass to air, respectively.

There are various methods of determining the refractive index of a given material. One important method is carrying out an experiment described in Activity 3.9. Another method of estimating the refractive index of liquids is based on a concave mirror method described through Activity 3.10.

Activity 3.10

Aim: To determine the refractive index of water using a concave mirror method

Materials: concave mirrors of different focal lengths (10 cm, 15 cm, 20 cm and 25 cm), water, retort stand, optical pin, clamp, meter rule

Procedure

1. Place a concave mirror on a bench.
2. Clamp an optical pin on the retort stand above the concave mirror.
3. Position one eye vertically above the pin and look in the concave mirror for the real inverted image of the pin.
4. Adjust the position of the optical pin by moving it up or down until the pin image coincides with the real pin as shown in Figure 3.29. Ensure that there is no parallax.
5. Measure and record the height, *h* of the head of the optical pin from the pole of the mirror.

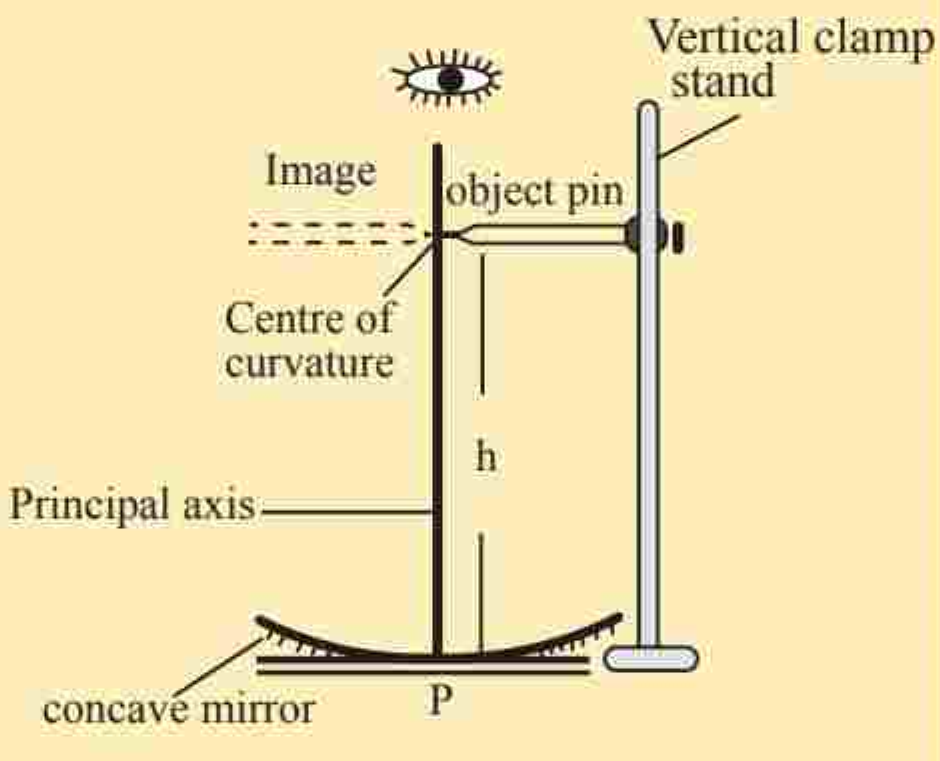


Figure 3.29

6. Fill the concave mirror with enough water to overcome the curvature of the concave mirror and the water surface tension. Wait for some time until the water is still.

7. Repeat steps 3 and 4.
8. Measure and record the new height, h' of the optical pin from the pole of the mirror.
9. Repeat the experiment using other concave mirrors of different focal lengths.
10. Record your results in a table similar to Table 3.5 and calculate the average refractive index of water.

Table 3.5

Focal length (f) (cm)	Height (*h*) (cm)	Height (h') (cm)	Refractive index $\eta = h' / h$
10			
15			
20			
25			

Questions

(a) Which mirror gives the most accurate value of the refractive index of water?

(b) What are the possible sources of uncertainties in determining the refractive index of water using the concave mirror method?

In this activity, the refractive index of water has been measured using a concave mirror method. As the known refractive index of water is 1.33, this method gives a good estimate of the refractive index of water. Therefore, the method can be useful in determining the refractive indices of different liquids.

Task 3.2

Use the apparent depth method to determine the refractive index of water. How accurate is your value of refractive index?

Example 3.7

Find the index of refraction for a certain medium, assuming light in air enters the medium at an incident angle of 30º and the angle of refraction is 22º.

Solution

Given, incident angle $(i) = 30°$; angle of refraction $(r) = 22°$, $\eta = ?$

$${}_a\eta_m = \frac{sin(i)}{sin(r)} = \frac{sin\,30°}{sin\,22°} = 1.33$$

Therefore, the refractive index of the medium is 1.33.

Total internal reflection of light

It has been established that, when light travels from a lighter medium to a denser one ($\eta_1 < \eta_2$), the refracted light ray bends towards the normal. Conversely, when light travels from a denser medium to a lighter one ($\eta_1 > \eta_2$), the refracted light ray bends away from the normal. It is also known that the angle of refraction and hence the amount by which light bends away from the normal depends on the angle of incidence. Therefore, the larger the angle of incidence, the more the light bends away from normal. Yet, one could ask, when light travels from denser medium to less dense medium, how much away from normal is it possible for the light to bend?

The answer to this question is that, the bending of light can only get as far as 90° without leaving the medium. Therefore, there is an angle of incidence of light for which the angle of refraction is 90°. This angle of incidence is known as the critical angle and is normally denoted by a letter c. When the angle of incidence becomes greater than the critical angle, light is not refracted anymore; instead, light is reflected back into the same medium where it is coming from. This phenomenon is known as total internal reflection.

To understand the total internal reflection phenomenon, let us suppose that a laser beam is submerged in a tank of water (don't do this at home) and pointed upwards towards the water-air boundary. If the angle at which the beam is directed upwards is slowly altered, starting from small angles of incidence, we would observe both reflection and refraction at the water-air interface. However, the intensity of the reflected and refracted rays changes as the angle of incidence changes. At angle of incidence close to 0°, most of the light energy is transmitted across the boundary and very little of it is reflected. As the angle of incidence is increased, the angle of refraction would eventually reach a 90°. If the angle of incidence is increased further, the refracted ray disappears and only the reflected ray is observed as shown in Figure 3.30.

Therefore, total internal reflection is a physical phenomenon in which all the light travelling in an optically denser medium is reflected back upon striking the interface with an optically less dense medium.

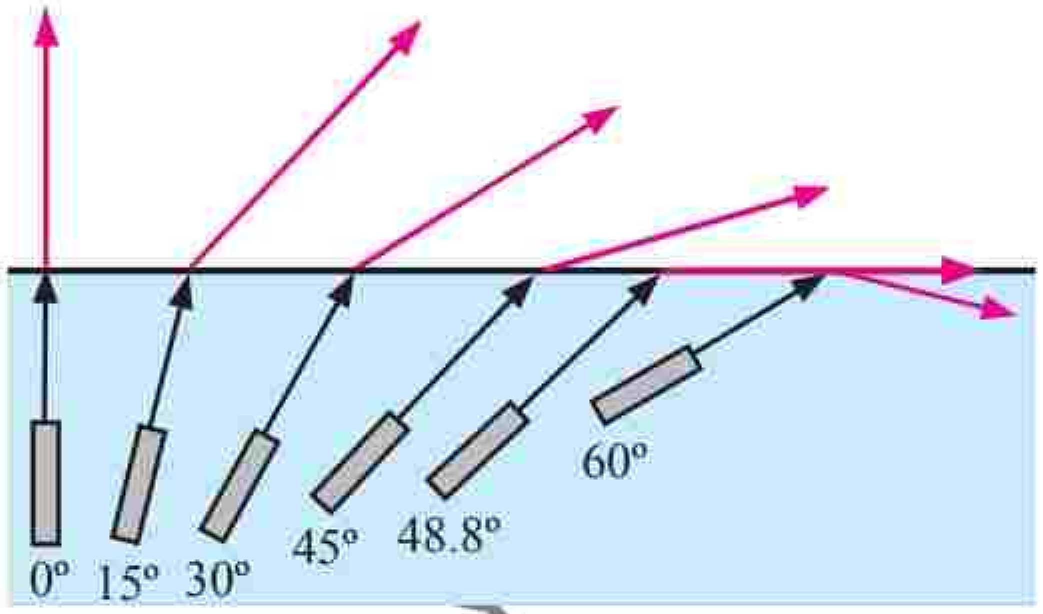


Figure 3.30: *Occurance of the total internal reflection*

Total internal reflection (TIR) is the phenomenon that involves the reflection of all the incident light off the boundary. It only takes place when both of the following two conditions are met:

1. Light travels from an optically denser medium towards a less dense medium.
2. The angle of incidence is greater than the critical angle.

Total internal reflection will not take place unless the incident light is travelling within the more optically dense medium towards the less optically dense medium. TIR will happen for light travelling from water towards air, but it will not happen for light travelling from air towards water. On the other hand, TIR occurs only if the angle of refraction exceeds 90°. When the angle of incidence equals the critical angle, the angle of refraction is 90° (Figure 3.31 (a)). Also, when the angle of incidence is greater than the critical angle, all the light undergoes total internal reflection as shown in Figure 3.31 (b).

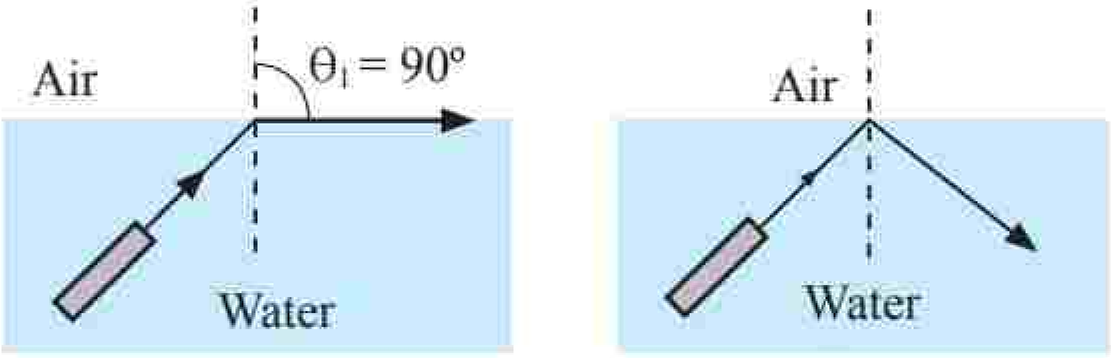


Figure 3.31: *Critical angle and total internal reflection*

Example 3.8

What must be the angle of incidence for total internal reflection to occur when a ray travels from glass to water? Use $\eta_g = 1.52$ and $\eta_w = 1.33$.

Solution

For total internal reflection, $(r) = 90°$

From

$$\eta_1 sin(i) = \eta_2 sin(r)$$

$$\eta_g sin(i) = \eta_w sin(r)$$

$$i = sin^{-1}\left(\frac{\eta_w \sin(90°)}{\eta_g}\right)$$

$$= sin^{-1}\left(\frac{1.33 \times 1}{1.52}\right) = 61.04°$$

Therefore, the critical angle is $61.04°$.

For total internal reflection to occur the angle of incidence must be greater than the critical angle. That is, $i > 61.04°$.

Mirages

In the previous discussion, it has been established that, light tends to bend when it strikes an interface between two media that have different refractive indices. After passing the boundary, light continues to travel in a straight line within the new medium. However, the discussion has clearly presumed that, the media in which light travels are uniform media. This means that, the optical density of each medium is uniform everywhere within the medium. Nevertheless, not every medium is uniform and even air can sometimes form a non-uniform medium. When the medium is optically non-uniform, light refraction can occur as the light travels within the medium. This leads to an interesting phenomenon; the formation of mirages. A mirage is an optical phenomenon that produces illusion images due to the refraction of light through a non-uniform medium.

Mirages can commonly be observed when you walk down a tarmac roadway on a sunny day. As you walk down the roadway, there appears to be a puddle of water on the road several metres (about 100 m) ahead of you. Yet, when you arrive at the perceived location of the puddle, you recognize that the puddle is not there. Instead, the puddle of water appears to be another one-hundred metre in front of you. The appearance of the puddle of water is simply an illusion because of mirage.

Mirages occur on sunny days because the sun heats the roadway to high temperatures. The heated roadway in turn heats the surrounding air, keeping the air just above the roadway at a higher temperature than the air above. Hot air tends to be optically less dense than cooler air. Thus, the layers of air just above the roadway are optically less dense than the air further above the roadway. This creates a non-uniform optical media. Consequently, as light travels from the sun, it passes through the cooler air layers towards the hot air layers. This means, light travels from an optically denser medium to a less dense medium. Thus, light will be refracted upon striking the boundary between the two layers of air. Therefore, light bends upwards. As the light continues to traverse different air layers, it is bent more upwards to an extent that it reaches your eyes instead of hitting the road. Your brain traces the light straight back along the direction from which it comes to your eyes. Therefore, you see the image of the sky that appears as a puddle of water just above the roadway as shown by Figure 3.32. Also, the mirage phenomenon can occur on the waterbodies such as seas, lakes, and ponds, and in the desert.

Figure 3.32: *Road mirage*

Task 3.3

Organize a walk with your friends down the roadway (if any) on a sunny day and try to observe the occurrence of mirage. Discuss the condition necessary for the occurrence of mirage.

Exercise 3.2

1. A coin is at the bottom of a trough containing three immiscible liquids of refractive indices 1.3, 1.4 and 1.5, poured one above the other at the heights 30 cm, 16 cm, and 20 cm respectively. What is the apparent depth at which the coin appears to be when observed from air medium outside? In which medium will the coin be seen?
2. Light is incident on an air-water interface at an angle of 40°. Given that, the refractive index of water is 1.33, determine the angle of refraction of the light in the water.
3. A stone is lying at the bottom of a pool of water 3 m deep. What would be the stone's dept as seen by an observer standing new the pool? Use $\eta_w = 1.30$.
4. The speed of light in water and in air is 2.8×10^8 m/s and 3.0×10^8 m/s, respectively.
   (a) Determine the refractive index from air to water.
   (b) A ray of light travelling from air to water is incident at the surface of water at an angle of 30°. Calculate the angle of refraction in the water.
5. A swimming pool appears to be 1.5 m deep. If the refractive index of water is 1.3, determine the real depth of the pool.

6. Diamond has a refractive index of 2.42. Given that, the speed of light in vacuum is 3.0×10^8 m/s, determine:
   (a) the speed of light in diamond.
   (b) the critical angle for diamond.
7. The critical angle for a beam of light travelling between water and air is 49°.
   (a) A beam strikes a water or air boundary and undergoes total internal reflection. Will the beam stay in the air or in the water?
   (b) Explain what happens when a beam of light from the air strikes the surface of a calm lake at an angle of 50° from the normal.

Refraction of light by a glass prism

A prism is made up of a glass block with two incident faces. When white light passes through a prism whose incident faces are not parallel, it is deviated from its original path and dispersed into a coloured spectrum. When light enters a prism with parallel faces, it is refracted at the entrance boundary and then continues to travel in a straight line. The refraction of light by a prism, and the measurement of the refractive index of its material can be achieved by the passage of a ray of light as shown in (Figure 3.33).

Refraction of light by a rectangular glass prism

Activity 3.8 introduced you to the refraction of light by a rectangular glass block, which is geometrically the same as a rectangular glass prism. Through this activity, you learnt how to trace the passage of a light ray as it travels from air to a glass prism. It was generally observed that, when light enters a glass block (rectangular glass prism) it is refracted at the entrance boundary and then continues to travel in a straight line. It is important to note that, light emerges on the other side of the rectangular prism and continues to travel in air following the same direction that it had before entering the rectangular glass prism as shown in Figure 3.33. This is possible because the sides of the rectangular glass prism are parallel to each other. Therefore, there is no deviation in the path of light.

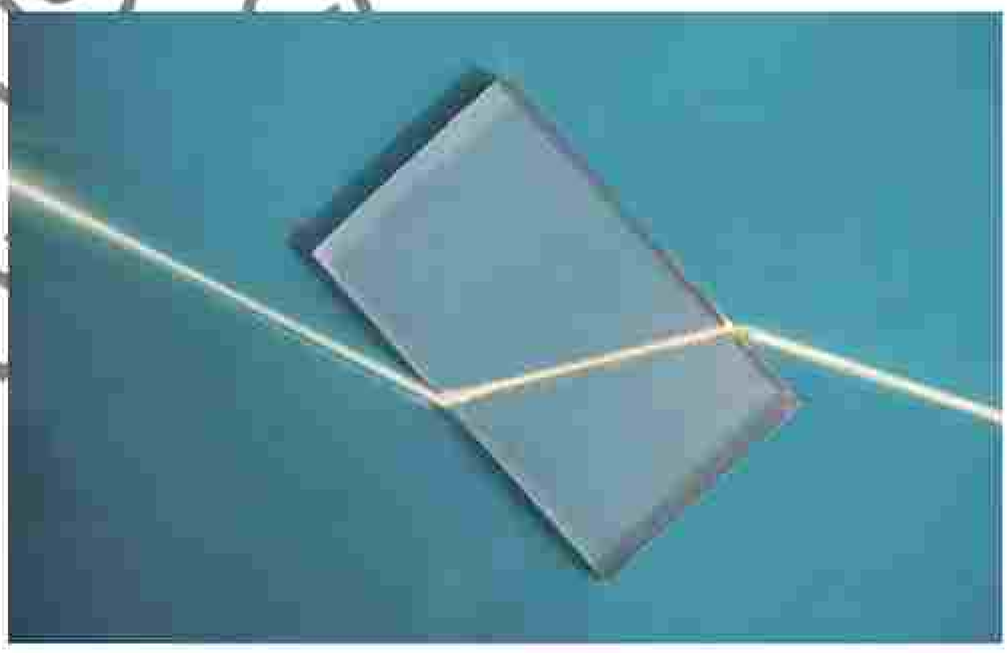


Figure 3.33: *Light refraction by a rectangular glass block*

What happens to the path of a light ray when the two opposite sides of a prism are not parallel? To answer this question, one needs to observe the path of light through a triangular glass prism.

Refraction of light by a triangular glass prism

A triangular glass prism is a transparent object having two triangular and three rectangular faces. The refraction of light by a triangular glass prism is different from the refraction by a rectangular glass prism. For triangular glass prism, the emergent ray of light is not parallel to the incident ray of light. This is because, when a ray of light enters the glass prism, it gets deviated two times. First, light is refracted when it enters the glass prism and then refracted for the second time when it comes out of the prism as shown in Figure 3.34. This is possible because the refracting surfaces of the prism are not parallel to each other, therefore when the ray of light passes through the prism it bends towards the normal which is towards its base. The amount by which light bends is dependent on the angle of incidence, wavelength of the light, and refractive indices of the materials at which light travels through.

Angle of deviation

The angle of deviation is a measure of how much the incident ray has been deflected from its original direction by the prism. Consider a ray of light incident on a glass prism as shown in the Figure 3.34.

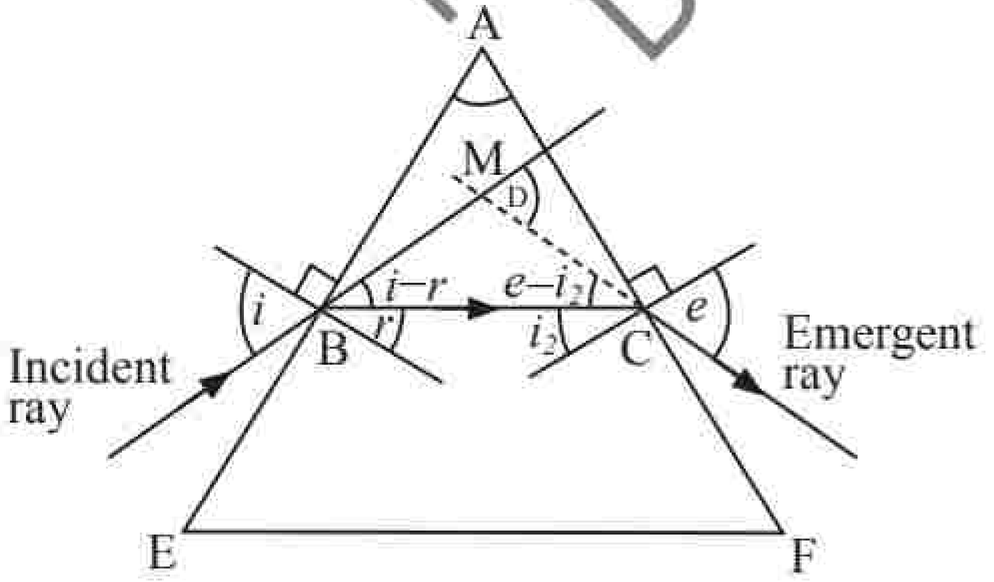


Figure 3.34: *Deduction of the angle of deviation*

where $B\hat{A}C =$ angle of prism or apex angle

$i =$ angle of incidence

$r =$ Angle of refraction at the first surface

$i_2 =$ Angle of incident at the second surface

$e =$ Angle of emergence from the prism

$D =$ Angle of deviation. This is the angle between the initial incident direction and the final emergent direction.

Consider triangle ABC in Figure 3.34.

The sum of internal angles is 180°

$$A\hat{B}C + B\hat{A}C + A\hat{C}B = 180°$$

$$90° - r + A + 90° - i_2 = 180°$$

$$180° + A - r - i_2 = 180°$$

$$A = r + i_2 \text{ ....} \quad \text{(i)}$$

Now consider triangle MBC

$$M\hat{B}C + M\hat{C}B + B\hat{M}C = 180°$$

$$i - r + e - i_2 + 180° - D = 180°$$

$$i - r + e - i_2 = D \text{ ....} \quad \text{(ii)}$$

r and e can be calculated by Snell's law.

Consider the first surface, by Snell's law

$$\eta = \frac{\sin i}{\sin r}$$

In order to determine the angle of deviation D, we must consider very small angles, that is, $\sin i \approx i$ and $\sin r \approx r$.

Now $\frac{\sin i}{\sin r} = \frac{i}{r} = \eta$

Then, $i = \eta r$ .... (iii)

Consider the second surface, by Snell's law

$$\eta = \frac{\sin e}{\sin i_2}$$

This implies that,

$$\eta = \frac{e}{i_2} \text{ and } e = \eta i_2 \ldots\ldots\ldots (iv)$$

Substitute equation (iii) and (iv) in equation (ii) we have,

$$\eta r - r + \eta i_2 - i_2 = D$$

$$r(\eta - 1) + i_2(\eta - 1) = D$$

Simplifying this equation we have,

$$D = (r + i_2) \times (\eta - 1) \ldots\ldots\ldots\ldots (v)$$

Combining equation (i) and (v) we finally have, $D = A \times (\eta - 1)$.

Therefore, the angle of deviation is given by $D = A \times (\eta - 1)$.

Activity 3.11

Aim: To trace the path of the rays of light through a triangular glass prism

Materials: a white sheet of paper, soft board, thumb pins, triangular prism, pencil, protractor, drawing board

Procedure

1. Fix a white sheet of paper on a drawing board using drawing pins.
2. Place the triangular prism resting on its triangular base on the paper. Using a pencil, draw the outline of the prism.
3. Draw a line NEN′ perpendicular to the face of the prism AB (Figure 3.35). This is the normal at point E.
4. Draw a line PE such that it makes an angle between 30° and 60° with the normal NEN′. This is an incident light ray.
5. On the line PE, fix two pins at a distance of 5 cm from each other and mark these as points P and Q. Replace the prism back on the outline.

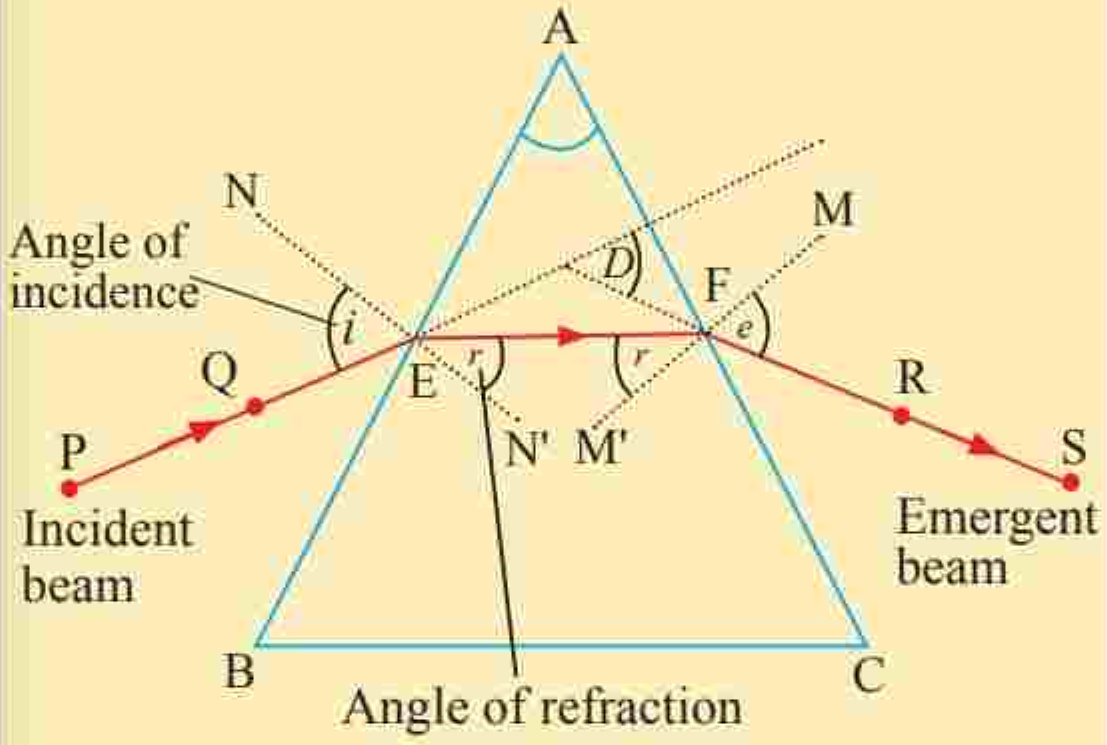


Figure 3.35

6. Look for the images of the pins at P and Q through the AC face of the prism.
7. Fix two pins at R and S such that they appear as a straight line as that of the P and Q when it is viewed from the AC face of the prism.
8. Remove the pins and the prism.
9. Join and produce a line joining points R and S, let this line meet the prism at point F.
10. Extend the direction of the incident ray PQE till it meets the face AC. Also, extend (backwards) the emergent ray SRF so that these two lines meet at a point D.

11. Mark the angle of incidence (i), the angle of refraction (r), the angle of emergence (e) and the angle of deviation (D).
    (a) What happens to the incident ray when enters the prism?
    (b) What are the factors on which the angle of deviation through a prism depends?

The incident ray bends towards the normal when it enters the prism and bends away from the normal when it exits the prism. Moreover, the angle of deviation decreases with the increase in the angle of incidence.

Angle of minimum deviation of a prism

The value of angle of deviation D depends on the angle of incidence, i. As the angle of incidence increases, the angle of deviation decreases to a minimum value called angle of minimum deviation (D_m), and then starts to increase again as shown in Figure 3.36.

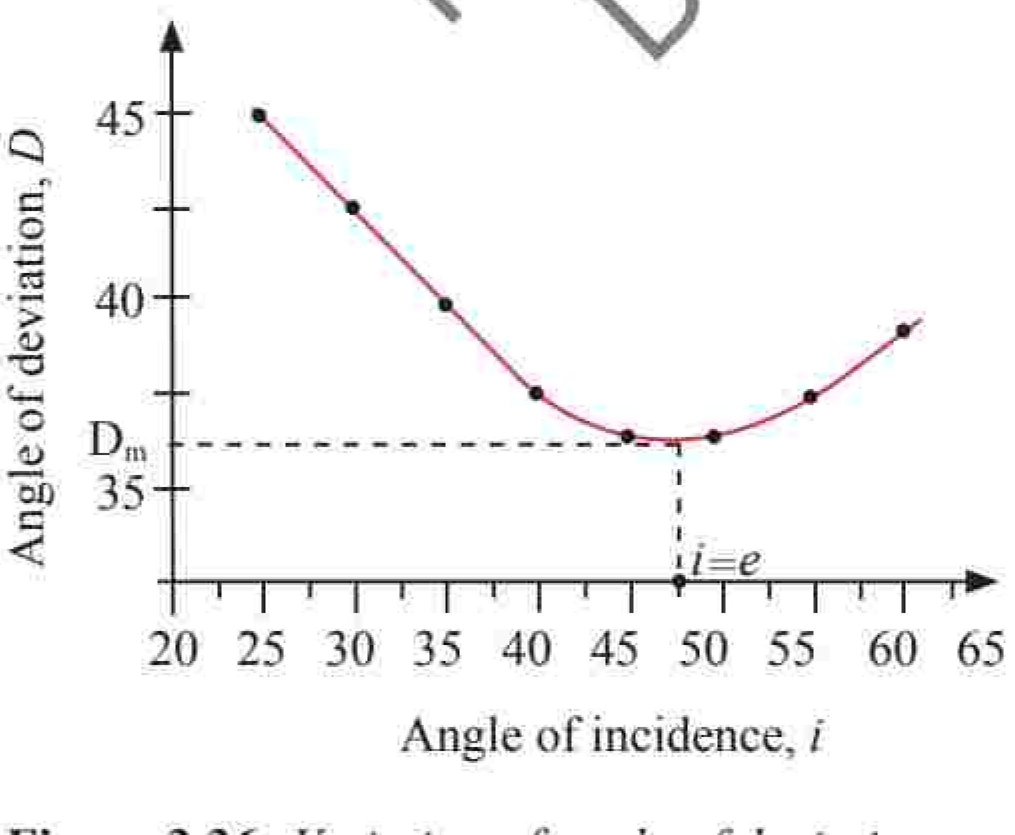


Figure 3.36: *Variation of angle of deviation with the angle of incidence*

At the angle of minimum deviation, the angle of incidence and the angle of emergence are equal. That is, $i = e$. At the angle of minimum deviation, the refracted ray from the first surface travels through the prism perpendicular to the bisector of the apex angle A as shown in the Figure 3.37.

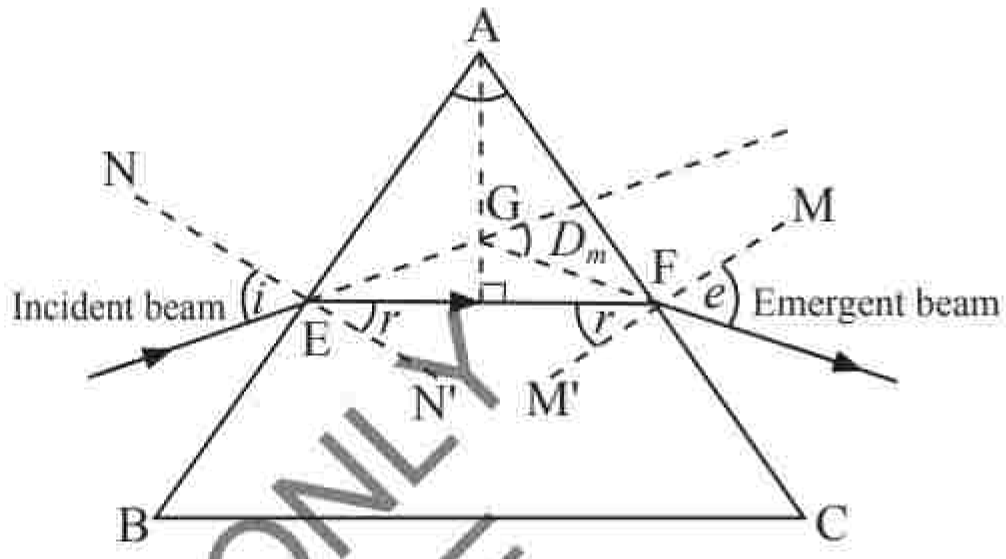


Figure 3.37: *Deducing the angle of minimum deviation*

Consider triangle AEF,

$$A\hat{E}F + E\hat{A}F + E\hat{F}A = 180°$$

$$90° - r + A + 90° - i_2 = 180° \quad \ldots\ldots\ldots\ldots \text{(vi)}$$

But we know that at minimum angle of deviation, $i_2 = r$, then equation (vi) can be written as

$$90° - r + A + 90° - r = 180°$$

$$A = r + r = 2r, \text{ implying that } r = \frac{A}{2}$$

Now consider triangle GEF,

$$G\hat{E}F + G\hat{F}E + E\hat{G}F = 180°$$

$$i - r + e - r + 180° - D_m = 180°$$

But, $i = e$

By simplifying we have,

$$D_m = i - r + i - r = 2i - 2r.$$

This becomes $D_m = 2i - A$, as $r = \frac{A}{2}$.

Therefore:

$$i = \frac{A + D_m}{2} \quad \text{............(vii)}$$

From Snell's law

$$\eta = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Activity 3.12

Aim: To determine the angle of minimum deviation of an equilateral triangular glass prism

Materials: a white sheet of paper, a drawing board, an equilateral triangular glass prism, drawing pins, a metre rule, pencil, office pins, graph paper, protractor

Procedure

1. Place the white sheet of paper on the drawing board and fix it with the help of drawing pins.
2. Place an equilateral glass prism on top of the paper and trace its outline.
3. Remove the prism and label the outline as ABC.
4. On the side AC just above the centre of the outline, draw a perpendicular line NQ.
5. Measure the angle of incidence of 25° at the point of the perpendicular line and draw a line RQ.
6. Insert two pins P_1 and P_2 on the line RQ and replace the glass prism.
7. On the other side of a prism, trace the pins P_3 and P_4 which appear to be in line with P_1 and P_2, and draw line TS as shown in Figure 3.38.

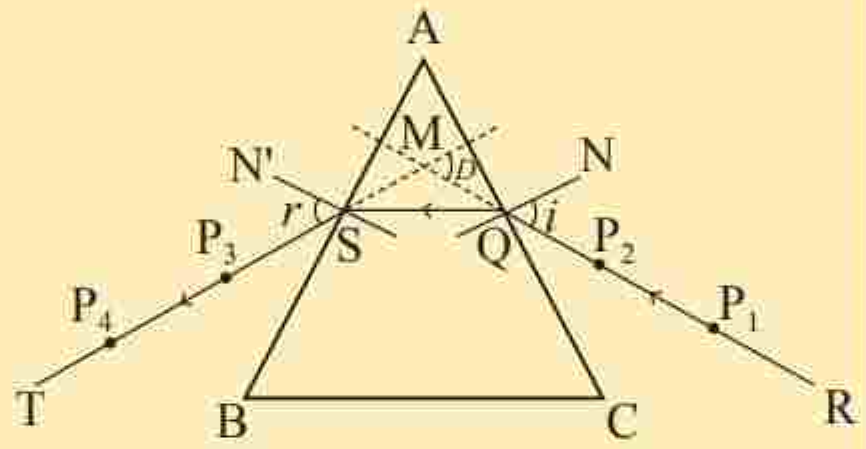


Figure 3.38

8. Extend line RQ and line TS with dotted lines to meet at a point M.
9. Measure the angle D.
10. Repeat steps 5 to 8 for other angles of incidence of 30°, 35°, 40°, 45°, 50°, 55° and 60°.
11. Record your results in a table similar to Table 3.6.
12. Plot a graph of the angle of deviation (y-axis) against the angle of incidence (x-axis) and use the graph to deduce the angle of minimum deviation, D_m. This corresponds to the lowest point on the graph.

PHYSICS FORM III.indd 83 09/03/2022 18:14:21

Table 3.6

Prism angle, A	
Angle of incidence (i)	Angle of deviation (D)
25°	
30°	
35°	
40°	
45°	
50°	
55°	
60°	

Questions

(a) When light passes through a triangular glass prism, what happens to the angle of deviation when the angle of incidence changes?

(b) Does the angle of minimum deviation (D_m) change when a different prism is used?

The angle of deviation first decreases to attain the minimum value of D_m and then increases as the angle of incidence increases. The angle of minimum deviation is important for calculating the refractive index of the glass, using the equation:

$$\eta = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Exercise

1. Define the angle of minimum deviation produced by a triangular glass prism. How is the angle of minimum deviation related to the angle of incidence and angle of prism?

2. There is no dispersion of light by a rectangular glass block. Why?

3. Light travelling through transparent oil enters into glass of refractive index 1.5. If the refractive index of glass with respect to the oil is 1.25, what is the refractive index of the oil?

4. The refractive index of the material of a prism with 60^{o} angle for yellow light is $\sqrt{2}$. At the position of minimum deviation, calculate the angle of:
   (a) minimum deviation;
   (b) incidence; and
   (c) refraction.

5. A monochromatic light is incident on an equilateral glass prism at an angle of 30º and emerges at an angle of 75º. What is the angle of deviation produced by the prism?

6. A light ray falls at normal incidence on the first face of an equilateral prism such that, the angle of incidence is 45^{o}, and emerges from the second face by making the same angle with the normal. What is the refractive index of the material of the prism?

7. The angle of minimum deviation for a prism is 37º. If the angle of the prism is 60º, find the refractive index of the material of the prism.

8. A rectangular glass block of thickness 10 cm and refractive index 1.5 is placed over a small coin. A beaker filled with water of refraction index $^4/_3$ to the height of 10 cm is placed over the block.

(a) Find the apparent position of the coin when viewed normally.

(b) If the eye is slowly moved away from the normal, then at a certain position the object is found to disappear due to total internal reflection. At what surface does it happen and why?

Colours of light

The sun is the major source of light for the earth. What is the colour of sunlight? Sometimes we see in the sky a rainbow consisting of different colours. In addition, during sunset or sunrise, the sky appears orange or red. Can we conclude from these observations that sunlight is made of different colours?

Dispersion of white light

White light is known to consist of seven colours. If a beam of white light is passed through a triangular glass prism, it splits into a band of seven colours. Figure 3.39 shows a band of colours formed when white light passes through a triangular glass prism. This band is known as the spectrum of the white light. Colours that form the spectrum of white light are red (R), orange (O), yellow (Y), green (G), blue (B), indigo (I) and violet (V). For the sake of remembering the arrangement of these colours, an abbreviation ROYGBIV is commonly used.

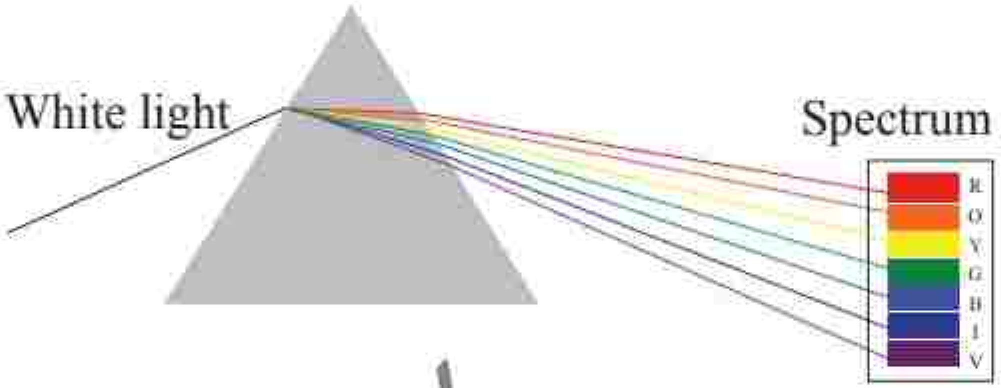


Figure 3.39: *Dispersion of white light by a triangular glass prism*

Dispersion of light is a phenomenon in which white light splits into its seven colour components upon passing through a triangular glass prism. This phenomenon happens because when a beam of white light enters a transparent medium, each colour component is refracted at a different angle of refraction, resulting in different angles of deviation for each colour. For example, red colour deviates least while violet colour deviates most. The result is the splitting of the white light to its components to form a spectrum with red colour being at the upper part of the spectrum and violet colour is at the bottom of the spectrum as shown in Figure 3.39. Each component has a different wavelength, which falls well within the visible part of the electromagnetic spectrum.

The rainbow

Light dispersion can also occur when light passes through some other transparent materials such as water droplet and soap bubbles. One of the consequences of light dispersion by water droplets in the atmosphere is the formation of rainbow. Rainbow is a natural phenomenon of dispersion of white light. It is caused by refraction and reflection of white light by falling water droplets.

Formation of rainbow

The rainbow phenomenon can be observed when there are water droplets in the atmosphere as the sun shines from behind the observer at low altitude. The rainbow's appearance is caused by dispersion of sunlight as it is refracted by nearly circular rain drops. The white light from the sun is first refracted as it enters the surface of the rain drops.

When light travels from less denser medium to light denser medium bends towards the normal. However, not all colour bend equally. As in glass prism, red light bends the least while violet bends the most. At the back of the rain drops, some of dispersed light is reflected back over a wide range of angles and again refracted as it leaves the rain drop. In most cases, the violet light leaves the rain drop at an angle of 40° relative to incident sunlight whereas the red light leaves at 42° as shown in Figure 3.40 (a). As a result, the observer sees an image of well-arranged coloured bands in the atmosphere called rainbow as shown in Figure 3.40 (b).

(a) (b)

Figure 3.40: *Formation of a rainbow*

Spraying water in the direction of sunlight produces water drops that can result to the formation of a rainbow.

Activity 3.13

Aim: To observe the formation of a rainbow by spraying water in air

Materials: water, a hose that can spray water, a sprinkler or a fountain

Procedure

1. Bring the water hose to an open space.
2. Open the water tap so that the hose sprays water into the air.
3. Stand near the spray of water drops with your back directed towards the sun.
4. Move around until you locate your shadow.
5. Look for a rainbow in the spray of water.

Note that, if you do not see a rainbow, try the following steps: look for the shadow of your head. Hold both arms straight out in front of you. Spread your hands as wide as they will stretch with your thumbs touching, head-to-head. Place the head of one little finger so that its shadow falls in the centre of the shadow of your head. Keeping that finger in place, look at the sunlit drops that line up with your other little finger. You should see rainbow right there.

Questions

(a) Why does the sunlight shining through water drops make a rainbow?

(b) Why do you have to stand with your back directed towards the sun to see a rainbow?

(c) Can a person standing beside you also see the rainbow?

When you look at a rainbow, you see seven colours. They are always in the same order. That is red, orange, yellow, green blue, indigo, and violet. An easy way to remember the colours and the order is to think of the name ROYGBIV, spelled from the first letter of each colour.

Recombining colours of white light

We have seen that upon falling on a transparent medium such as a triangular glass prism or water droplet, white light splits into its colour components as shown in Figure 3.39. What happens when the components of white light are passed through a second prism that is identical to the first prism?

If the second prism is arranged similarly to the first prism, no further splitting of white light can be observed. But, if the second prism is inverted with respect to the first prism, a beam of white light is observed to emerge from the other side of the second prism. This means that, the dispersed colours of white light can be recombined to form white light, as illustrated by Figure 3.41. This phenomenon is known as the white colour recombination.

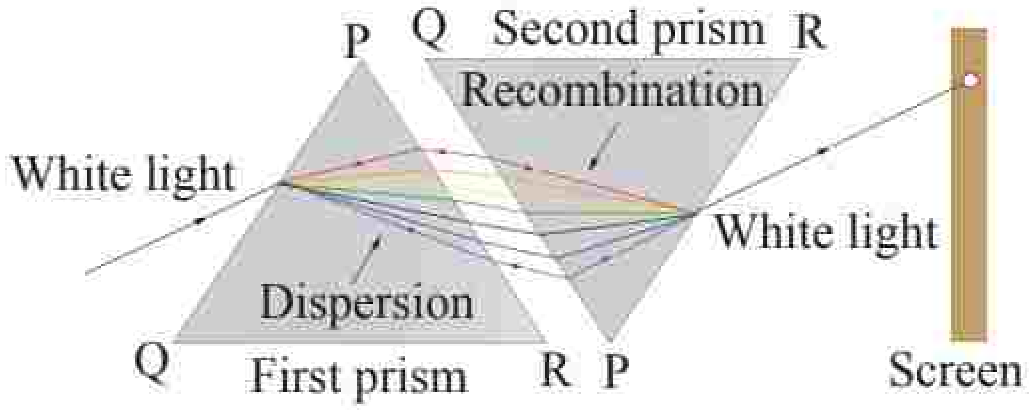


Figure 3.41: *Dispersion and recombination of white light*

The experiment of recombination of the spectrum of light can be extended to observe the recombination of two or more individual colours. This was for the first time done by Sir Isaac Newton using prisms and mirrors. He discovered that when the light from the red, green, and blue regions of the spectrum are recombined, they regenerate white light. He therefore called red, green and blue, the primary colours.

Newton also observed that, when any two of the primary colours are combined, other colours are formed. For example, when blue and green lights are combined, cyan light is formed. Furthermore, green and red lights combine to give yellow light while red and blue lights combine to form magenta light. Newton named the colours that resulted from the recombination of two primary colours as secondary colours. Secondary colours include yellow, cyan and magenta. He finally organized his findings in a colour wheel showing the three "primary colours" separated by the three "secondary colours" as shown in Figure 3.42. This colour wheel is famously known as the Newton colour wheel or disc. When the disc is rotated about its axis at high speed, all the colours blend to form a white colour. However, as the disc slows down, individual colours are seen again.

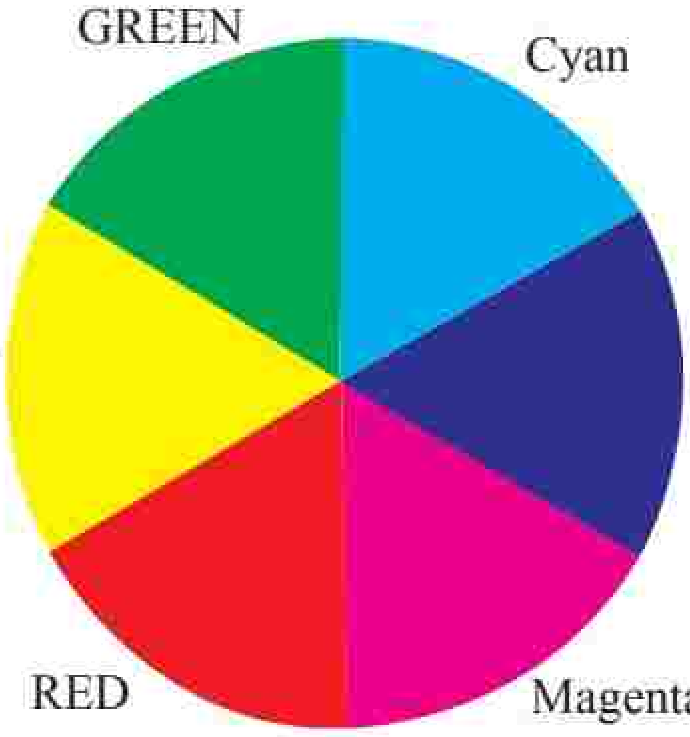


Figure 3.42: *A Newton's colour wheel*

Since magenta was not a part of the light spectrum, its origin posed a mystery. This mystery was resolved by Hermann von Helmholtz, who established that the human eye consists of three types of colour receptors. One of the primary colours (red, blue and green) can stimulate only one type of a receptor. Moreover, the eye perceives all of the variations in colour by internally combining the signals from these receptors. Therefore, when red, blue and green light enter the eye, white light is seen. However, when both red and blue light but no green light enters the eye, magenta is seen even though the light is not magenta. Similarly, when red and green light enters the eye, yellow light is seen and when blue and green light enter the eye, a cyan light is seen.

Task 3.4

Use a white board to make a Newton's colour wheel and observe the colours as the wheel spins at different speeds.

Appearance of objects under white light

When one looks at an object, the object is seen when it reflects light into the eye of the observer. Objects may absorb certain colours of the light falling on them and reflect other colours. Therefore, the colour appearance of objects depends on the colour of the light that the object reflects or absorbs and the colour of the light falling on the object. The selective absorption or reflection of light by an object gives the object its characteristic colour as perceived by an observer.

Coloured objects under white light

When white light falls on an object and the object reflects the entire colour spectrum in the light, the object appears white. Conversely, the object appears black if it absorbs all colours of the white light falling on it. If the object reflects some of the colours and absorbs the others, the object appears coloured. For example, some flowers in Figure 3.43 appear yellow because they absorb all other colours except the yellow colour, which is reflected into the observer's eyes. Other flowers also absorb all the colours of white light except their respective colours, as seen in the Figure 3.43.

Figure 3.43: *Coloured flowers*

White objects under coloured light

An object is seen white because it reflects all colours of the white light falling on it. Therefore, when a white object is viewed under the coloured light, it will reflect the coloured light and hence appear to have the colour of the light. For example, when a white object is viewed under the blue light, it appears as a blue object. Similarly, an object will appear red or green when viwed under red or green light respectively.

Activity 3.14

Aim: To investigate the colours of a white object under coloured lights

Materials: colour filters (red, yellow, green and blue), a white object

Procedure

1. Take filters of different colours (red, yellow, green and blue).
2. Let the light fall on the white object through a colour filter.
3. View the white object under the filtered light.
4. Observe the appearance of the white object under the filtered light and record your observation.

Questions

(a) Describe the appearance of the white object as seen through each light filter.

(b) What happens when light from the blue filter is passed through a green filter?

When white light is transmitted through a colour filter, the filter allows only a specific light colour to pass through it. Therefore, the filtered light will have a colour depending on the colour of the filter. For example, the light will be yellow, green, blue or red if the filter used is respectivelly yellow, green, blue or red. Consequently, if light from one filter is passed through another filter of different colour, the light is absorbed by the second filter. Now, when the light through a filter of a given colour falls on the white object, the object reflects that light into the observer's sight. Therefore, the object appears to have the colour of the filtered light. Figure 3.44 shows the appearance of a white object under different coloured lights.

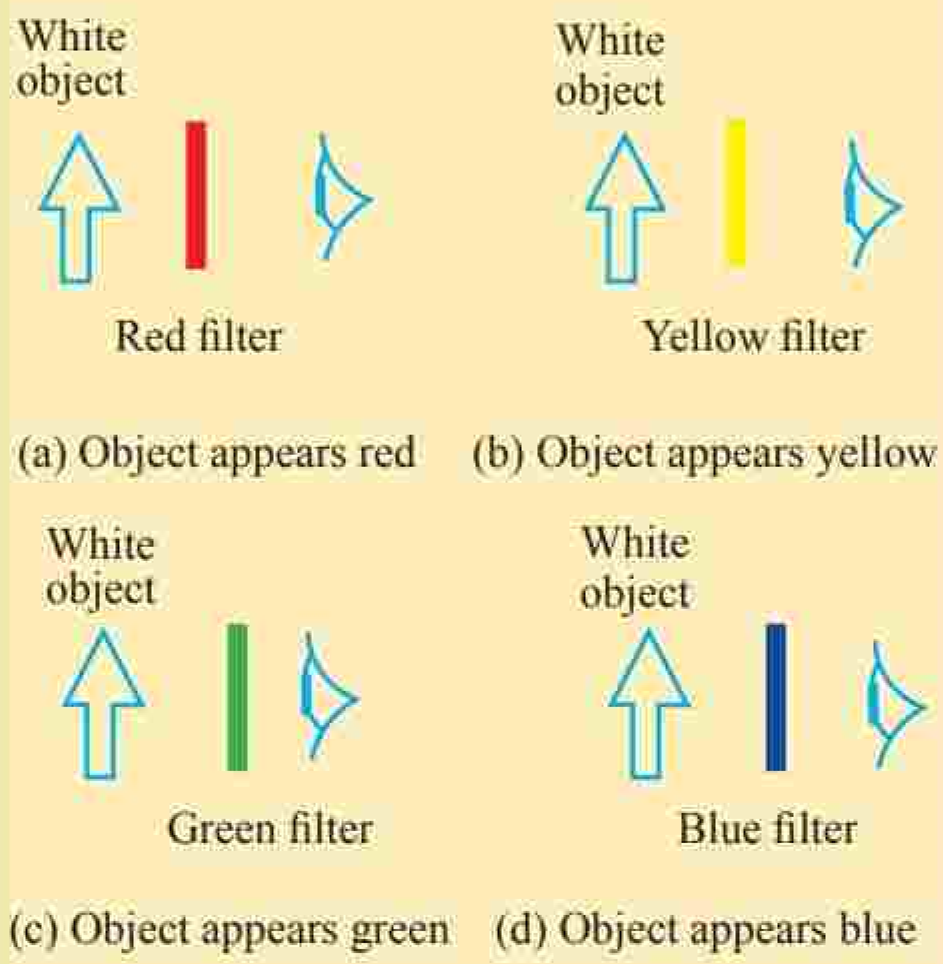


Figure 3.44: *Appearance of a white object under coloured lights*

Primary, secondary and complementary colours of light

What happens when a physicist simultaneously casts the blue, green and red light beams on a spot on a white screen?

Experiments with beams of different coloured lights have shown that, most colours of the light spectrum can be described in terms of the combination of other light colours. For example, when light from one end of the light spectrum is combined with light from the middle region of the spectrum in various proportions, all colours in the half of the spectrum can be obtained. Similarly, when lights from the opposite ends of the light spectrum are combined with light from the middle region, all colours in the half of the spectrum that lies in between them can be generated.

White light can also be produced by combining only three distinct colours of light, provided that they are widely separated on the visible light spectrum. Any three colours of light that produce white light when combined with the correct intensity are called primary colours. These include red (R), blue (B) and green (G) as shown in Figure 3.45. Moreover, mixing together of two or all of these three primary colours of light with varying degrees of intensity can produce a wide range of other light colours. Many television sets and computer monitors produce a wide range of colours on the monitor by using red, green and blue light-emitting phosphors.

Red	Blue	Green

Figure 3.45: *Primary light colours*

Light colours that are formed by the combination of the primary colours are referred to as secondary colours. These include cyan, magenta and yellow as shown in Figure 3.46.

Cyan	Magenta	Yellow

Figure 3.46: *Secondary light colours*

On the other hand, white light can sometimes be formed by mixing any two colours of the light spectrum. Therefore, the two colours that mix in definite ratios to form white light are termed as complementary colours. Examples of complementary colours are blue and yellow, red and cyan as well as green and magenta.

Activity 3.15

Aim: To investigate the mixing of primary colours of light

Materials: three torches, light colour filters, a white screen, dark room

Procedure

1. Cover the opening of the first torch with a green filter, the second with a blue filter and the third with red filter.
2. Switch on the torches and ensure that the first torch gives green light, the second gives blue light and the third gives red light.

3. Direct the torches to the white screen such that circles of green, blue and red light can be seen on the screen.
4. Slowly move the torches such that the blue and red circle intersect by half of their size and observe the image on the screen. Record your observation.
5. Repeat step 4 for different combinations of light circles on the screen.
6. Now, combine all the three circles such that there is a region of intersection for all three circles.

   Observe the image on the screen.

Questions

(a) What is the resulting colour in each case?

(b) What is the colour at the region of intersection when all the three colour circles are combined?

(c) What do your observations mean?

Different combinations of primary colours of light produce different secondary colours. For example, green and blue lights combine to produce cyan, which is the complement of red. Green and red lights combine to produce yellow, which is the complement of blue, while blue and red combine to produce magenta, which is the complement of green. Thus, two primary colours combine to produce the complement of the third primary colour, whereas the three primary colours combine to produce white light. One can easily remember the combinations of primary colours of light by observing the circles in Figure 3.47.

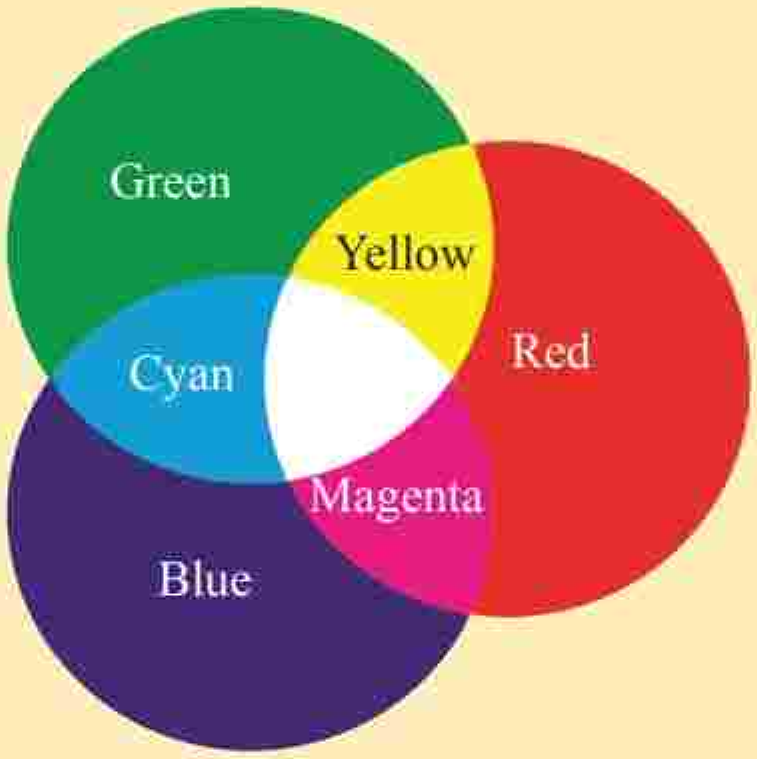


Figure 3.47: *Mixing primary colours*

Addition and subtraction of light colours

When you look at an object and perceive a distinct colour, you are not necessarily seeing a single colour of light rather a combination of different colours. Consider for instance, that you are looking at a shirt that appears yellow. There may be several colours of light with varying intensity that strike your eye after being reflected by the shirt. Yet your eye-brain system interprets the colours that strike your eye and the shirt is perceived to be yellow. Understanding how different colours are perceived requires the knowledge of adding and subtracting light colours.

Addition of light colours

The production of various colours of light by the mixing of the three primary colours of light is known as colour addition. The colour addition principles can be used to make predictions of the colours that would result when different coloured lights are mixed. For example, red (R) light and green (G) light add together to produce yellow (Y) light. Red light and blue light add together to produce magenta (M) light. Green light and blue (B) light add together to produce cyan (C) light. Finally, red light, green light and blue light add together to produce white light. Moreover, addition of complementary colours of light produces white light. The addition of light colours can be described by the following equations;

$$R + G = Y$$

$$R + B = M$$

$$G + B = C$$

$$R + C = R + (G + B) = W$$

$$B + Y = B + (G + R) = W$$

$$G + M = G + (R + B) = W$$

Note that, the diagram in Figure 3.48 can conveniently summarize the colour addition equations.

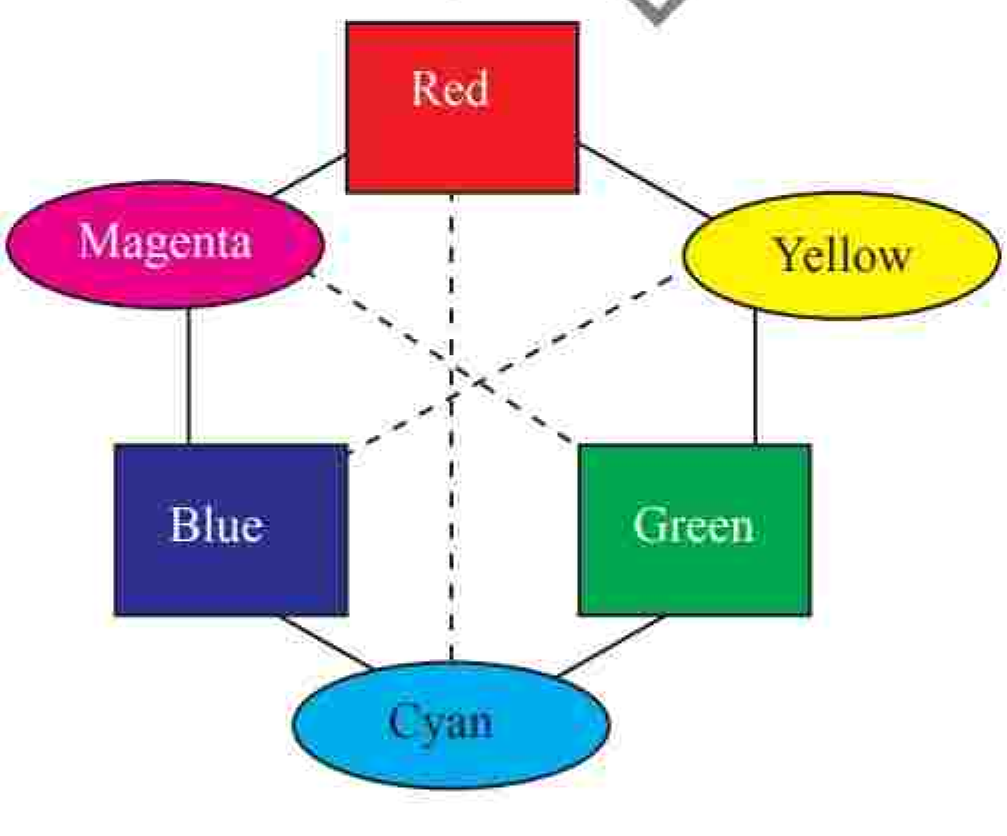


Figure 3.48: *Addition of light colours*

Subtraction of light colours

The principles of colour addition govern the perceived colour resulting from the mixing of different colours of light. However, understanding the colour perceptions cannot be complete without an understanding of the principles of colour subtraction. Some materials contain atoms and molecules that are capable of selectively absorbing one or more colours of light. Therefore, if a beam of white light falls on such a material, some light colours are absorbed and some are reflected to the observer's eye. This is the basis of subtraction of light colours. Figure 3.49 illustrates light colour subtraction. Yellow filter absorbs blue light, magenta filter absorbs green light, and cyan filter absorbs red light.

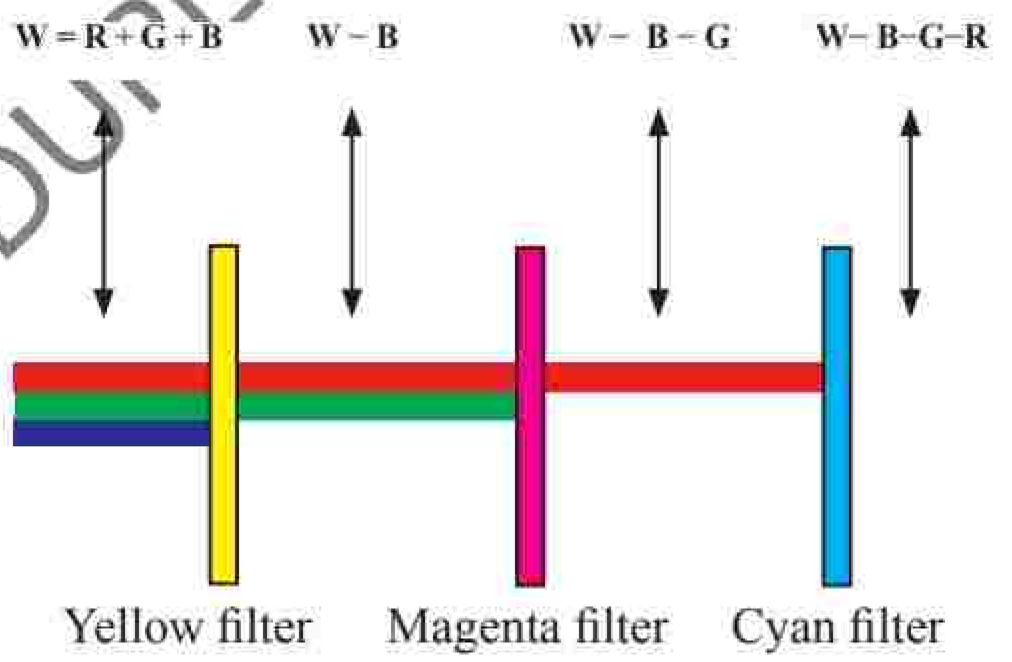


Figure 3.49: *Subtraction of light colours*

The process of colour subtraction

Consider a piece of cloth that is made of materials capable of absorbing blue light. Such a cloth will absorb blue light and reflect the other colours. What appearance will such a cloth have if illuminated with white light and how can we account for its appearance?

Suppose that a beam of white light made

of blue, green and red colours falls on a shirt. If the shirt absorbs blue light, it will then reflect only red and green light. Therefore, a combination of red and green light will strike the observer's eye. Since $R+G=Y$, the shirt will appear to be yellow in colour. This example illustrates the process of colour subtraction. In this process, the ultimate colour appearance of an object is determined by beginning with a single colour or mixture of colours and identifying which colour or colours of light are subtracted from the original set. The process can be depicted visually by the diagram in Figure 3.50.

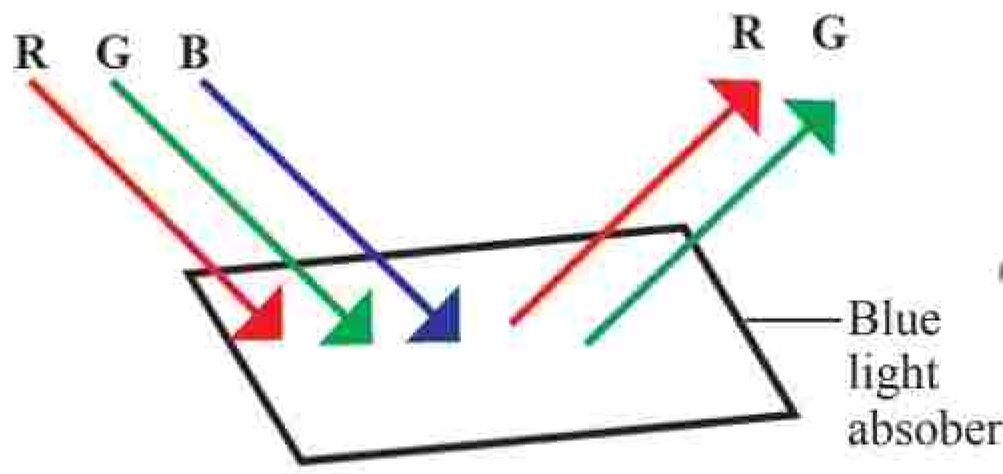


Figure 3.50: *Process of subtraction of light colours*

The colour subtraction process illustrated by Figure 3.50 can alternatively be depicted using the colour subtraction equation:

$$W-B=\left(R+G+B\right)-B=R+G=Y.$$

Suppose that the same piece of cloth is illuminated by a cyan light. Since cyan consists of blue and green light, and the cloth is blue, then a blue light is subtracted. This means that the cloth will appear to be green in colour. This process is represented by the following equation:

$$C-B=(G+B)-B=G$$

This observation emphasizes the fact that, the colour of an object does not reside in the object itself. Rather, the colour is in the light that shines upon the object and that ultimately becomes reflected to our eyes. Hence, a cloth appears yellow because it reflects green and red lights, which must be present in the incident beam. Generally, the rules of subtraction of light colour are:

1. Red colour absorbs green and blue, and reflects red colour.
2. Blue colour absorbs green and red, and reflects blue colour.
3. Green colour absorbs blue and red while reflecting green colour.
4. Cyan colour absorbs red and reflects cyan colour (blue + green).
5. Magenta colour absorbs green and reflects magenta colour (red + blue).
6. Yellow colour absorbs blue and reflects yellow colour (red + green).

These rules are very important concepts for understanding combinations of coloured pigments. Figure 3.51 summarizes the colour subtraction rules.

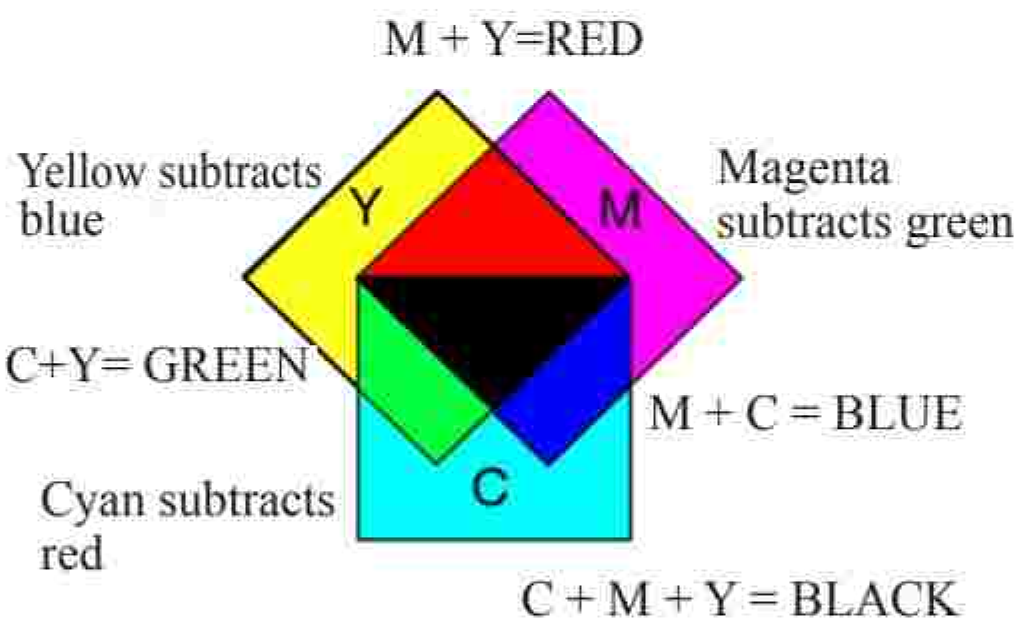


Figure 3.51: *Light colour subtraction using idealized primary filter*

Additive and subtractive mixing of colours

Following our discussion in the addition and subtraction of light colours, you may have realized that objects can have different colours depending on the combinations of light that strike our eyes. This idea is very useful in the synthesis of colours, which we refer as colour mixing. Since colour-mixing uses the concepts of addition and subtraction of light colours, there are two main types of colour mixing. These are additive colour mixing and subtractive colour mixing.

Additive colour mixing

Additive colour mixing is the process of creating a new colour by adding one set of colours to another set of colours. Additive colour mixing happens when lights of different colours are mixed. When we add all of the different colours of sunlight, we see white light rather than many individual colours. Thus, this type of colour mixing is called additive colour mixing because all of the wavelengths still reach our eyes but the observed colour depends on the combinations of different colours. Normally, primary colours used in additive colour mixing are red, blue and green and secondary colours are cyan, magenta and yellow. Even addition of all of these colours yields white light, which means all colours are reflected. Additive colour mixing is commonly used to generate colours on televisions and computer monitors using a system known as RGB (red, green and blue) system. Activity 3.15 demonstrates the process of additive mixing of colours.

Subtractive colour mixing

Subtractive colour mixing is a process of creating a new colour by the removal of some colours from a broad light spectrum. Subtractive colour mixing occurs when we mix paint pigments. When colour pigments are mixed, both paints still absorb all of the colours as they did before being mixed, thus what we are left with is only the colours that both paints reflect. This process is therefore called subtractive mixing of colours because when the pigments mix, colours are deleted from what we see because each paint pigment will absorb some colours that the other paint pigment reflects. This leaves a lesser number of colours that reach the observer's eye. Since we only see light that is reflected by the paints, the primary colours in the subtractive colour mixing are cyan, magenta and yellow while secondary colours are red, blue, and green. Even addition of primary colours gives black colour, which simply means all colours have been absorbed and no light is reflected. Subtractive colour mixing is commonly used in coloured printers and production of different paints using a system known CMY (cyan, magenta and yellow) system.

It is important to note that, each primary colour of paint pigment absorbs one primary colour of light. The colour absorbed by a primary colour paint pigment is the complementary colour of that paint pigment. That is,

(a) Magenta paint absorbs green light.

(b) Cyan absorbs red light.

(c) Yellow absorbs blue light.

On the other hand, each secondary colour paint pigment absorbs two secondary colours of light. That is,

(a) Blue paint absorbs cyan and magenta.

(b) Green paint absorbs yellow and cyan.

(c) Red paint absorbs magenta and yellow.

Activity 3.16

Aim: To explore the subtractive colour mixing

Materials: three colours of paint (blue, red and yellow), plain white papers, a stick, a dropper

Procedure

1. Put a few drops of blue paint on the white paper.
2. Using a clean dropper put a similar number of yellow paint drops on the same paper. This makes an even mixing ratio.
3. Gently mix the paints and observe the resulting colour. Record this observation.
4. Repeat the procedure for different paints, different number of paint and different mixing ratios.

Question

Explain how new colours have been generated by mixing different paints.

When different paints are mixed, each paint reflects the same light colours that it was reflecting before being mixed. However, in the mixture, each paint absorbs some of the colours that are reflected by the other paint. Consequently, only the remaining colours of light strike the observer's eye giving the mixture of paints a different colour appearance.

Note that, both additive and subtractive colour mixing are of paramount importance for understanding our perception of different colours.

Differences between additive and subtractive colour mixing

The easy way to remember the difference between additive and subtractive colour mixing is that, additive colour mixing is what happens when we mix lights of different colours whereas subtractive colour mixing occurs when we mix paints or other coloured material. Table 3.7 summarizes the differences between the two-colour mixing processes.

Table 3.7: Comparison between additive and subtractive colour mixing

Criteria	Additive mixing	Subtractive mixing
Definition	A process of creating a new colour by adding one set of colours to another set of colours	A process of creating a new colour by the removal of some colours from a broad light spectrum
Primary colours	red, green and blue	cyan, magenta and yellow
Secondary colours	cyan, magenta and yellow	red, blue and green
Colour combination	green + red = yellow blue + red = magenta blue + green = cyan red + blue + green = white	white – red = cyan white – green = magenta white – blue = yellow white – red – green – blue = black
Application system	RGB	CMY
Applications	Used in colour televisions and computer monitors	Used in colour printers, paint pigments

Exercise 3.4

1. Determine the colour appearances of objects in the following conditions;
   (a) Yellow light falling on a shirt that can absorb blue colour.
   (b) Yellow light falling on a sheet of paper that can absorb cyan colour.
2. Suppose that light from a magenta spotlight and light from a yellow spotlight are mixed together, will white light be produced? Explain.
3. What is dispersion?
4. How are rainbows formed?
5. Why do clouds appear white?
6. Explain why an object which appears yellow in day light, appears red when illuminated with red light and black when illuminated with a blue light?
7. What is the difference between secondary colour and complementary colour in light?
8. Explain the appearance of a red tie with blue spots when observed in
   (a) red light.
   (b) green light.
9. Briefly explain the two ways of combining the colours of the spectrum into white.

Refraction of light by lenses

A piece of glass or other transparent material may be shaped in such a way that, parallel incident light rays passing through it would either converge to a point or appear to diverge from a point. A piece of glass that has such a shape is referred to as a lens. In other words, a lens is a transparent or a translucent medium that alters the direction of light passing through it. Though most lenses are made of glass, it is possible to make lenses from various transparent materials. In recent times, the use of plastic for making lenses has been increasing. A lens is therefore a carefully molded piece of transparent material that refracts light rays in such a way that an image is formed. Lenses can be thought of as a series of tiny refracting prisms, each of which refracts light to produce their own image. When these prisms act together, they produce a bright image focused at a point as shown in Figure 3.52.

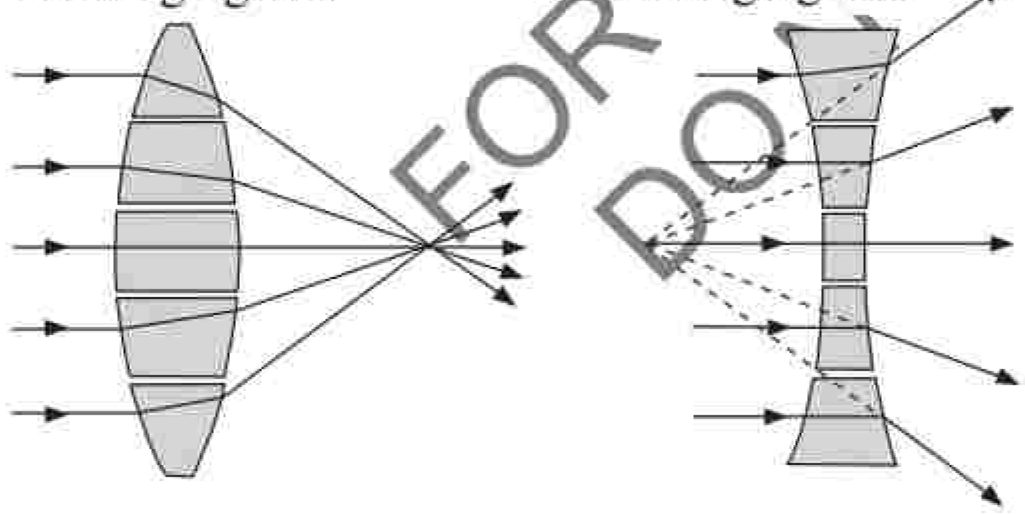


Figure 3.52: *A set of prisms packed to converge or diverge light rays*

Convex and concave lenses

Lenses differ from one another in terms of their shape and the materials from which they are made. Based on their shapes, there are two major types of lenses. These are the converging (convex) lenses and diverging (concave) lenses.

A convex lens is a lens that converges rays of light that pass through it. Converging lenses are relatively thick across their middle and thin at their edges as shown in Figure 3.53. At least one side of a converging lens curves outwards, but the common name is convex lenses. Convex lenses can be further categorized depending on their specific shapes. Types of convex lenses include bi convex lenses, plano-convex lenses and converging meniscus lens. These types of convex lenses are shown in Figure 3.53.

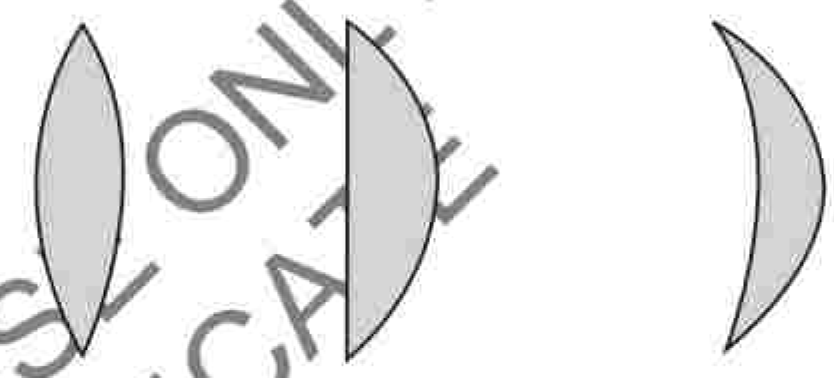


Figure 3.53: *Type of convex lenses*

Task 3.5

Arrange a convex lens and sheet of paper on a sunny day such that the sunlight is directed through the convex lens and converged at specific point on the sheet of paper. Hold the lens at the same position for some minutes. Observe what happens. Discuss with your classmates on your observation.

Caution: Avoid looking at the Sun directly or even into a refracted sunlight.

On the other hand, a concave lens is a lens that diverges rays of light that pass through it. Diverging lenses are relatively thin across their middle and thick at their edges. At least one side of a diverging lens curves inwards, but the common name is concave lenses. Types of concave lenses include biconcave lens, plano-concave lens and the diverging meniscus. Figure 3.54 shows the types of concave lenses.

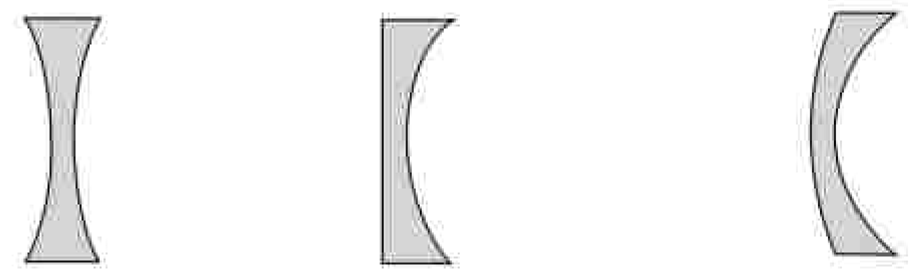


Figure 3.54: *Type of diverging lenses*

Terminologies related to lenses

The discussion of refraction of light rays and the formation of images by lenses requires the use of a variety of terms. These terms describe the various parts of a lens and include words such as the principal axis, vertical axis, focal point, focal length, optical centre, radius of curvature and centre of curvature.

If one face of a lens is thought of as being a portion of a spherical surface, then the imaginary line passing through the centre of the sphere and the exact centre of the lens is called the principal axis of the lens. A lens also has an imaginary vertical line that bisects it symmetrically into two halves. This line is the vertical axis of the lens. The point of intersection between the vertical axis and the principal axis is called the optical centre of the lens.

Light rays incident towards the face of a lens and travelling parallel to the principal axis will either converge or diverge. If the light rays converge, then they will converge to a point. This point is known as the focal point or principal focus of the converging lens. On the other hand, if the light rays diverge, then the diverging rays can be traced backwards until they intersect at a point. This point of intersection is known as the focal point or principal focus of a diverging lens. In both cases, the focal point is denoted by the letter, F. The distance between the focal point and the optical centre is the focal length, denoted by the letter, f.

Note that, unlike mirrors, lenses allow light to pass through either face. Therefore, a lens has two focal points- one on each side of the lens. A lens has also an imaginary point called centre of curvature which is the centre of the sphere of which a given surface of a lens is a part. This is the point on the principal axis whose distance from the optical centre is twice the focal length. The lens has two centres of curvatures, on each side of the lens. Note that, the principal focus (focal point), F is real for a convex lens and virtual for a concave lens.

Other common terms used in are; real image, virtual image, image distance, v, object distance, u and magnification, m. All these terms take the same meaning as they were defined in the case of image formation by curved mirrors. Figure 3.55 illustrates the terms used in discussing lenses.

It is important to note that, for thin lenses, the pole and the optical centre merely coincide. Moreover, the plane through the principal focus, which is perpendicular to the principal axis, is called the focal plane.

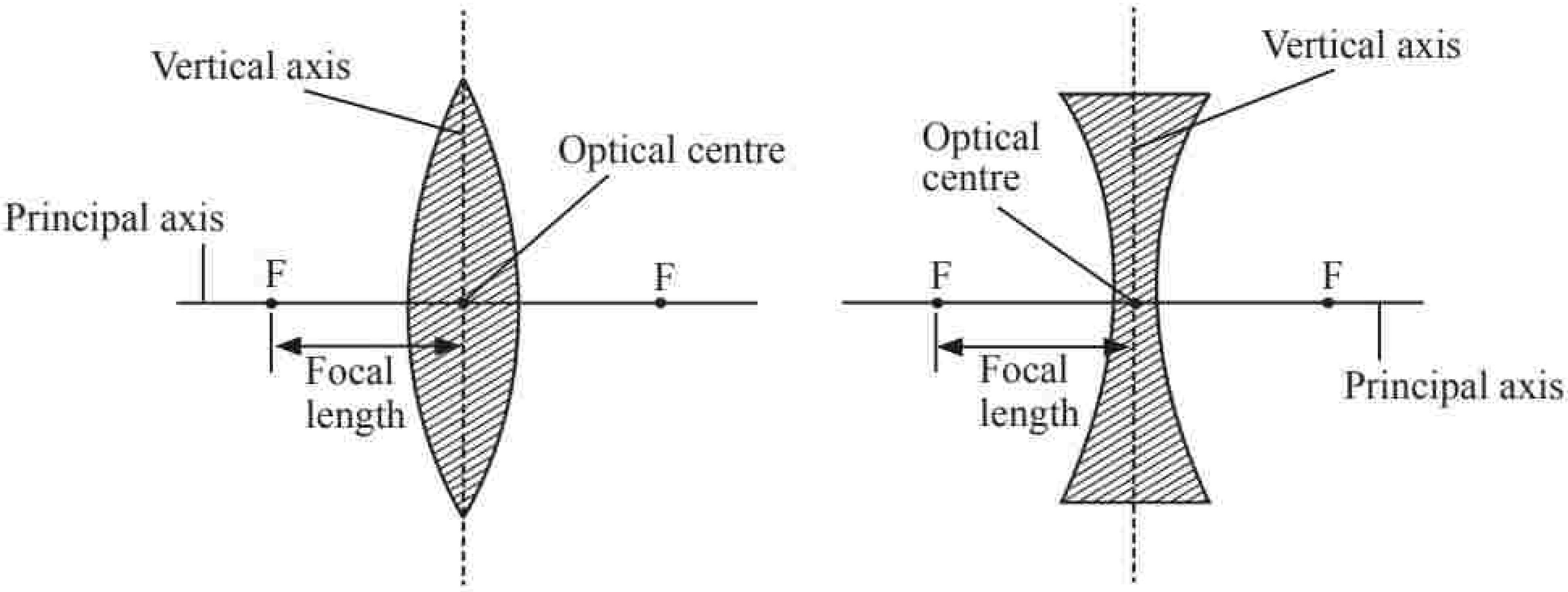


Figure 3.55: *Features of thin lenses*

Images formed by lenses

Image formation by lenses is a result of the refraction of light at each boundary of the lens. As a ray of light enters a lens, it is refracted; and as the same ray of light exits the lens, it is refracted again. The net effect of the refraction of light at these two boundaries is the change in direction of the ray of light. Because of the special geometric shape of a lens, the light rays are refracted such that they intersect at a certain point, thus, forming an image.

Construction of ray diagrams

The way in which lenses form images of objects can be shown by means of ray diagrams. In ray diagrams, sometimes lenses are represented by vertical lines with an appropriate indication to show whether it is a converging lens or a diverging lens. Figure 3.56 shows the representation of converging lens and diverging lenses in ray diagrams.

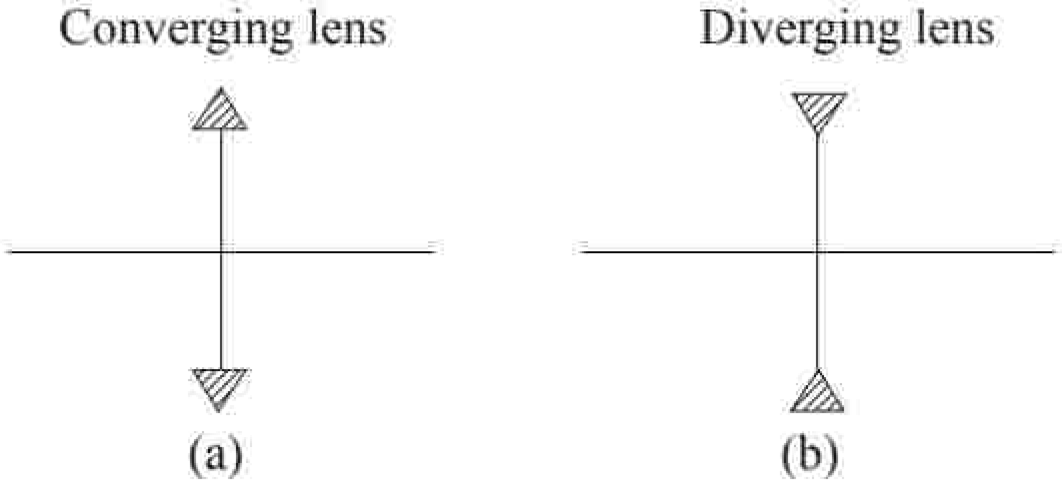
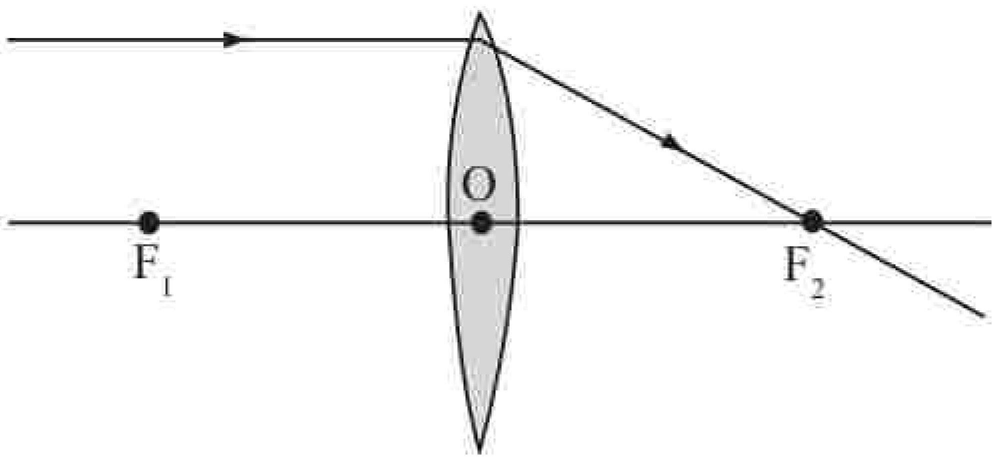


Figure 3.56: *Representation of converging and diverging lenses in ray diagrams*

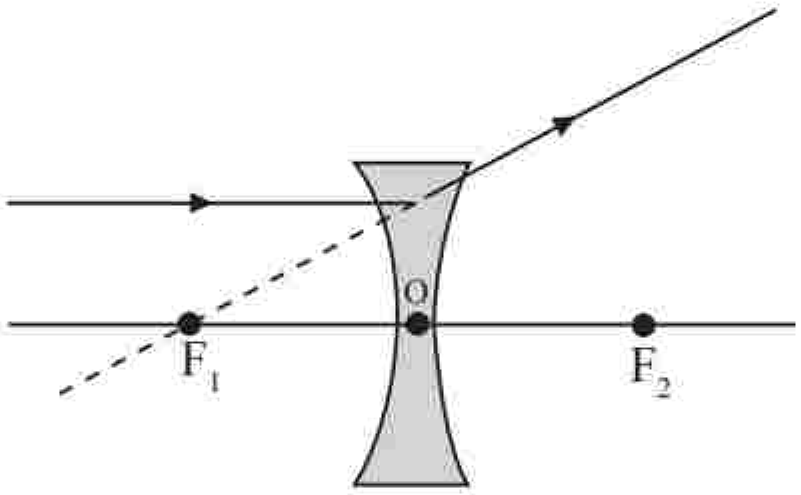
Some rules are to be followed when one constructs ray diagrams for both convex and concave lenses. These rules originate from the principles of refraction of light. When a ray of light strikes the first face of a lens, it is travelling from a less dense to a denser medium, thus it bends towards the normal. Upon exiting on the other side of the lens, the ray of light is now travelling from the denser medium to a less dense medium. Therefore, the ray bends away from the normal as it exits the lens. It is because of the change in directions of the light rays by a lens. Three important rules are useful in constructing ray diagrams. The rules are:

1. *Convex lens:* A ray of light travelling parallel to the principal axis of a convex lens (Figure 3.57(a)) is converged to the principal focus.

   Concave lens: A ray of light travelling parallel to the principal axis of a concave lens (Figure 3.57(b)) is diverged in such a way that it appears to be coming from the principal focus.



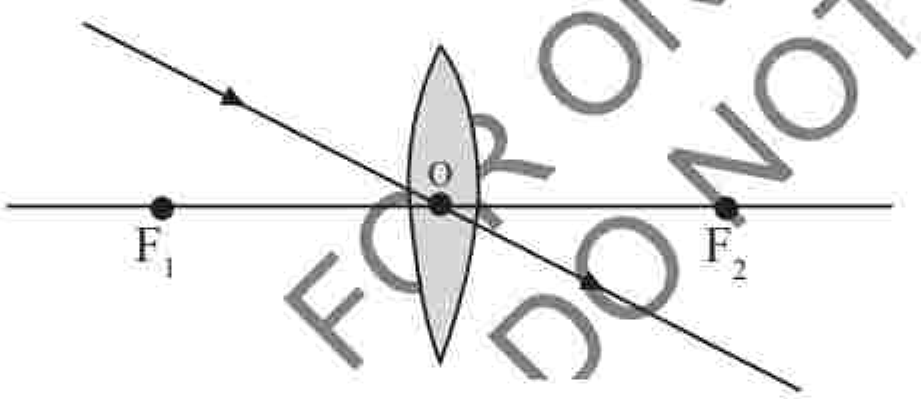
(a)



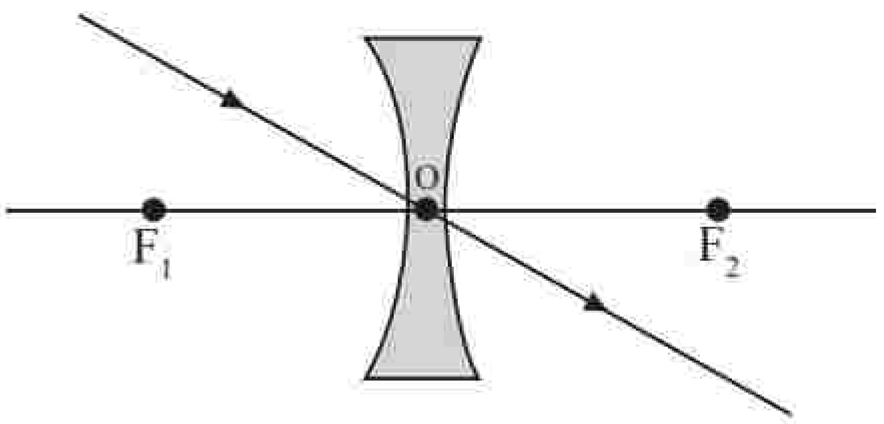
(b)

Figure 3.57: *Refraction of incident light rays that are parallel to the principal axis*

2. A ray of light travelling through the optical centre continues to travel along the same path for both convex and concave lenses as seen in Figure 3.58.



Convex lens

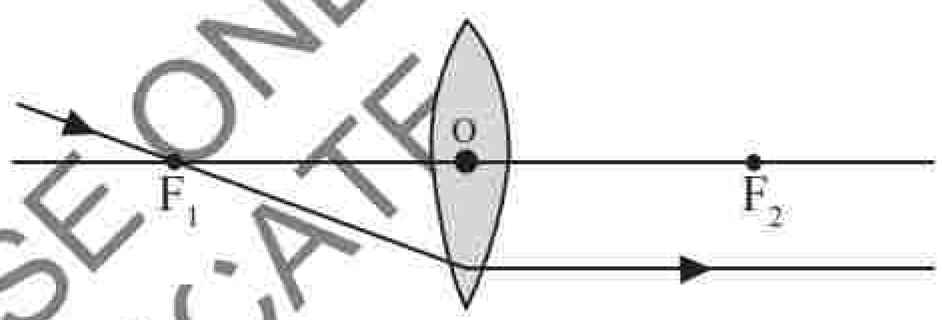


Concave lens

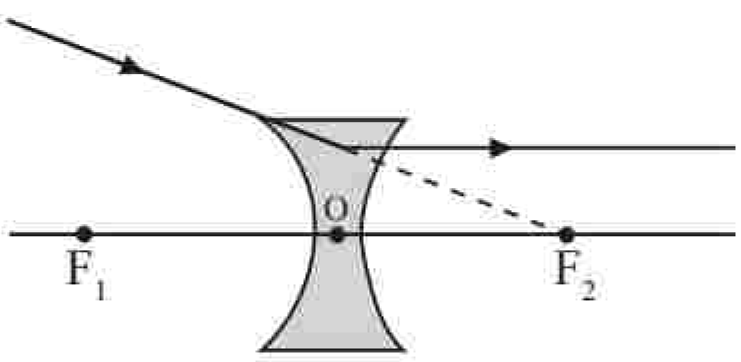
Figure 3.58: *Refraction of light rays incident through the optical centre*

3. *Convex lens:* An incident ray of light travelling through the principal focus of a converging lens is refracted parallel to the principal axis of the convex lens as shown in Figure 3.59 (a).

   Concave lens: An incident ray of light that appears to travel through the principal focus of a diverging lens is refracted parallel to the principal axis. Figure 3.59 (b) shows the path of a ray through the principal focus for concave lenses.



(a)



(b)

Figure 3.59: *Refraction of light rays incident through the principal focus*

Like in the case of spherical mirrors, any two of the light rays are enough to locate the image formed by a lens. The refraction of light produced by a lens is treated as occurring at a plane through the optical centre, that is, perpendicular to the principal axis of the lens. Whenever rays diverge after passing through the lens, they are extended backwards to locate the virtual images.

Images formed by convex lens

The location and characteristics of images formed by a convex lens vary depending on the position of the object relative to the optical centre of the lens. One can determine the characteristics of images formed by a convex lens when the object is placed at various locations relative to the optical centre of the lens by performing Activity 3.17.

Note: The image of an object can be obtained by drawing any two of the ray diagrams.

Activity 3.17

Aim: To determine the locations and characteristics of images formed by a convex lens for various object positions

Materials: graph paper, sharp pencil, ruler

Procedure

1. Choose an appropriate scale so that the ray diagrams fits on the available space on the graph paper.
2. Draw a convex lens at the centre of the graph paper.
3. Draw a horizontal line passing through the optical centre of the lens. This is the principal axis.
4. Locate the points F_1, $2F_1$, F_2 and $2F_2$ on the principal axis as shown in Figure 3.60. Choose the focal point such that there is a considerable distance between the optical centre, O and the point $2F_1$.
5. Using the chosen scale, draw an upright arrow at a point beyond $2F_1$ point.
6. Choose a point at the head of the object arrow and draw a light ray from this point parallel to the principal axis. Refract this ray according to the rules of refraction.
7. From the same point, draw another ray that passes un-deflected through the optical centre. You may also choose to draw a ray through the focal point and refract it according to the refraction rules.
8. Locate the point at which the refracted rays intersect and draw an arrow from the principal axis to the point of intersection. Figure 3.60 shows the image formation by a convex lens for an object beyond point $2F_1$.
9. Measure the object distance, the image distance, the object size and the image size. Calculate magnification.
10. Repeat steps 5 to 9 for different positions of the object. That is, object at point $2F_1$, object between $2F_1$ and F_1, object at F_1 and object between F_1 and O.
11. Summarise your results using a table as one shown in Table 3.8.

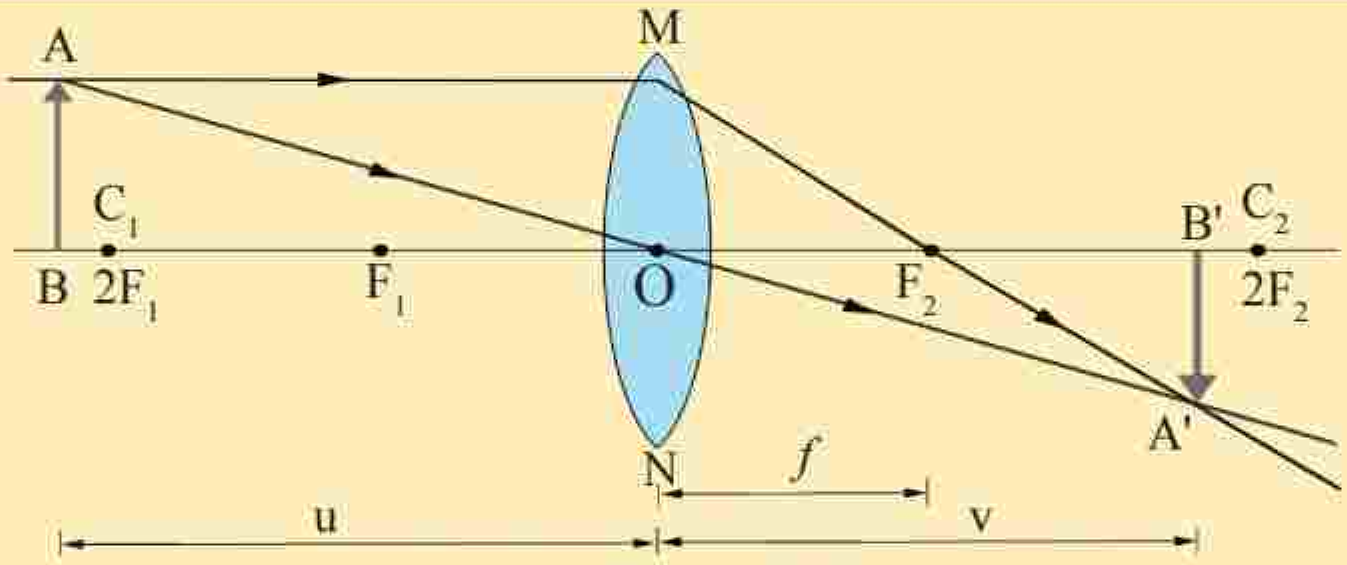


Figure 3.60

Questions

(a) What are the positions (image distance) of the images formed for different object positions?

(b) Describe the nature of the images formed. That is, are the images for various object positions real or virtual?

(c) Are the images for objects at different positions enlarged, the same or diminished?

Table 3.8

Object position (cm)	Image position (cm)	Object height (cm)	Image height (cm)	Nature of the image (real or virtual)	Magnification ($m = h_i / h_0$)
Beyond C					
At C					
Between F and C					
At F					
Between F and O					

The activity focused on the refraction of light and the formation of images by convex lens. It has been observed that, depending on the object position, an image formed by a convex lens can be virtual or real, upright or inverted and magnified or diminished.

Task 3.6

Use a ray diagram to observe what happens when the object is located very far away from the 2F point of a convex lens. Discuss your observations with your classmates.

Images formed by concave lens

To characterize the images formed by a concave lens, perform Activity 3.18.

Activity 3.18

Aim: To determine the locations and characteristics of images formed by a concave lens for various object positions

Materials: graph paper, sharp pencil, ruler

Procedure

1. Choose an appropriate scale so that the ray diagram fits on the available space and draw a concave lens at the centre of the graph paper.
2. Draw a horizontal line passing through the centre of the concave lens. This is the principal axis. Locate the points F_1, $2F_1$, F_2 and $2F_2$ on principal axis as shown in Figure 3.61 such that there is a considerable distance between point $2F_1$ and the optical centre, O.
3. Using the chosen scale, draw an upright arrow at a point just beyond $2F_1$. The arrow represents an object.
4. Choose a point at the head of the object arrow and draw a ray from this point towards the focal point on the opposite side of the lens. This ray will strike the lens before reaching the focal point; stop the ray at the point of incidence with the lens.
5. Draw the second ray such that it travels exactly parallel to the principal axis. Remember to place arrowheads upon the rays to indicate their direction of travel. Draw the third ray to the exact centre of the lens.
6. Once these incident rays strike the lens, refract them according to the three rules of refraction for the double concave lenses. That is, the ray that travels towards the focal point will be refracted through the lens and travel parallel to the principal axis. The ray that travels parallel to the principal axis on the way to the lens will be refracted and travel in a direction such that its extension passes through the focal point on the object's side of the lens. The ray that travels to the exact centre of the lens will continue to travel in the same direction.
7. Using dashed lines, extend the refracted rays on the opposite side of the lens until they intersect. The point of intersection is the image point of the top of the object. Note that, any two rays are enough to locate the position of an object.
8. Now, choose a point at the bottom of the object and repeat steps 4 to 7. This way you can locate the image of the bottom of the object. Use an arrow to draw the image by joining the top and bottom points of intersection as shown in Figure 3.61.

9. Measure the image distance v, the object distance u, the image height, h_i and the object height, h_o. Note that, distances measured behind the lens are considered to be negative.
10. Repeat steps 3 to 9 for different positions of the object.

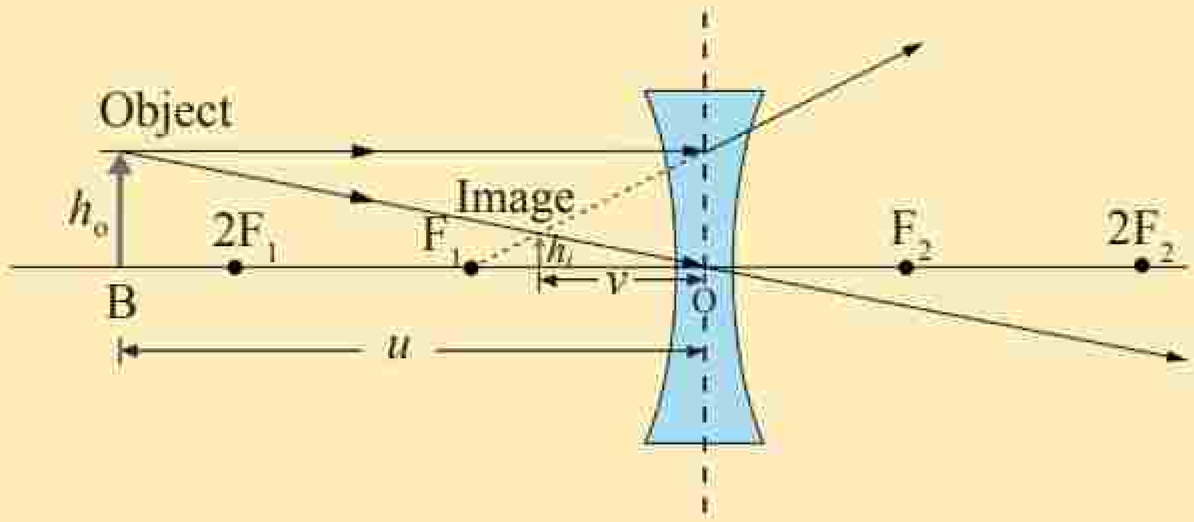


Figure 3.61

Questions

(a) What are the characteristics of the images formed by a concave lens?

(b) Do the characteristics of the images formed by a concave lens vary with the change of object position?

This activity illustrates that the image positions for an object in front of a concave lens will be located at different positions between F_1 and optical centre, O. Furthermore, in case of the diverging lens, always erect, diminished and virtual images are formed.

Example 3.9

An object 5 cm in length is placed at a distance of 25 cm away from converging lens of focal length 10 cm. Use a ray diagram to determine the position, size and nature of the formed image.

Solution

Choose a suitable scale for your ray diagram. Take a scale that, 1 cm represents 5 cm.

Therefore, the height of the object is represented by 1 cm, object distance is represented by 5 cm and focal length is represented by 2 cm.

1. Drawing the ray diagram for the provided information appears as shown in Figure 3.62.

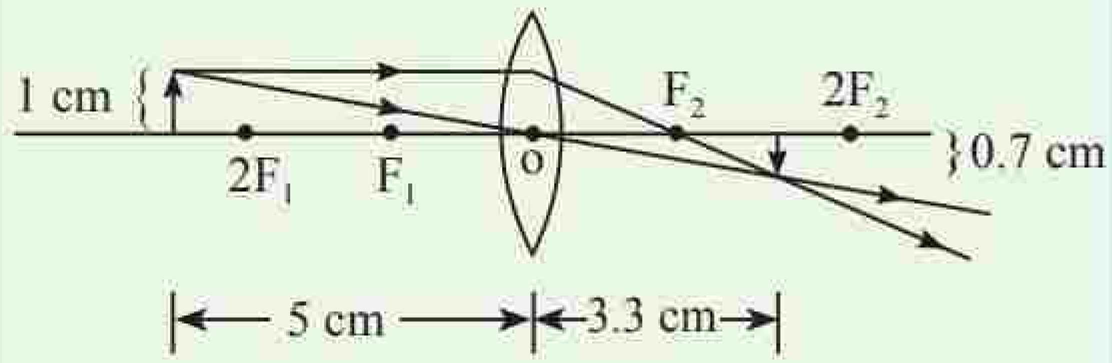


Figure 3.62

2. Thus, measuring distance of image, it is 3.3 cm on the opposite side of lens (real image) and height of image is 0.7 cm inverted.

3. Converting using the scale we have:

(a) Distance of image

$$= \frac{3.3 \text{ cm} \times 5 \text{ cm}}{1 \text{ cm}} = 16.5 \text{ cm}$$

(b) Height of image

$$= \frac{0.7 \text{ cm} \times 5 \text{ cm}}{1 \text{ cm}} = 3.5 \text{ cm}$$

Thin Lens formula

If we represent the object distance by letter *u*, the image distance by letter *v*, and the focal length by letter *f*, then the general formula relating the three quantities is,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This equation is called the thin lens formula or lens equation. It is valid for both converging and diverging lenses. The equation is a simpler and more accurate alternative for locating the image formed by either convex or concave lens. The formula can be verified experimentally as demonstrated in Activity 3.19.

Real-is-positive convention

To determine the values of *u* and *v* using the formula, a sign convention is adopted. The convention is referred to as the real-is-positive convention. When using this convention, all distances are measured from the optical centre. Distances of real objects and real images are treated as positive whereas distances of virtual objects and images are taken to be negative. Because the principal focus of a concave lens is virtual, concave lenses have negative values of focal length.

Activity 3.19

Aim: To verify the lens equation

Materials: an illuminated object, screen, metre rule, a convex lens of a known focal length, lens holder

Procedure

1. Set up the apparatus as shown in the Figure 3.63.

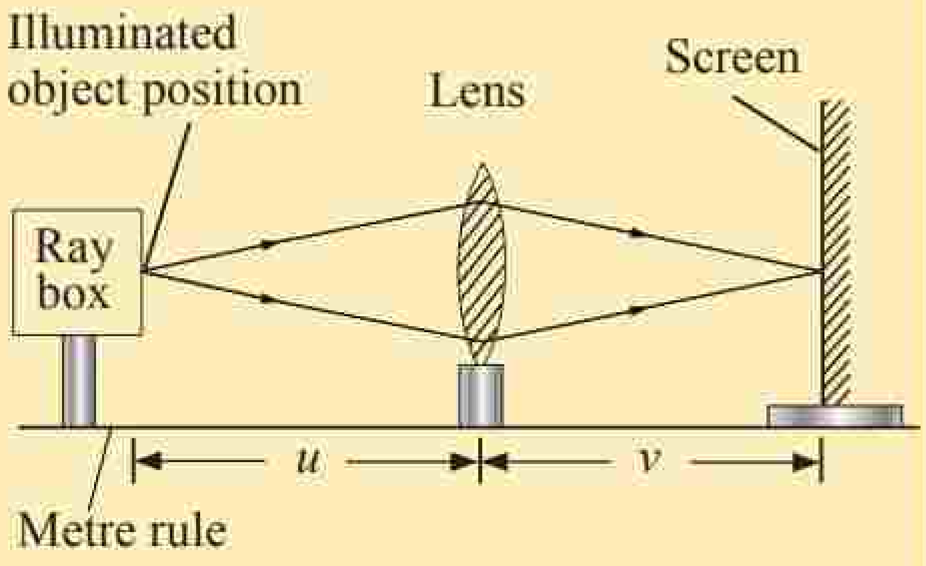


Figure 3.63

2. Place the object and the screen at a reasonable distance apart so that a real image is formed on the screen.
3. Adjust the position of the lens along the metre rule until a sharp image is formed on the screen.
4. Measure the object and the image distances from the lens. By changing the position of the object, obtain several values of *u* and the corresponding values of *v*. Record your results in a table as the one shown in Table 3.9.

Table 3.9

f(cm)	$\frac{1}{f}(\text{cm}^{-1})$	u(cm)	$\frac{1}{u}(\text{cm}^{-1})$	v(cm)	$\frac{1}{v}(\text{cm}^{-1})$	$uv(\text{cm}^2)$	$\frac{uv}{u+v}$(cm)

Questions

(a) Determine the average value of $\frac{uv}{u+v}$.

(b) Compare the value obtained in (a) above with the focal length of the lens.

(c) Plot a graph of $\frac{1}{v}$ against $\frac{1}{u}$.

(d) Explain how you can obtain the value of the focal length from the graph.

The average value obtained from the formula $\frac{uv}{u+v}$ is the focal length of the lens. This is in agreement with the lens formula. Note that, from the formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{v+u}{uv}$$

Therefore,

$$f = \frac{uv}{v+u}$$

Alternatively, the value of f may be obtained by plotting a graph of $\frac{1}{v}$ against $\frac{1}{u}$. The graph is a straight line with an intercept on both axes, as shown in Figure 3.64. From the graph, we may use the formula, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ to realize that, at y-intercept, $\frac{1}{u} = 0$. So, $\frac{1}{f} = \frac{1}{v}$ or $f = v$. The focal length may therefore be obtained by using the y-intercept of the graph and finding its reciprocal.

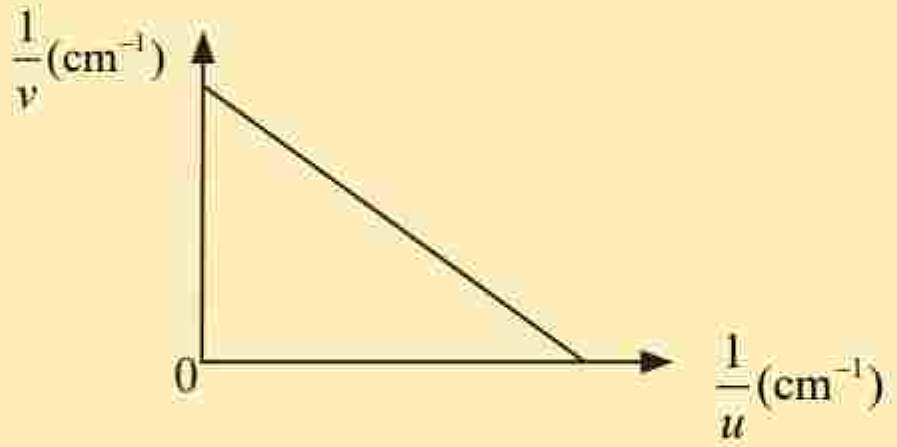


Figure 3.64: *A graph of* $\frac{1}{v}$ *against* $\frac{1}{u}$

Focal length of a convex lens

The focal length of a converging lens can be determined experimentally using various methods. Activity 3.20 describes one of the methods.

Activity 3.20

Aim: Estimating the focal length of a converging lens by focusing a distant object on a screen

Materials: a converging lens, lens holder, metre rule, plain paper (screen)

Procedure

1. Fix the lens on the holder and the screen on a stand.
2. Place the screen and the lens on a table in such a way that the light from a distant object passes through the lens as shown in Figure 3.65.
3. Move the screen closer or away from the lens until a sharp image of the distant object is focused on the screen.

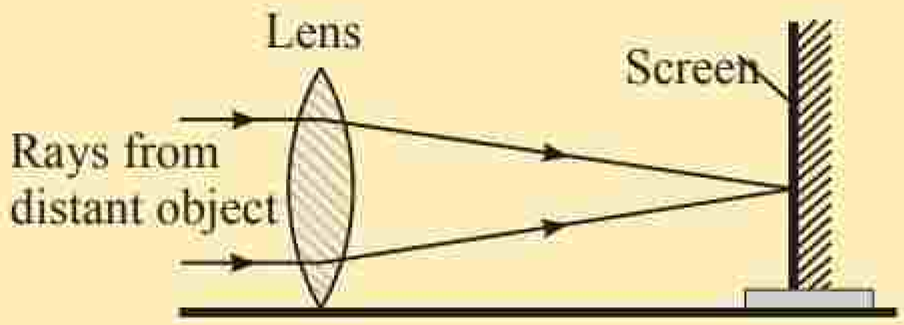


Figure 3.65

4. Measure the distance between the screen and the lens.

Question

What is the distance between the screen and the lens?

The image of a distant object is formed at the focal point of a convex lens. Therefore, the distance between the screen and the lens is the focal length of the lens. Note that, this is not a very accurate method but it is a quick way of estimating the focal length of a convex lens.

Example 3.10

An object is placed 12 cm from a convex lens of focal length 18 cm. Using the lens formula, find the position of the image.

Solution

For a convex lens, the value of focal length is positive. Therefore, $f = +18$ cm and $u = +12$ cm.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{18\text{ cm}} - \frac{1}{12\text{ cm}}$$

$$= \frac{2\text{ cm} - 3\text{ cm}}{36\text{ cm}^2}$$

$$= \frac{-1}{36\text{ cm}}$$

$$v = -36\text{ cm}$$

Because the value of v is negative, the image is virtual (on the same side as the object) and is 36 cm from the lens. Because the image is virtual, it is also erect.

Example 3.11

An object is placed 10 cm from a concave lens of focal length 15 cm. Using the lens formula, determine the nature and the position of the image.

Solution

$$u = +10 \text{ cm}$$

$$f = -15 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-15 \text{ cm}} = \frac{1}{10 \text{ cm}} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{-1}{15 \text{ cm}} - \frac{1}{10 \text{ cm}}$$

$$\frac{1}{v} = \frac{-2 \text{ cm} - 3 \text{ cm}}{30 \text{ cm}^2} = \frac{-5 \text{ cm}}{30 \text{ cm}^2}$$

$$v = -6 \text{ cm}$$

A virtual image is formed 6 cm from the lens, on the same side as the object.

Magnification by a lens

Magnification is a measure of the extent to which an optical system changes the size of an image in relation to the object. The linear or lateral magnification produced by a lens is the ratio of the height of the image to the height of the object. If the image size is bigger than the object size, then the image is said to be enlarged. On the other hand, when the image size is smaller than the object size, the image is said to be diminished. Magnification is usually denoted by the letter *m*. Thus,

$$m = \frac{\textit{image height}\,(h_i)}{\textit{object height}\,(h_o)}$$

Linear magnification is also given by the negative of the ratio of the distance of the image from the lens, v to the distance of the object from the lens, u. Hence,

$$m = -\frac{v}{u}$$

Example 3.12

An object 2 cm high is placed 24 cm from a converging lens. An erect image, which is 6 cm high, is formed. Determine the focal length of the lens.

Solution

Using the lens formula:

$$h_o = 2 \text{ cm}, \quad h_i = 6 \text{ cm}, \quad u = 24 \text{ cm}$$

$$m = -\frac{v}{u} = -\frac{v}{24 \text{ cm}}$$

Also, $m = \frac{h_i}{h_o}$

$$= \frac{6 \text{ cm}}{2 \text{ cm}} = 3$$

Thus, $m = -\frac{v}{24 \text{ cm}} = 3$

$$v = -72 \text{ cm}$$

Because the image is erect, it must be virtual. Using the lens formula:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{24 \text{ cm}} + \frac{1}{-72 \text{ cm}}$$

$$\frac{1}{f} = \frac{3 \text{ cm} - 1 \text{ cm}}{72 \text{ cm}^2} = \frac{2 \text{ cm}}{72 \text{ cm}^2}$$

$$f = 36 \text{ cm}$$

Therefore, the focal length of the lens is 36 cm.

Example 3.13

A vertical object, 10 cm high, is placed 30 cm in front of a diverging lens. An image of the object is formed 7.5 cm in front of the lens. Determine the focal length and the magnification of the lens.

Solution

$u = 30$ cm, $v = 7.5$ cm, $h_o = 10$ cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$f = \frac{uv}{u+v} = \frac{30\text{ cm} \times (-7.5\text{ cm})}{30\text{ cm} + (-7.5\text{ cm})} = -10\text{ cm}$$

$f = -10$ cm (Negative sign indicates that f is measured against the incident ray)

$$m = -\frac{v}{u} = \frac{-(-7.5\text{ cm})}{30\text{ cm}} = 0.25$$

$m = 0.25$

The positive value of magnification indicates that the image is erect or upright. Therefore, the focal length, f is 10 cm and the magnification, m is 0.25.

Uses of lenses in life

Lenses have different applications in our daily life. Convex or concave lenses are used in medical eyeglasses for correcting long sightedness and short sightedness respectively. Convex lenses are also used in binoculars, cameras, telescopes, microscopes and other optical instruments. On the other hand, concave lenses are used in flashlights to magnify the light produced by a bulb. These lenses are also used in pinhole cameras of house doors to provide a view of people or objects outside the door. Generally, lenses are useful in designing and constructing optical instruments; these will be discussed in detail in the succeeding chapter.

Exercise 3.5

1. (a) Draw ray diagrams to show the nature and position of images formed by convex lens with a focal length of 30 cm for the following object distances: (i) 15 cm (ii) 30 cm (iii) 45 cm.
   (b) Use the lens formula to determine the nature and position of the image formed by the objects in (a).
2. A vertical object 10 cm high, is placed 30 cm in front of a diverging lens. An image of an object is formed 7.5 cm in front of the lens. Determine the focal length of the lens by using a ray diagram.
3. An object 2 cm high is placed 9 cm from a convex lens of focal length 6 cm. Determine the magnification, position and nature of the formed image.
4. A converging lens forms an upright image that is four times the size of the object. Given that the focal length of the lens is 20 cm, determine the object distance.
5. An object is placed 36 cm from a converging lens of focal length 24 cm. If a real image which is 4 cm high is formed, calculate the height of the object.

6. A lens of focal length 15 cm forms an upright image four times the size of the object. Calculate the distance of the image from the lens.
7. An object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. Find magnification of the image formed.

Chapter summary

1. The images formed by curved mirrors can be real or virtual, erect or inverted, and enlarged or reduced (diminished).
2. A convex mirror always produces a virtual, erect and diminished image.
3. When light encounters the boundary between two different optical media, a portion of the light is reflected and the remainder is transmitted into the second medium. The transmitted light undergoes a change in speed that causes a change in its direction. This phenomenon is called refraction.
4. The change in speed and direction of light depends on the refractive indices of the two media.
5. Light travelling from a medium of low refractive index to a medium of high refractive index is bent toward the normal.
6. When light passes through a medium of high refractive index to one of lower refractive index, total internal reflection occurs if the angle of incidence is greater than the critical angle.
7. The use of prisms in binoculars relies on their ability to totally reflect the incident light internally.
8. Refraction of light through a glass prism obeys Snell's law.
9. The angle formed by the intersection of the incident ray and the emergent ray directions is called the angle of deviation.
10. The minimum angle of deviation by a triangular prism occurs when the angle of incidence produces a refracted ray that travels parallel to the base of the prism.
11. Dispersion of light is the splitting of white light into its component colours. The split light forms a spectrum of colours.
12. The spectrum colours of white light are red, orange, yellow, green, blue, indigo and violet.
13. A rainbow is formed as a result of refraction and reflection of the colours of light in rain-drops.
14. Colours of objects result from reflection and absorption of certain colours by objects.
15. Mixing of pigments is called subtractive mixing of colours. Subtractive complementary colours combine to produce black.
16. Mixing of coloured lights is called additive mixing of colours.

17. Thin lenses form images by refracting light.
18. There are two main types of thin lenses. These are convex lenses and concave lenses. A concave lens is thicker at its edges than its centre, while a convex lens is thicker at its centre than at its edges.
19. A convex lens converges light rays while a concave lens diverges the incident light rays.
20. A concave lens always produces a virtual, erect and reduced image.
21. Ray diagrams can be used to determine the nature and position of an image formed by lenses.

Revision exercise 3

1. Choose the letter of the correct answer from the given alternatives:
   (i) Which of the following statements is TRUE about the images formed by curved mirrors?
      A All real images are reduced.
      B All virtual images are inverted.
      C All inverted images are enlarged.
      D All real images are inverted.
   (ii) A concave mirror has a focal length of 32 cm. When an object is placed in front of the mirror, a real, inverted and half way diminished image is formed. Which of the following is likely to be the distance of the object from the mirror?
      A 16 cm
      B 35 cm
      C 64 cm
      D 96 cm
   (iii) A pin at the bottom of a basin full of water appears to be 6 cm from the surface. Given that the refractive index of the water is 4/3, what is the actual distance of the pin from the surface?
      A 1.33 cm
      B 4.85 cm
      C 7.33 cm
      D 8.0 cm
   (iv) Figure 3.66 shows a ray of light travelling from air to water.

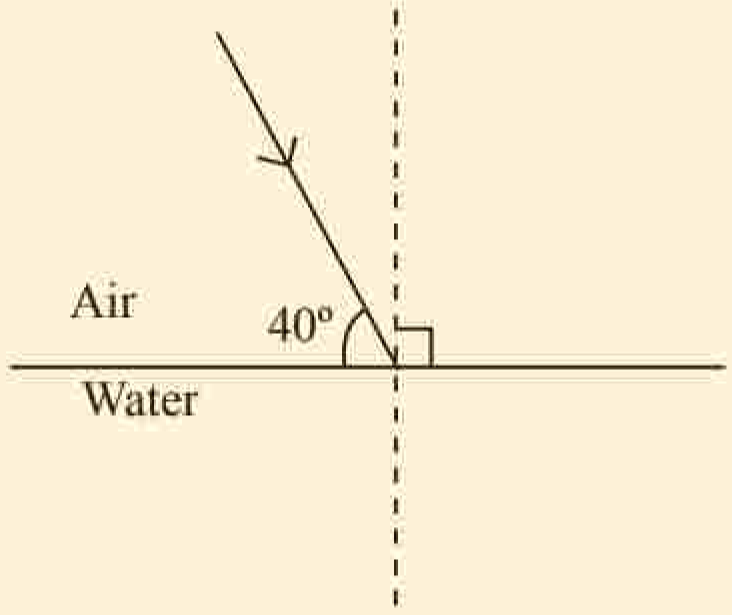


Figure 3.66

Given that the refractive index of water is 4/3, what is the angle of refraction of the ray of light?

A $\sin^{-1}\left(\frac{4}{3}\sin(40°)\right)$

B $\sin^{-1}\left(\frac{3}{4}\sin(50°)\right)$

C $\sin^{-1}\left(\frac{4}{3}\sin(50°)\right)$

D $\sin^{-1}\left(\frac{3}{4}\sin(40°)\right)$

(v) The Figure 3.67 shows a ray of light travelling through three different media.

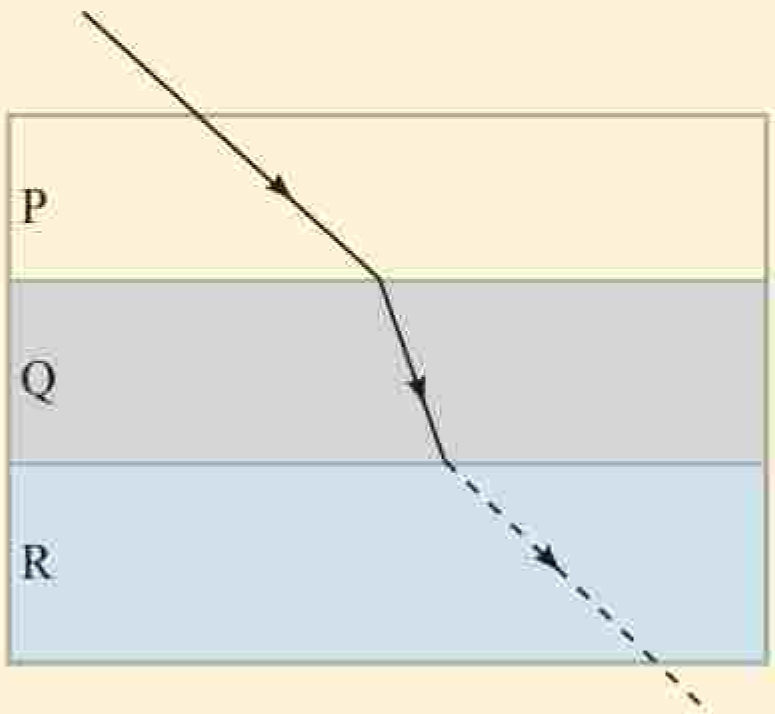


Figure 3.67

Given that the three media are air, glass and water (not in order), which medium is represented by letters P, Q and R, respectively?

A Air, glass, water.
B Air, water, glass.
C Glass, air, water.
D Glass, water, air.

(vi) The radius of curvature of curved surface of a thin plano-convex lens is 10 cm and the refractive index is 1.5. If the plane surface is silvered, then the focal length will be:

A 5 cm
B 10 cm
C 15 cm
D 20 cm

(vii) A ray of light travelling in a transparent medium of refractive index η, falls on a surface separating the medium from air at an angle of incidents of 45º. The ray can undergo total internal reflection for the following:

A $\eta = 1.25$
B $\eta = 1.33$
C $\eta = 1.4$
D $\eta = 1.5$

(viii) An air bubble in glass slab of refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness of the slab is,

A 8 cm
B 10 cm
C 12 cm
D 16 cm

2. Using a ray diagram, show that if an object is placed at the focal point of a concave mirror, the image will be formed at infinity.

3. Copy and complete the diagrams in Figure 3.68 and show the images formed.

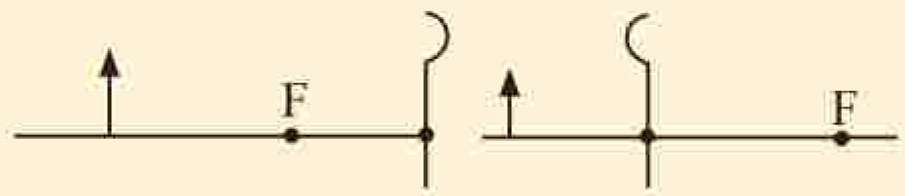


Figure 3.68

4. Parallel light rays from a distant star are incident on a concave mirror with a radius of curvature of 120 cm. How far from the mirror will the star's image be formed?

5. An object is placed 18 cm from a concave mirror. An image that is twice the size of the object is formed. Determine the image distance and the focal length of the mirror.

6. A convex mirror produces an image that is 22 cm behind the mirror when an object is placed 34 cm in front of the mirror. What is the focal length of the mirror?

7. A concave mirror has a focal length of 40 cm. How far from the mirror must an object be placed to produce an image that is:
   (a) twice the size of the object?
   (b) half the size of the object?
   (c) 40 times the size of the object?

8. Two objects A and B when placed in turn in front of a concave mirror of focal length 7.5 cm gives images of equal size. If the size of the object A is three times the size of B and is placed 30 cm from the mirror, find the distance of object B from the mirror.

9. A motor car is fitted with a convex driving side mirror of focal length 20 cm. A second motor car 2 m wide and 1.6 m high is 6 m away from the first car. Calculate the position and size of the image as seen in the mirror of the first car.

10. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed (you may take the distance of the near surface of the slab from the mirror to be 1 cm).

11. What happens to the image formed by a convex mirror when the object distance is decreased?

12. Show that, to obtain an image with a magnification of m using a concave mirror with a focal length f, the object distance, u, is given by

$$u = \frac{(m+1)f}{m}$$

13. A rectangular glass block 5 cm thick is placed on top of the page of a book. If the refractive index of the glass block is 1.53, calculate apparent depth of the letters on the book.

14. A ray of light is incident at an angle of 60° on a block of glass of refractive index 1.5. Determine the angle of refraction of the ray.

15. A small coin was placed at the bottom of a tall glass containing some water and viewed from above. The real and apparent depths of the coin were then measured. By varying the depth of the water in the jar, the following readings were obtained:

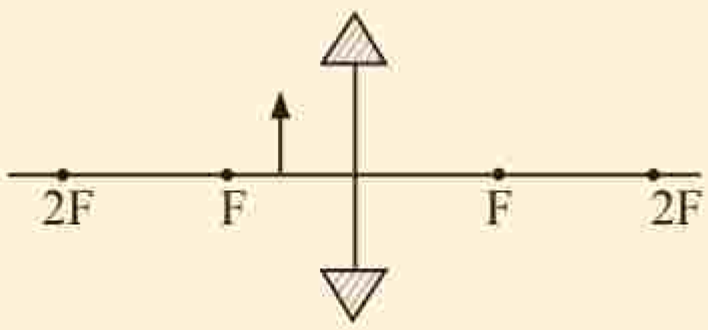
Real depth (cm)	8.1	12.0	16.0	20.0
Apparent depth (cm)	5.9	9.0	12.0	15.1

By plotting an appropriate graph from the results, determine the refractive index of the water.

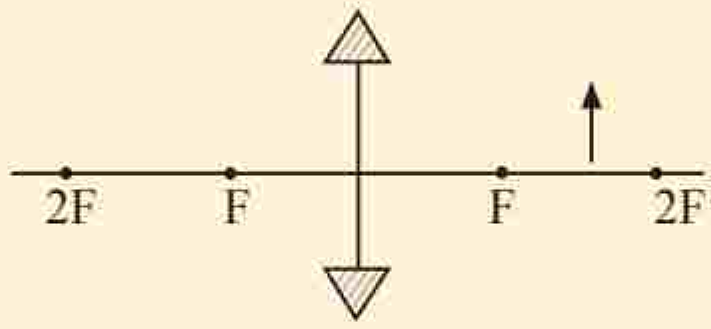
16. State whether it is possible or not to obtain a critical angle when light travels from:
   (a) air to glass.
   (b) water to air.
   (c) glass to air.
   (d) air to water.

17. The refractive index of water is 1.33 and that of glass is 1.5. Calculate the critical angle for:
   (a) a glass–air interface.
   (b) a water–air interface.
   (c) a glass–water interface

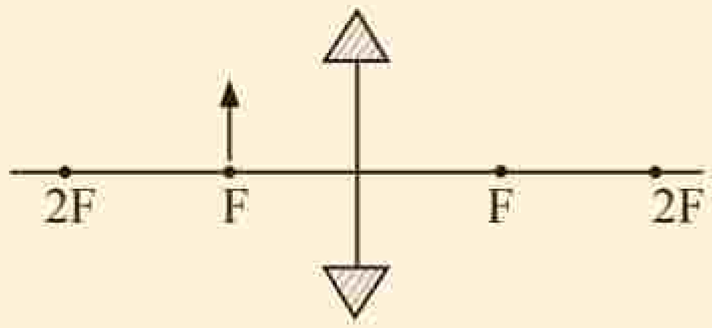
18. Figure 3.69 shows some objects placed in front of or behind lenses.



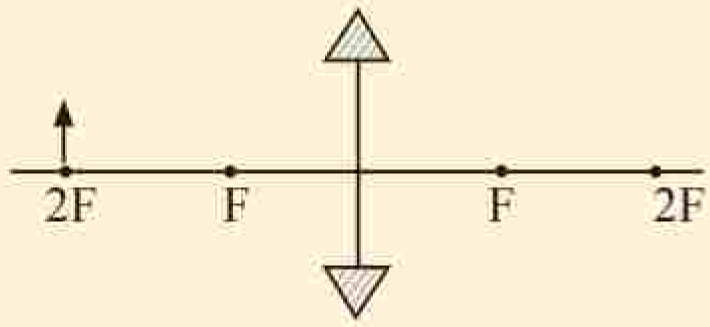
(a)



(b)



(c)



(d)

Figure 3.69

By construction, find the nature and position of the image formed by each object.

19. An object 5 cm high is placed 25 cm from a convex lens with a focal length of 20 cm. Using the lens formula, determine the position, size and nature of the formed image.

20. When an object is placed 30 cm from a lens, a virtual and erect image which is half the size of the object is formed.
   (a) Explain whether the lens is a concave or a convex lens.

(b) What is the focal length of the lens?

21. When an object is placed 25 cm from a convex lens, an inverted image which is twice as large as the object, is formed. How far from the lens must the object be placed to obtain an image four times the size of the object?

22. A beam of light consisting of red, green and blue is incident on a right-angled prism as shown in Figure 3.70. The refractive indices of the material of the prism for red, green and blue colours are 1.39, 1.44 and 1.47 respectively. Which colour suffers total internal reflection? Explain.

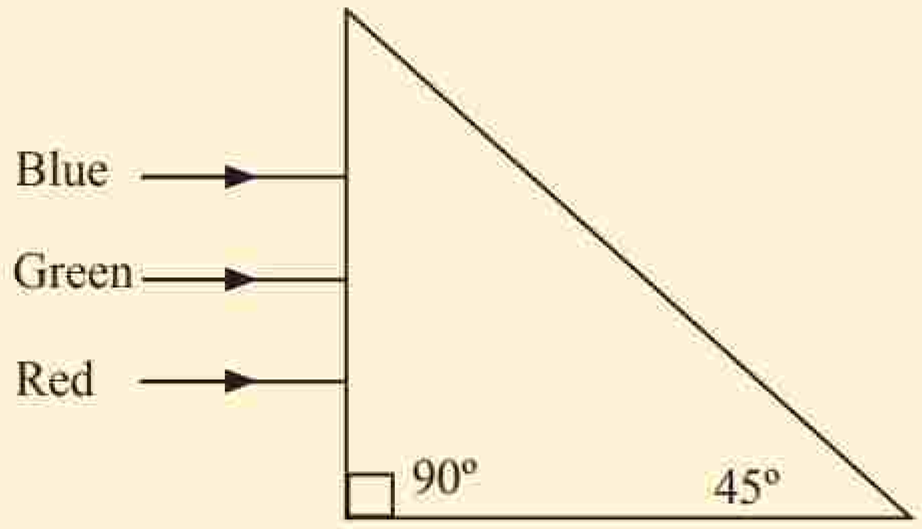


Figure 3.70

23. Rays of light from luminous object are brought to a focus at point Q. A convex lens of 24 cm focal length is then placed 12 cm from Q so as to intercept the rays before they meet at Q. If now they meet at P, find the distance QP.

24. A glass block whose refractive index is 1.564 for sodium light is to be used to construct a prism such that the angle of minimum deviation for such light shall be equal to the angle of the prism. What is the angle of the prism?

Chapter Four

Optical instruments

Introduction

In the modern era, one can view the stars that are billions of kilometres away from the earth's surface and cannot be seen by naked eyes. Biologists can see micro-organisms such as bacteria, which are normally invisible by the naked eye. But how are these activities possible? The abilities of the human eye can be greatly improved when the eye is aided by optical instruments. These devices are used to help the human eye to see small or distant objects clearly. Optical instruments are designed to use light to form images. They use various combinations of lenses, mirrors and prisms to produce enhanced images of objects. Common optical instruments include microscopes (simple and compound microscopes), telescopes, binoculars, cameras and projectors. The human eye itself is the principal and natural optical device. In this chapter, you will learn the principles of action of different optical instruments. The competencies developed will enable you to make correct decisions on the choice of the right optical instruments for various purposes, properly use various optical instruments and construct your own simple optical instruments.

Simple microscope

The size of an object as viewed by the eye is determined by the size of its image formed on the retina. The size of the image depends on the distance from the object to the observer's eye and the angle subtended by the object at the eye, called angular size (θ). If you want to look closely at a small object such as an insect, you bring the object close to your eye. By doing so, you increase the angular size and hence make the image on the retina as large as possible. However, your eyes cannot focus sharply on objects that are closer than a point at about 25 cm from the eye. This point is known as the near point of the eye. Moving an object closer to the near point of the eye, increases the size of image on your retina. But if you still move the object closer than the near point, the image will be fuzzy or blurred because the lens of the eye will no longer focus an image on the retina. Therefore, the closest comfortable distance for viewing an image is when an object is at this point. Thus, an object cannot be brought closer to the eye beyond the near point. To overcome this limitation, a converging lens can be used to enlarge the image of an object. The object can then be moved even closer to the eye, resulting in a large image formed on the retina. A lens used in this way forms a device known as a simple microscope.

Structure of a simple microscope

A simple microscope is made of a biconvex lens normally held by a round-shaped frame with a handle. It can therefore be hand-held and moved according to the user's needs. A simple microscope is sometimes called a magnifying glass or simply a magnifier. Figure 4.1 shows a simple microscope.

Figure 4.1: *A simple microscope*

Mode of action of a simple microscope

When using a simple microscope, you can simply put it over the object to be viewed. For the comfortability of the observer's eye, the position of the magnifying glass with respect to the object is adjusted so that the object is at the focal point of the lens. This produces a virtual, upright and magnified image of the object. Note that, the largest image can be formed on the retina of a naked eye when the object is at the near point ($D \approx 25$ cm). Therefore, for the clear and maximum size of the image to be formed on the retina, the magnifier must have a focal length of approximately 25 cm. The image is formed at infinity when the position of the magnifying glass or the object itself is adjusted so that the object is at the focal point of the magnifying glass. In this way, the eyes see a magnified image such that smaller features can be observed. Figure 4.2 illustrates the action of a simple microscope.

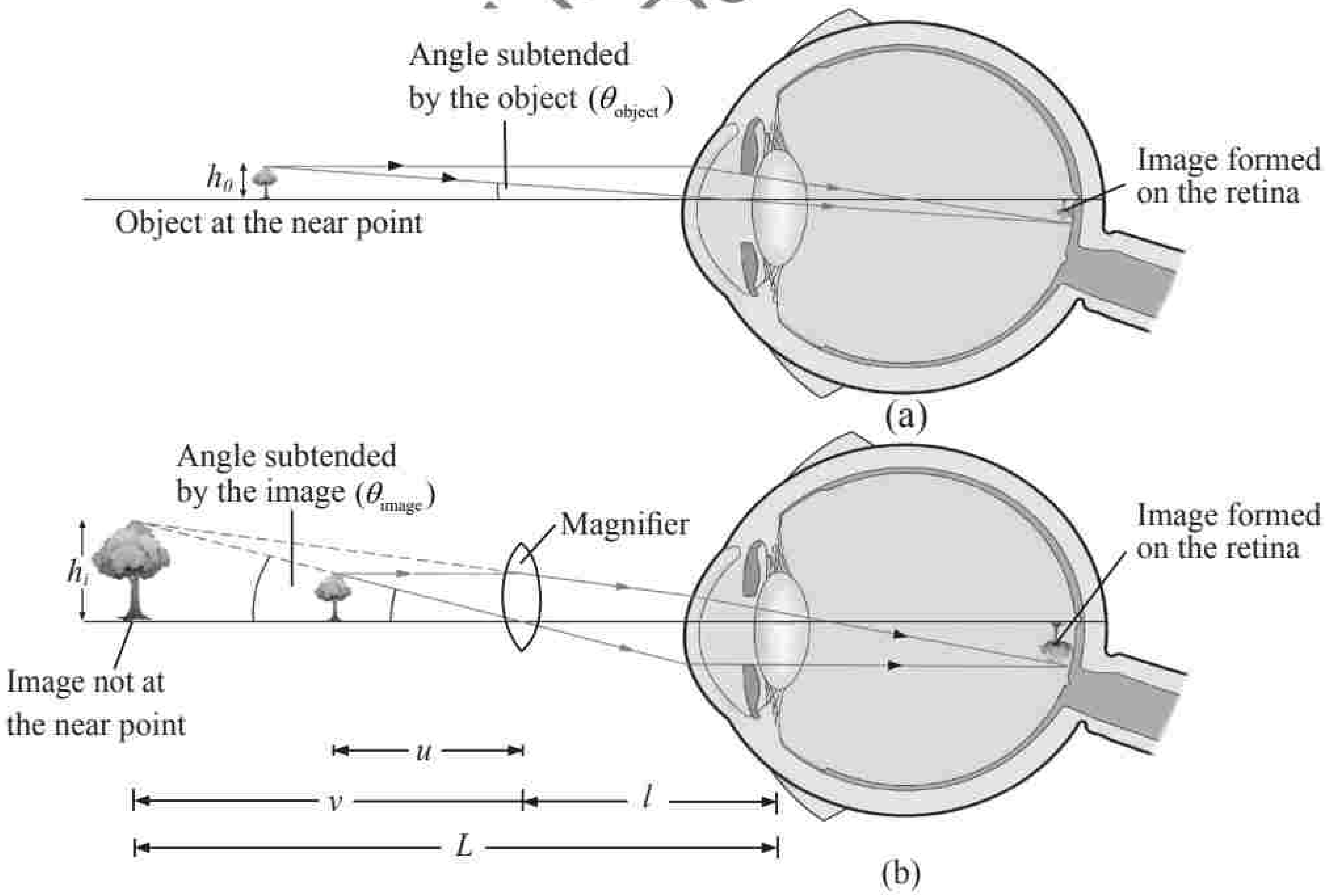


Figure 4.2: *Viewing an object (a) with a naked eye (b) with the aid of a magnifier*

Magnification

A simple microscope magnifies an object by increasing its angular size. The magnification produced by the simple microscope depends on the focal length of the lens. Lenses of short focal length give larger magnification than those with long focal length. In Figure 4.2(a), the object is at the near point, where it subtends an angle θ_{object} to the eye. On the other hand, in Figure 4.2(b), the magnifier in front of the eye forms an image far away from the lens; the image subtends an angle θ_{image} at the magnifier. The usefulness of the magnifier is given by the ratio of the angle subtended by the image (θ_{image}) to the angle subtended by the object (θ_{object}). This ratio is called the angular magnification, M, and is given by,

$$M = \frac{\theta_{image}}{\theta_{object}}$$

The angular magnification should not be confused with the linear or lateral magnification, m, of a simple lens which is the ratio of the image height, h_i, to the object height, h_o, given by,

$$m = \frac{h_i}{h_0}$$

Linear magnification is also given by the ratio of the image distance, v, to the object distance, u. That is:

$$m = \frac{v}{u}$$

It follows that, $\frac{v}{u} = \frac{h_i}{h_0}$ and thus $v = \frac{uh_i}{h_0}$.

To find the value of angular magnification, let the angles subtended by the object and the image be very small such that, $\sin\theta_{object} = \tan\theta_{object} \approx \theta_{object}$ and

$\sin\theta_{image} = \tan\theta_{image} \approx \theta_{image}$. Suppose the object in Figure 4.2, has a height h_o, and it is placed at the focal point, $f = D = 25$ cm, where it subtends an angle θ_{object}. If the angle θ_{object} is very small, then,

$$\tan\theta_{object} \approx \theta_{object} = \frac{h_o}{D}$$

The virtual image formed by the lens subtends a small angle θ_{image} such that,

$$\tan\theta_{image} \approx \theta_{image} = \frac{h_i}{v}$$

But, $v = \frac{uh_i}{h_o}$

Therefore,

$$\tan\theta_{image} \approx \theta_{image} = h_i \times \frac{h_o}{uh_i} = \frac{h_o}{u}$$

Now,

$$M = \frac{\theta_{image}}{\theta_{object}} = \frac{h_o}{u} \div \frac{h_o}{D} = \frac{h_o}{u} \times \frac{D}{h_o} = \frac{D}{u}$$

Hence, angular magnification,

$$M = \frac{D}{u}$$

Using the lens formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

The angular magnification is maximum when the image is at the near point of the eye.

Then, $\frac{1}{f} = \frac{1}{u} + \frac{1}{25}$

Using real is positive, $\frac{1}{f} = \frac{1}{u} - \frac{1}{25}$

This implies that, $u = \dfrac{25f}{25+f}$

Since $M = \dfrac{D}{u}$, and D = v,

then, $M = \dfrac{25}{\left(\dfrac{25f}{25+f}\right)} = 1 + \dfrac{25}{f}$

$$M = 1 + \frac{25}{f}$$

This is the expression for the angular magnification of the simple microscope.

The eye is most relaxed when the image is at infinity. In this case, the object has to be at the focal point of the lens. Thus,

$$\theta_{\text{object}} \approx \frac{h_o}{25} \text{ and } \theta_{\text{image}} \approx \frac{h_i}{f}$$

Here the magnification is said to be minimum, given by,

$$M = \frac{\theta_{\text{object}}}{\theta_{\text{image}}} = \frac{h_i}{f} \div \frac{h_o}{25} = \frac{25 \text{ cm}}{f}$$

Angular magnification is also known as the magnifying power of the microscope. The angular magnification of a simple microscope may be made as large as possible by decreasing its focal length f. However, decreasing the focal length is limited by the imperfection of the lens. The limitation of the simple microscope can be overcome by using a compound microscope.

Example 4.1

A simple microscope with a focal length of 5 cm is used to read the division of scale 1.5 mm in size. How large will the size of the divisions be as seen through the simple microscope?

Solution

$f = 5$ cm, $h_0 = 1.5$ mm

$M = \dfrac{25}{f} + 1 = 1 + \dfrac{25 \text{ cm}}{5 \text{ cm}} = 6$

$M = \dfrac{h_i}{h_0}$

$h_i = h_0 M$

$h_i = 1.5 \text{ mm} \times 6 = 9 \text{ mm or } 0.9 \text{ cm}$

Each division will appear to have a size of 9 mm when viewed through the simple microscope.

Construction of a simple microscope

One can construct a simple microscope for various simple uses. Materials that are important in constructing a simple microscope include, a biconvex lens and a hard box.

Task 4.1

Construct a simple microscope

Task 4.2

1. Fill a transparent plastic bottle with water and seal it.
2. Place the bottle filled with water on a page with texts. Observe what happens to the letters on the page.

Uses of a simple microscope

A simple microscope has a range of applications. The following are some of the uses of a simple microscope:

1. It is used to view specimen in a laboratory.
2. It is used by watchmakers to view small components of a watch.
3. It is used to read small prints.
4. It is used to concentrate light rays.

Exercise 4.1

1. A small object is placed 3 cm from the lens of a simple microscope. The focal length of the lens is 5 cm.
   (a) Find the linear magnification produced by the simple microscope.
   (b) How far from the lens should you place the object to obtain maximum magnification of the image?
2. A magnifying glass of focal length 5 cm is used to magnify a small object held 4 cm from the optical centre of the lens. Determine the position and magnification of the formed image.
3. The magnifying power of a simple microscope is 8. What will be the focal length?
4. A student with a near-normal point (25 cm) reads a story book with small fonts using a thin convex lens of focal length 5 cm.
   (a) What is the closest and the farthest distance the student can read when viewing through the magnifying glass?
   (b) What are the maximum and minimum angular magnification (magnifying power) possible using the above simple microscope?

Compound microscope

The magnification of a simple microscope is limited to $M \leq 9$ for realistic focal lengths. This means that, the object will appear 9 times larger when viewed using a simple microscope of maximum magnification. For example, an object whose size is 1 mm will be seen to be 9 mm when viewed through a magnifier. However, the size of bacteria, for example, is much smaller than 1 mm. Thus, to see a bacteria, a magnification larger than 9 is required. To achieve this, a new microscope that uses two lenses in such a way that one lens compounds the effect of the other lens, is normally used. Such a microscope is called a compound microscope.

Structure of a compound microscope

A compound microscope is composed of a lens system, a lighting system and a focusing system. The lens system is made up of two convex lenses. A lens that is placed nearer the object is called the objective lens. This lens forms a real, inverted and magnified image of the object. The image formed by the objective lens serves as the object for the second lens, which is known as the eyepiece. The eyepiece functions as a simple microscope to produce the final image, which is enlarged and virtual. A compound microscope has a revolving nose piece that allows the user to change the objective lens according to needs.

The compound microscope consists also of the lighting system, which may be called the illumination system. It consists of a light source, which can be an electric bulb, a light-emitting diode (LED) or sometimes natural sunlight. There is also a mirror and a condenser which help to transmit light through an object for viewing.

On the other hand, the compound microscope has a focusing system. This system is made of coarse and fine adjustment knobs, which are essential for moving the objective lens away or towards the object in searching for a clear image. The system also has the inclination joint, which helps in tilting the microscope for a comfortable view.

Other parts of a compound microscope include the body tube, which separates the objective and the eyepiece and assures continuous alignment of the optics. A stage is used for holding the object to be viewed and the base supports the whole structure of the microscope. Figure 4.3 shows important parts of a compound microscope.

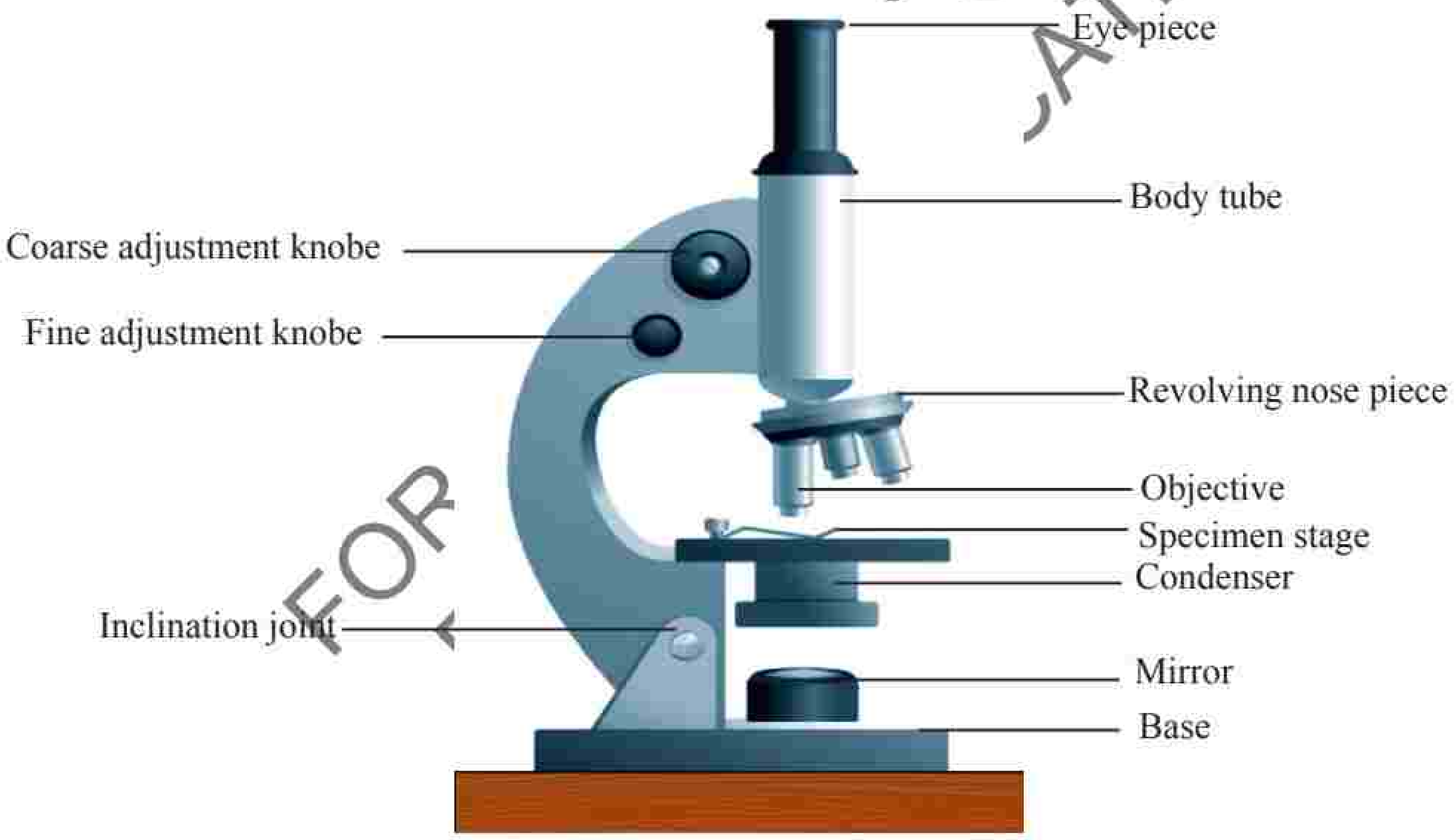


Figure 4.3: *A compound microscope*

Mode of action of a compound microscope

A compound microscope works by using the principle that, an image formed by one optical element can serve as the object for the second optical element. That is, an image formed by the objective lens is used as the object for the eyepiece. The object to be viewed is placed just beyond the first focal point F_o of the objective lens, which forms a real and enlarged image I_1 as illustrated by Figure 4.4.

In a properly designed compound microscope, the image formed by the objective lens lies just inside the focal point F_e of the eyepiece. The eyepiece (or ocular) acts as a simple magnifier and forms a final virtual image I_2. The position of I_2 may be anywhere between the near and far points of the eye.

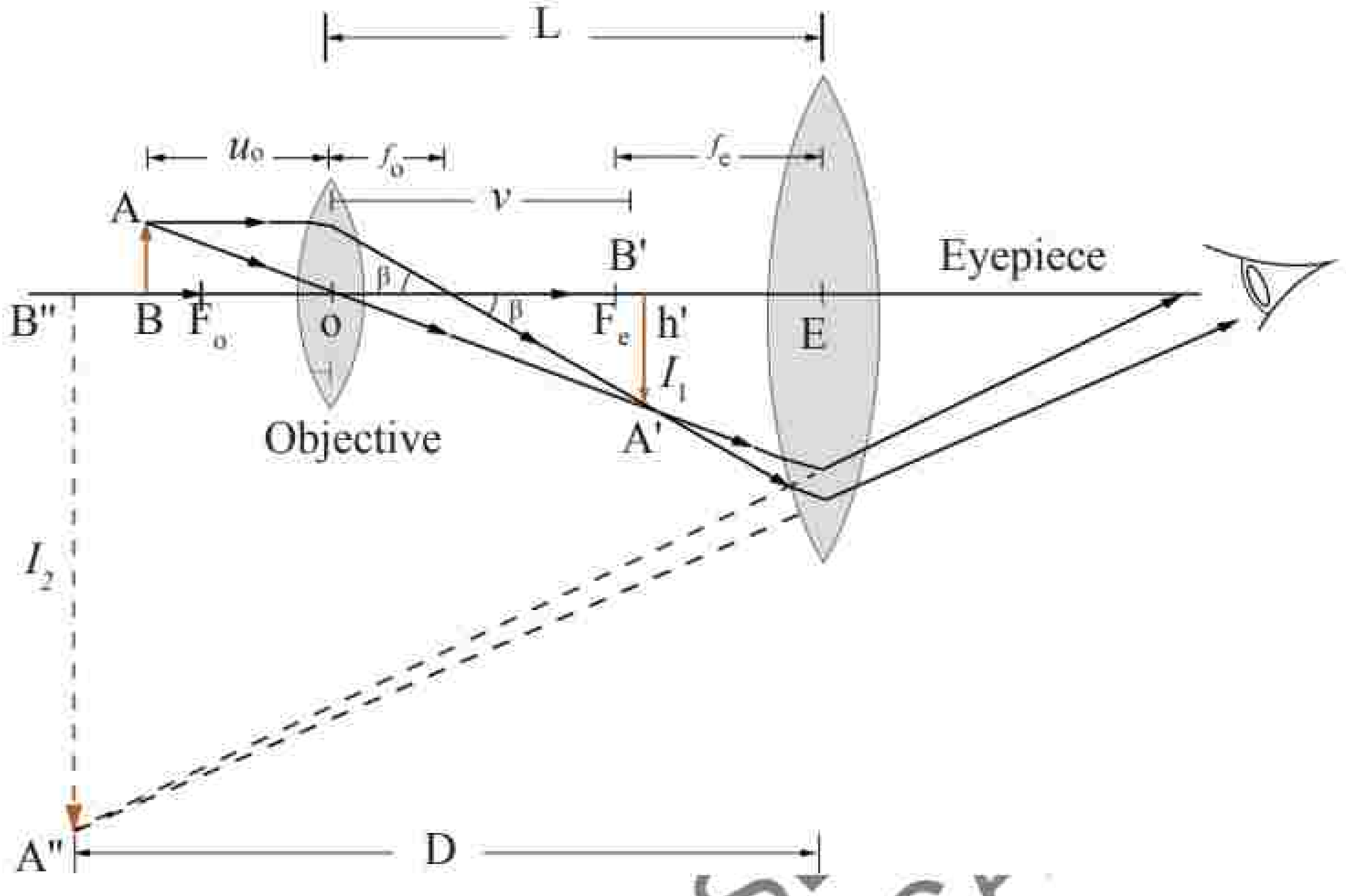


Figure 4.4: *Image formation by a compound microscope*

Magnification of a compound microscope

As for a simple magnifier, what matters when viewing through a microscope is the angular magnification M. The overall angular magnification of the compound microscope is the product of two factors.

The first factor is the lateral magnification m_o of the objective lens, which determines the linear size of the real image I_1. The second factor is the angular magnification M_e of the eyepiece, which relates the angular size of the virtual image seen through the eyepiece to the angular size that the real image I_1 would have if you view it without the eyepiece.

The first factor is given by the ratio of distance of the image formed by the objective lens to the distance between the object and the objective lens. According to Figure 4.4, the lateral magnification is,

$$m_o = \frac{v}{u_o}$$

The object is normally placed at a point very close to the focal point F_o of the objective lens, that is, $u_o \approx f_o$. Therefore, the lateral magnification of the objective lens is approximately taken as,

$$m_o = \frac{v}{f_o} \quad \text{but} \quad v \approx L - f_e$$

$$m_o = \frac{L - f_e}{f_o}$$

The real image I_1, formed by the objective lens is close to the focal point F_e of the eyepiece, that is, $u_e \approx f_e$. As a result, the image formed by the eyepiece appears at infinity. Although the eye can focus on an image formed anywhere between the near point and infinity, it is most relaxed when the image is at infinity. From the relation, $M_e = \frac{D}{u_e}$ then, angular magnification is,

$$M_e = \frac{D}{f_e},$$

where f_e is the focal length of the eyepiece.

Since D is approximately 25 cm, then,

$$M_e = \frac{25 \text{ cm}}{f_e}$$

The overall angular magnification M of a compound microscope is the product of the two magnifications; the lateral magnification of the objective lens and the angular magnification of the eyepiece. That is;

$$M = m_o M_e$$

$$M = m_o M_e = \frac{L - f_e}{f_o} \times \frac{25 \text{ cm}}{f_e}$$

To produce large magnification, f_e and f_o must be very small compared to L. Therefore, $L - f_e \approx L$.

$$M = m_o M_e = \frac{L}{f_o} \times \frac{25 \text{ cm}}{f_e}$$

Hence,

$$M = \frac{25L}{f_o f_e}$$

Note that; f_o, f_e and L are measured in centimetres (cm). This expression is used to determine the magnification produced by a compound microscope.

Task 4.3

Observe the image formed by a compound microscope, then state the nature of the formed image.

Example 4.2

A compound microscope consists of the objective lens and the eyepiece lens of focal length 12 cm and 4 cm, respectively. The two lenses are separated by a distance of 30 cm. The microscope is focused so that the image is formed at infinity. Determine the position of the object.

Solution

The final image is at infinity and two lenses are separated by 30 cm distance as shown in Figure 4.5.

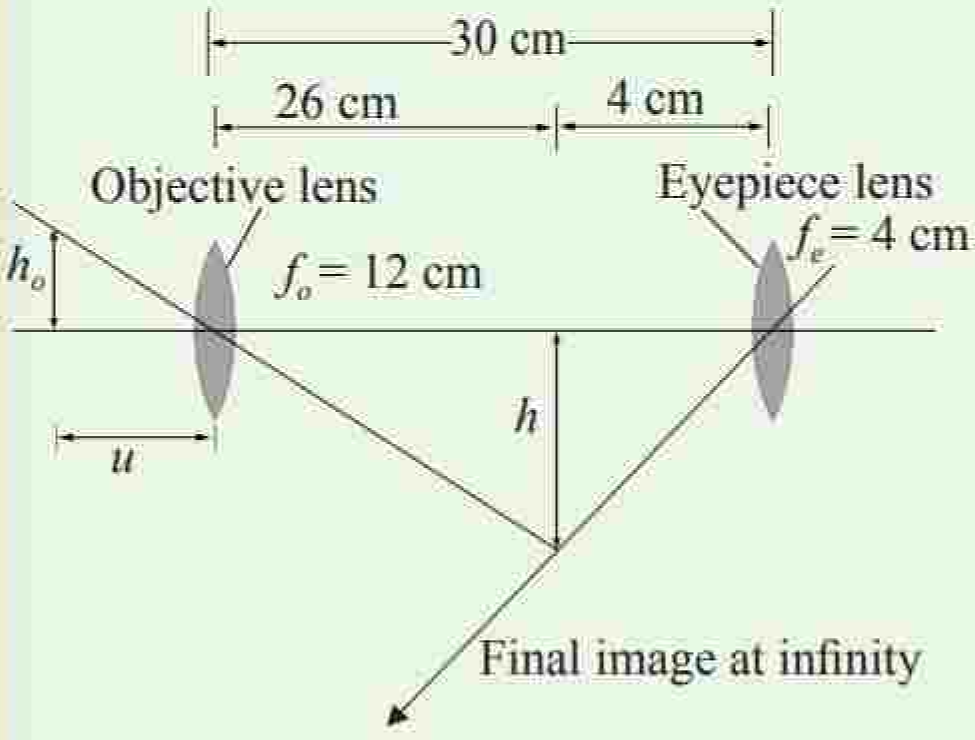


Figure 4.5

From $v = L - f_e$,

$$v = 30 \text{ cm} - 4 \text{ cm} = 26 \text{ cm}$$

Using lens formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}; \quad \frac{1}{12} = \frac{1}{u} + \frac{1}{26}$$

$$\frac{1}{u} = \frac{1}{12 \text{ cm}} - \frac{1}{26 \text{ cm}}; \quad \frac{1}{u} = \frac{26 \text{ cm} - 12 \text{ cm}}{312 \text{ cm}^2}$$

$$\frac{1}{u} = \frac{14}{312 \text{ cm}}$$

$$u = 22.3 \text{ cm}$$

Therefore, the object distance is 22.3 cm from the objective lens.

Uses of a compound microscope

Compound microscopes have a wide range of applications in different aspects of life. Some of the applications of compound microscopes include:

(a) Viewing laboratory small samples such as tissues and body fluids to check for infections caused by micro-organisms;

(b) Studying micro-organisms and cells in Biology experiments; and

(c) Observing the Brownian motion of fluid particles.

Figure 4.6 shows a student observing something using a compound microscope.

Figure 4.6: *Using a compound microscope*

Construction of a simple compound microscope

One can easily construct a simple optical compound microscope. Activity 4.1 illustrates the steps for constructing a simple compound microscope using local materials.

Activity 4.1

Aim: To construct a simple compound microscope

Materials: two convex lenses, hard manila sheets (coloured black), scissors, cutter, rubber disc, drill, a film canister or a small plastic container, plywood, a wooden stick

Procedure

1. Use the black manilla sheets to construct two tubes with slightly different diameters such that the smaller tube can slide into the larger tube. Each tube should be approximately 3 cm long.
2. Fix a lens to one end of each tube.
3. Slide the small tube inside the larger tube such that you have one tube with a lens on both sides of the tube.
4. Cover each side of the tube with a rubber disc to buffer the lenses.
5. Drill a hole at the bottom of a canister.
6. Insert the smaller tube into the open end of the canister to about half the length of the canister. Ensure that the lens coincides with the hole drilled on the canister.
7. Use a square piece of plywood or plastic to set your base. The square should have 10 cm sides. Attach

a white sheet at the middle of the base to act as the stage.

8. Use a wooden stick to create a vertical stand. Make the stand 2 cm tall.
9. Attach your microscope to the stand and try to observe different small objects using your own microscope.

Question

What is the nature of images of different objects when viewed through your microscope?

The intermediate image formed by a compound microscope is real, inverted and magnified.

Exercise 4.2

1. (a) Draw a labelled ray diagram showing the formation of a final image by a compound microscope at the least distance of distinct vision.

   (b) A compound microscope has an objective lens of focal length 2 cm and eye piece of focal length 6 cm. An object is placed 2.4 cm from the objective lens. If the distance between the objective lens and eyepiece is 17 cm, find the distance of the final image from the eye piece.

2. The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. The microscope is focused on a certain object. The distance between the objective and eyepiece is 14 cm. If the least distance of distinct vision is 20 cm, calculate the focal length of the objective and the eyepiece.

3. Suppose you have lenses A and B having focal lengths 100 cm and 4 cm, respectively. If asked to choose one as an eyepiece, which one would you opt for, and why?

4. (a) Draw a labelled ray diagram of a compound microscope. Explain briefly its working principle.

   (b) In a compound microscope, why should the objective lens form an image at a point around the focal point of the eyepiece?

Astronomical telescope

The human eye has limitations when viewing objects that are very far from the observer. When the object is at a distance far from the point 2F of the eye lens, its image is formed at the focal point of the eye lens. However, this image is extremely small and sometimes cannot be decoded by the brain. How can this limitation be overcome? Astronomers use a device named an astronomical telescope to observe the universe and its components, which are extremely far from our eyes.

Structure of an astronomical telescope

Just like the compound microscope, an astronomical telescope uses two convex lenses: the objective lens and the eyepiece. It also consists of a focusing system for adjusting the sharpness and clarity of the image. However, unlike the compound microscope, the objective lens of a telescope has a large focal length while the eyepiece has a much shorter focal length. Since the astronomical telescope uses a lens as its objective glass, it is also called a refracting telescope. Figure 4.7 shows some parts of the astronomical telescope.

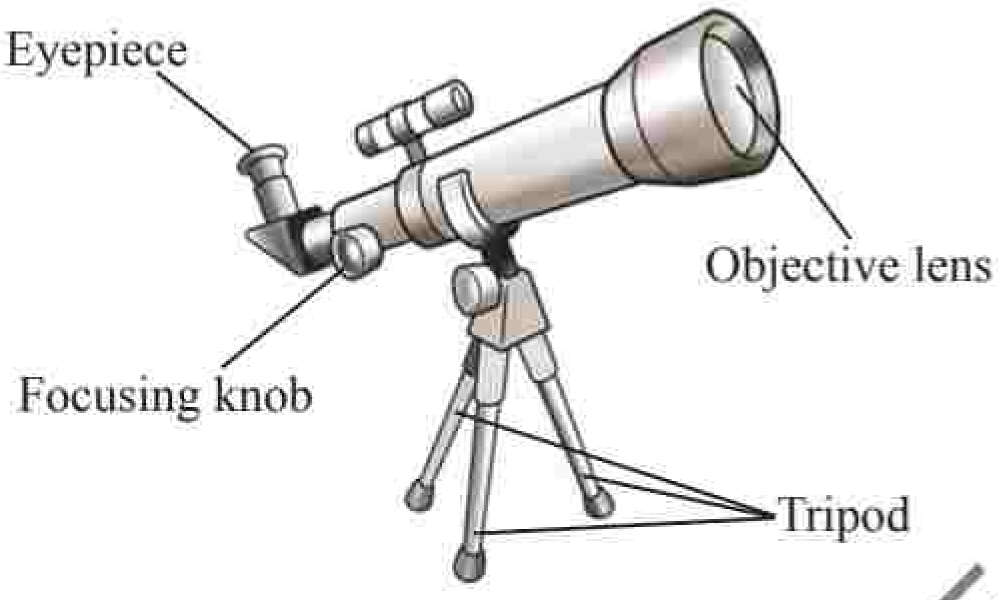


Figure 4.7: *Parts of an astronomical telescope*

Mode of action of an astronomical telescope

A telescope is used to provide angular magnification of distant objects. Light from a distant object enters the objective lens and a real image is formed in the tube at the second focal point of the objective lens. This image becomes the object for the eyepiece. The position of the eyepiece lens is adjusted until the image from the objective lens falls at the focal point of the eyepiece. The image of the eyepiece is inverted and magnified as illustrated in Figure 4.8.

Note that, the main considerations with an astronomical telescope are its light-gathering power and its resolution or resolving power. The light-gathering power depends on the area of the objective lens. However, the larger the diameter of the objective lens, the fainter the image observed. The resolving power, which is the ability to observe two close objects distinctly, also depends on the diameter of the objective lens. So the desire is to make astronomical telescopes with an objective lens of larger diameter. Nevertheless, lenses with big diameters tend to be very heavy and therefore, difficult to make and to be supported by their edges. Further, it is rather difficult and expensive to make such large-sized lenses that may form images that are free from any kind of distortions.

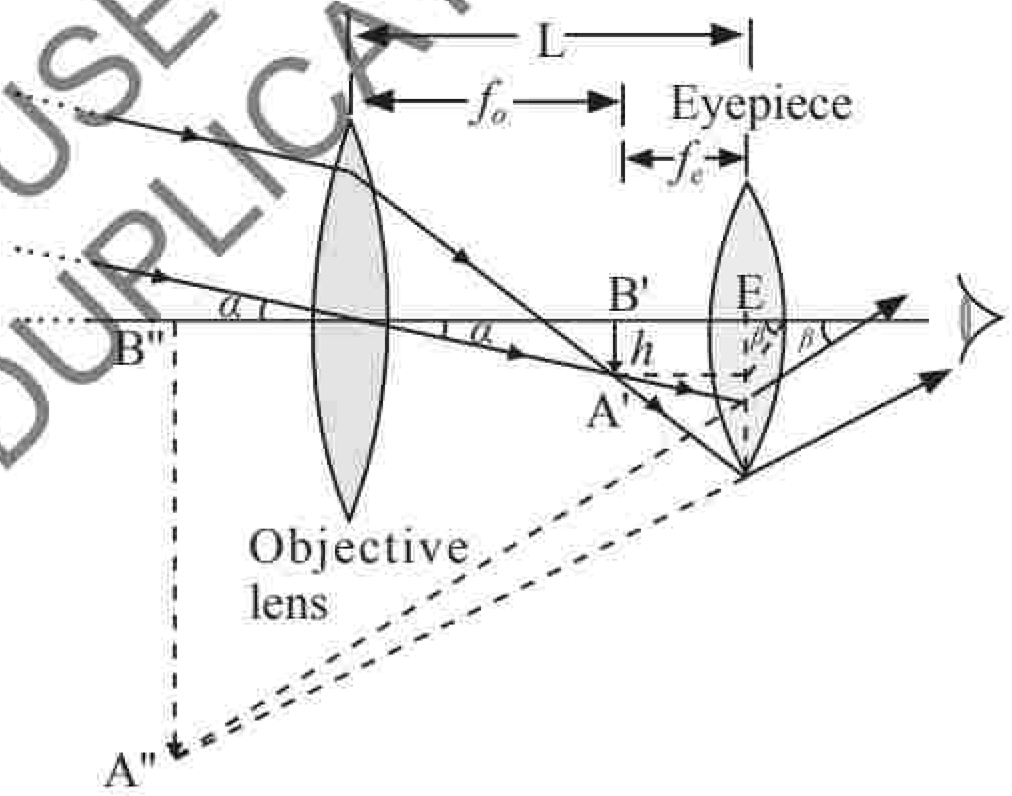


Figure 4.8: *Image formation by an astronomical telescope*

Magnification of an astronomical telescope

The magnifying power, *m,* of the astronomical telescope is given by the ratio of the angle β subtended by the final image at the eye and the angle α subtended by the object at the objective lens. That is;

$$m = \frac{\beta}{\alpha}$$

Assuming that the angles are very small, we have,

$$\tan\alpha \approx \alpha \text{ and } \tan\beta \approx \beta$$

The angle subtended by the object at the objective lens is the same as the angle subtended by the first image at the objective lens. Thus,

$$\tan\alpha \approx \alpha = \frac{h}{f_o}.$$

Similarly, the angle subtended by the final image at the eye is the same as the angle that a ray coming from the head of first image and travelling parallel to the principal axis makes with the principal axis after it passes through the lens. Then,

$$\tan\beta \approx \beta = \frac{h}{f_e}$$

Thus,

$$m = \frac{h}{f_e} \times \frac{f_o}{h} = \frac{f_o}{f_e}.$$

Therefore, the magnification produced by an astronomical telescope is the ratio of the focal length of the objective lens to that of the eyepiece lens. Note that the distance between the two lenses, $L = f_e + f_o$.

Example 4.3

Find the magnification of a telescope with an objective lens whose focal length is 100 cm and an eyepiece lens whose focal length is 2 cm.

Solution

$$m = \frac{f_o}{f_e}$$

$$m = \frac{100 \text{ cm}}{2 \text{ cm}} = 50$$

Uses of an astronomical telescope

A telescope is used to view distant objects such as stars and planets. It is one of the primary tools in the fields of astronomy and astrophysics

Construction of a simple astronomical telescope

One can easily construct a simple astronomical telescope using locally available materials.

Task 4.4

Construct a simple astronomical telescope using materials that are locally available in your surroundings.

Exercise 4.3

1. An astronomical telescope in normal adjustment has a total length of 78 cm and produces an angular magnification of 15. What is the focal length of the objective and eyepiece lens?
2. An astronomical telescope has its two lenses 78.0 cm apart. If the objective lens of the telescope has a focal length of 75.5 cm, what is the magnification produced by the telescope?
3. A homemade telescope has an objective lens of focal length 140 cm and an eyepiece of

focal length 4 cm. What is the magnifying power of this telescope? Determine the separation distance between the objective lens and the eyepiece.

4. How would the magnification of an astronomical telescope be affected if the focal length of the eyepiece and the objective lens are increased.

5. An observatory telescope has an objective lens of focal length 14 m. Suppose an eyepiece of focal length 2 cm is used,
   (a) What is angular magnification of the telescope?
   (b) If this telescope is used to view the moon, what is the diameter of the moon's image formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of the lunar orbit is 3.8×10^8 m.

Binoculars

When you visit a national park, you cannot get very close to dangerous animals, such as lions and leopards, for safety reasons. To closely watch such animals, one needs to use a device that can help the eye to see distant objects closely. An optical device used in such a situation is called binoculars.

Structure of binoculars

Binoculars can be considered as having two telescopes that are exactly the same and placed beside each other such that they accurately point in the same direction. This allows the observer to use both eyes when looking at distant objects. The main components of binoculars are the eyepiece, the objective lens and prisms. The objective lens focuses a distant object to a point near the focal point of the eyepiece, which magnifies that image. A pair of prisms inverts the image so that it can be seen properly upright by the eyes. Other parts are the focusing system which allows the lenses to be moved back and forth. Figure 4.9 shows some parts of binoculars.

Figure 4.9: *Parts of a pair of binoculars*

Mode of action of binoculars

Two objective lenses are situated at the ends of both telescopes making the binoculars. The purpose of the objective lens is to collect light from the object that the user is looking at and form the image of the object at or near the focal point of the eyepiece. The eyepiece can be thought of as a magnifying glass. It picks up the small image formed by the objective lens and magnifies it so that the observer can clearly see the object.

However, when light is refracted through the objective, the light rays cross over, resulting in an upside-down image. The eyepiece simply magnifies this image so that the viewer sees the object upside-down. This problem can be solved by deploying a pair of prisms. These prisms are essentially used to rotate and reflect the image using the principle of total internal reflection. To rotate the image 180 degrees, prisms are arranged in a way that each prism effectively rotates the image 90 degrees. Normally two types of prisms are used in binoculars: roof prisms and porro prisms. Figure 4.10 shows how image inversion is achieved in binoculars.

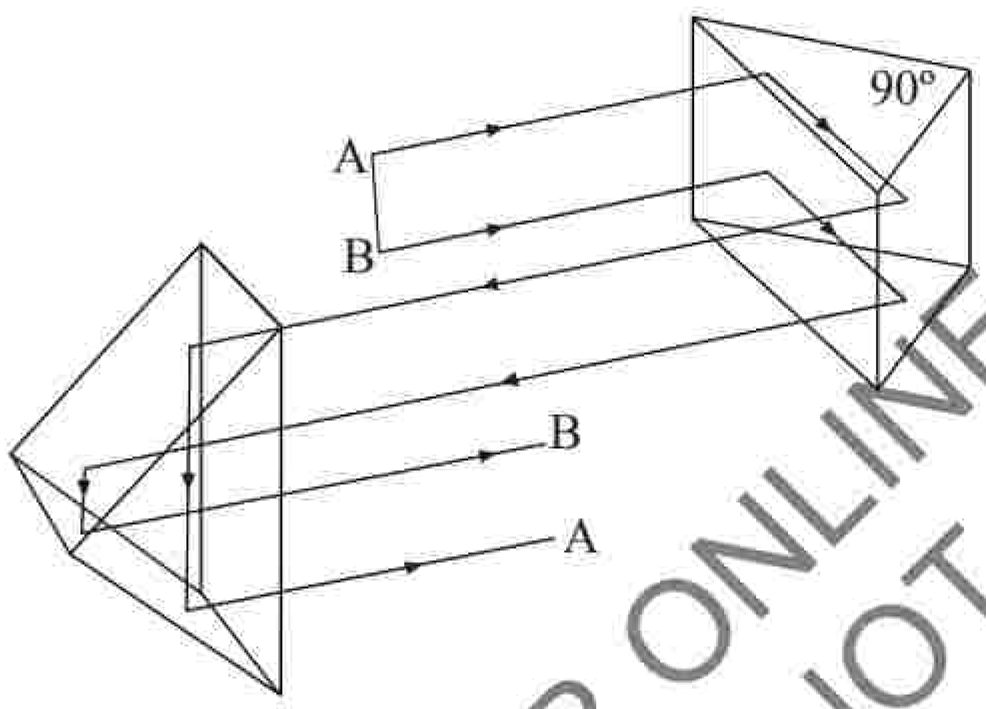


Figure 4.10: *Image inversion by prisms*

Magnification of binoculars

Since binoculars are made of two identical refracting telescopes, the magnifying power of the binoculars can be calculated as in the case of the astronomical telescope. That is;

$$m = \frac{f_o}{f_e}$$

where, f_o is the focal length of the objective lens and f_e is the focal length of the eyepiece.

The quality of images formed by binoculars is determined by the magnification power and the size of the objective lens. While the magnifying power affects the size of the image, the size of the objective lens affects the clarity or details of the image. Commercial binoculars are normally imprinted with information about the objective lens size and the magnification power.

Example 4.4

A binoculars, acting as a telescope, produces an angular magnification of 7.5. What is the eyepiece's focal length if the binoculars have an objective lens of a focal length 75.0 cm?

Solution

$$m = \frac{f_o}{f_e}$$

$$f_e = \frac{75.0 \text{ cm}}{7.5} = 10 \text{ cm}$$

Uses of binoculars

Being able to focus on distant objects, binoculars are used in various activities that require the observer to view distant objects in detail. For example, binoculars are commonly used by tourists to view animals. Also, soldiers use binoculars to view the enemy camp from a far point. Likewise, surveyors use binoculars to survey and observe the landscape and other features.

Construction of a simple binocular

A simple binocular can be constructed using locally available materials.

Task 4.5

Use the locally available materials to design and construct a simple binocular and use it to view various distant objects. Discuss with your class members, factors which may affect the performance of your binoculars.

Projection lantern

In modern era, you may see that, lesson notes are projected on a white screen so that everybody in a class can see them. Similarly, in movie theatres, movies are cast onto big screens for better view and enjoyment. The projection of lesson notes and movies is made possible by a device commonly known as a projector. Although recent projectors use different technologies, old projectors used incandescent light sources, a concave mirror and a set of convex lenses to project large images on a distant screen. Such projectors are therefore called projection lanterns.

Structure of a projection lantern

A projection lantern consists mainly of a concave mirror, a pair of plano-convex lenses, a biconvex lens and a light source. Figure 4.11 shows some important parts of a projection lantern. The lamp produces light which is concentrated by the concave mirror, called the reflector. The fan helps to cool the lamp as it would otherwise get very hot. The condenser, which is made of a couple of plano-convex lenses, refracts the light so that a whole slide is illuminated with parallel light rays. The projection lens forms the image of the slide on the screen while the focusing knob allows the adjustment of the focusing lens position.

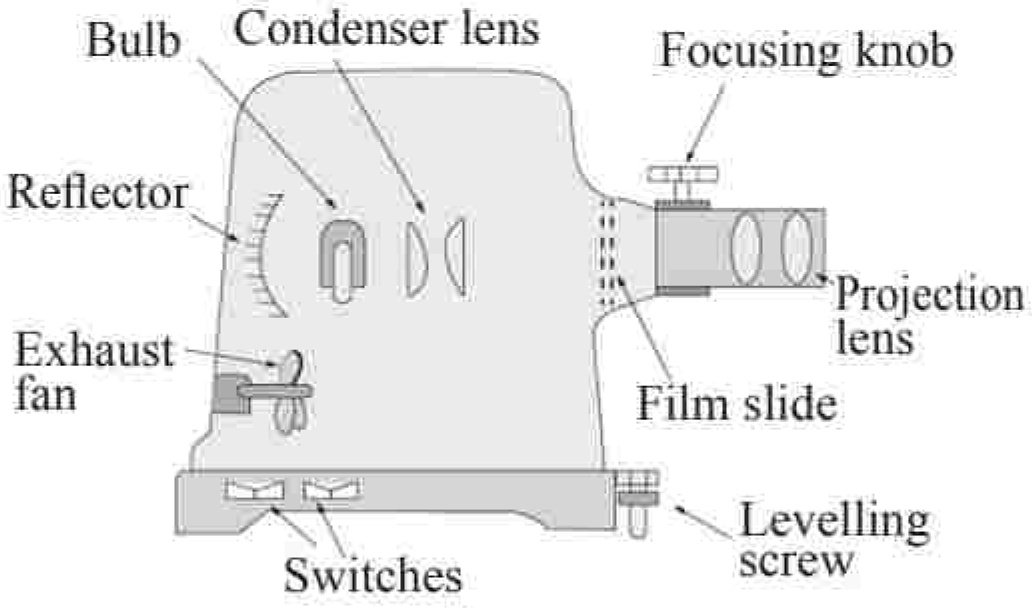


Figure 4.11: *Parts of a projection lantern*

Mode of action of a projection lantern

The light source is placed at the centre of curvature of the concave mirror. The mirror reflects the light back through the condenser. The condenser is made up of two converging lenses which refract the light such that, all parts of the slide are illuminated with parallel rays. Note that, the image must be real so that it can be projected on the screen. The slide (object) must be placed between F and 2F of the projection lens so as to produce a real, enlarged and inverted image on the screen. This is done by adjusting the projection lens in or out to change the distance of the lens from the slide. Because the image is inverted, the slide must be placed upside down and laterally inverted so that we the upright image can be seen on the screen. Figure 4.12 illustrates the working principle of a projection lantern. Note that, the size of the image formed on the screen is influenced by the size of the slide as well as the size of the screen on which the slide's image is projected.

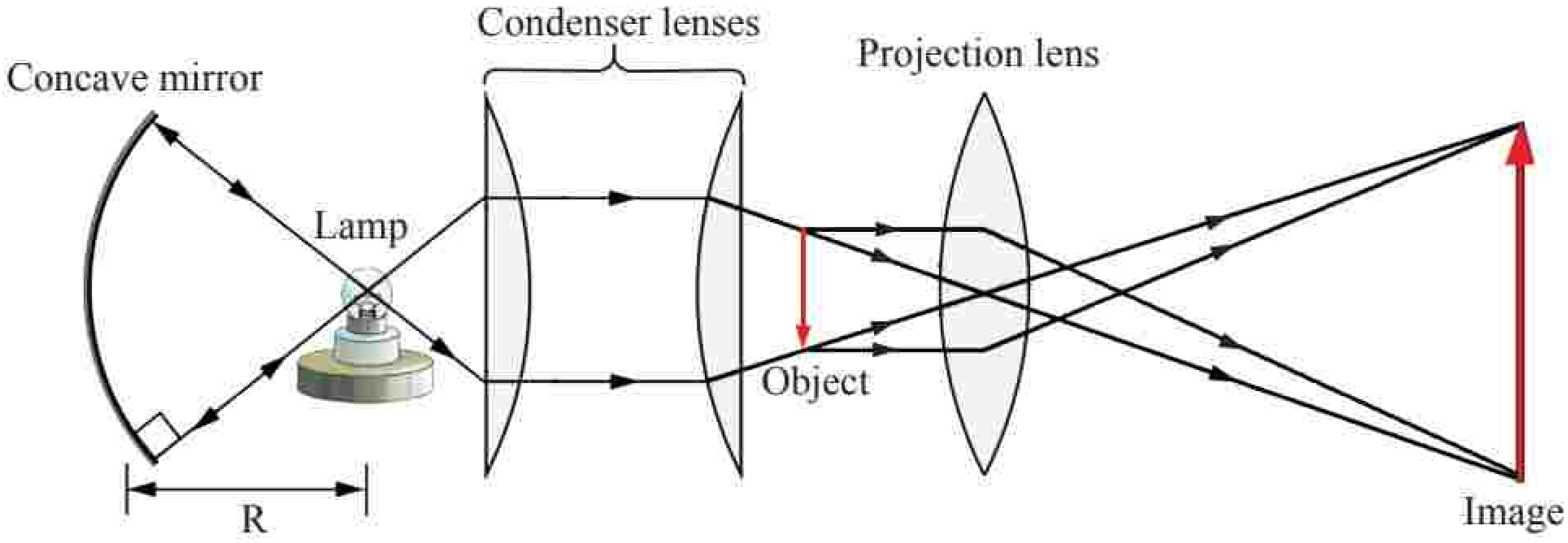


Figure 4.12: *Image formation by a projection lantern*

Magnification of a projection lantern

The magnification produced by a projection lantern is given by the ratio of the image height to the object height. That is,

$$m = \frac{h_i}{h_o}$$

The magnification can also be given by the ratio of the image distance to the object distance. That is,

$$m = \frac{v}{u}$$

Example 4.5

A projection lantern gives the image on the screen 24 times as large as the slide. If the screen is 7.2 m from the projecting lens, what is the position of the slide from the lens?

Solution

$$m = \frac{v}{u}$$

$$24 = \frac{7.2 \text{ m}}{u}$$

$$u = 0.3 \text{ m}$$

Therefore, the slide is at a distance of 0.3 m from the lens.

Uses of projection lantern

Lantern projectors are used for many purposes, including:

1. Displaying notes and diagrams during a class session.
2. Displaying pictures in art and entertainment halls.
3. When aided with some more accessories, movies can be projected to a large screen.
4. Projection lanterns are coupled with microscopes to project images of very small objects.

Construction of a simple projection lantern

A simple projection lantern can be constructed at home using some locally available materials. Some of the materials are boxes, bulbs, lenses from unused cameras and shaving mirrors.

Task 4.6

Use locally available materials in your environment to construct a simple projection lantern.

Task 4.7

Visit a nearby institution which uses projectors for teaching or other purposes. Observe how the projection is done.

(a) Point out the differences and similarities between current projectors and lantern projectors.

(b) Discuss with your teacher and fellow students the advantages of current projectors over the lantern projectors.

Exercise 4.4

1. A projector is to be used to focus the image of an object 35 mm high onto a screen measuring 2 m by 2 m. If the distance from the projector to the screen is 10 m, determine the focal length of the projection lens that should be used so that the image fits exactly on the screen.
2. A slide projector has a projection lens of focal length 15 cm. The projector is to be used to project an image of a slide measuring $5 \text{ cm} \times 5 \text{ cm}$ onto a screen measuring $1 \text{ m} \times 1 \text{ m}$. How far from the projection lens should the screen be placed?

The lens camera

The camera is one of the optical instruments that have been used since the discovery of lenses. The word camera is a Latin word that means, "a room or enclosure". Thus, the primary component of a camera is the light-tight enclosure or box. A camera is made up of different components contained inside the light-tight box. Depending on the components, cameras may take many forms including film cameras, digital cameras and video cameras. In this section, we discuss the simplest form of a camera, which is known as a lens camera.

Structure of a lens camera

Major components of a lens camera include a biconvex lens, a film, a diaphragm, a shutter and a mounting base. Figure 4.13 shows some parts of a lens camera.

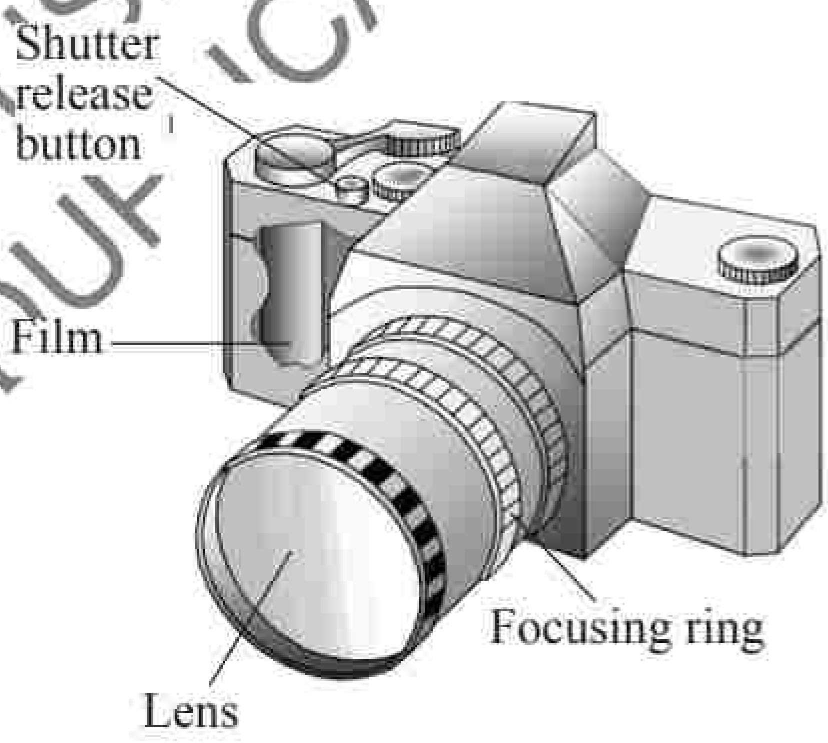


Figure 4.13: *Parts of a lens camera*

The lens is the image-forming device in a camera. The focal length of a lens determines the size of the image that will be formed on the film. Therefore, four basic categories of lenses exist. These are:

(a) **Normal lens** which allows the viewing field of approximately 50 degrees, giving a picture that is normal in size relative to the background and which looks natural to the viewer.

(b) **Wide angle lens** which enables a view of 90 degrees and is used when smaller objects are to be photographed.

(c) **Telephoto lens or long-focus lens** which has a wider field of view than the normal lens. It shows enlarged detail of the image over the same film area. The telephoto lens has a long reach, allowing you to capture an object that is far away. You have probably seen a photograph where an object is in focus but the background is blurred; this is often done with a telephoto lens.

(d) **Interchangeable lens** which offers the photographer opportunity to select a focal length that is best for a given situation. In recent years, variable focal length or "zoom" lenses have become very popular.

The diaphragm determines the amount of light that passes through the lens by changing the size of the aperture. The aperture is an opening whose diameter is adjustable. The size of aperture is measured in *f*-number; the larger the number the smaller the aperture. Most cameras use an iris-type diaphragm, which consists of a number of very thin metal blades. They are mounted so that the size of the lens opening can be changed by a rotating ring or moving lever.

The shutter is a mechanical device that acts as a gate, controlling the duration of time that light is allowed to pass through the lens and fall on the film.

A film is a light-sensitive surface of the camera. It is normally rolled to the back of the camera. In most cameras, the film is wound up on a spool with an interleaving light-tight backing paper. However, in a digital camera, an electronic detector called a charge-coupled device (CCD) array is used instead of a film. The digital information is stored on a memory chip for play back on the screen of the camera.

Mode of action of a lens camera

To take a photograph of an object, the image of the object must be sharply focused on the film. This is done by adjusting the distance of the lens from the film. After focusing and correctly setting the aperture size and shutter time, the shutter release button is pressed. The shutter opens to allow light to enter and expose the film to form an image of the object being photographed. The film is then developed to produce a photograph of the object. Figure 4.14 illustrates the image formation by a lens camera.

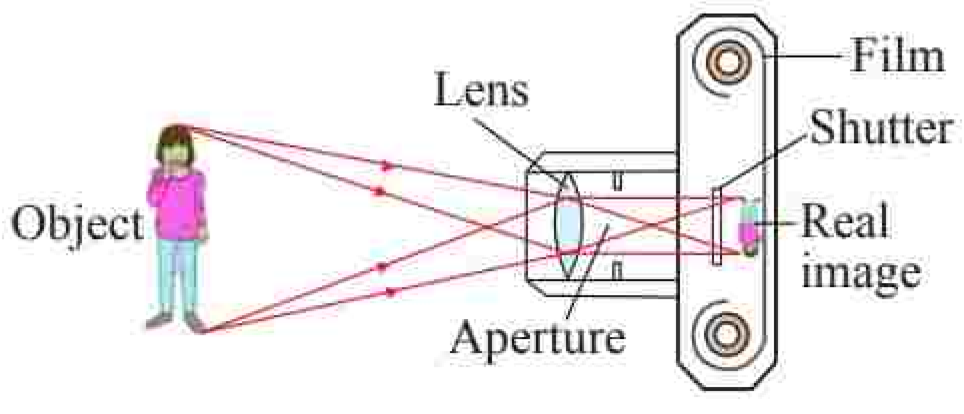


Figure 4.14: *Image formation by a lens camera*

Magnification of a lens camera

The magnification produced by a lens camera is given as;

$$m = \frac{\text{image height } (h_i)}{\text{object height } (h_o)} = \frac{\text{image distance } (v)}{\text{object distance } (u)}$$

Recalling the lens formula, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, one can write;

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u} \text{ and so, } v=\frac{uf}{u-f}, \text{ then,}$$

$$m=\frac{v}{u}=\frac{uf}{u-f}\times\frac{1}{u} = \frac{f}{u-f}$$

Hence, $m=\frac{f}{u-f}$.

Therefore, magnification produced by the lens camera depends on the focal length f of the convex lens used and the object distance u.

Example 4.6

A lens camera of focal length 10 cm is used to take the picture of a girl 1.5 m tall. Determine the magnification of the image if the girl is 11 m from the camera.

Solution

$f = 10$ cm $= 0.1$ m, $u = 11$ m

but $m=\frac{f}{u-f}$

$$m=\frac{0.1\text{ m}}{11\text{ m}-0.1\text{ m}}$$

$$=0.009$$

Construction of a simple lens camera

A simple lens camera can easily be constructed using some local materials including the lens from unused optical instruments. Perform the following activity to construct your own lens camera.

Activity 4.2

Aim: To construct a simple lens camera

Materials: a convex lens, box, a cardboard tube, a translucent piece of paper, a black or opaque paper, glue and optical pins, pencil, ruler and scissors

Procedure

1. Make a round hole on one side of the box and use tape to cover the hole with a translucent piece of paper as shown in Figure 4.15. The translucent paper works as a screen for the camera.

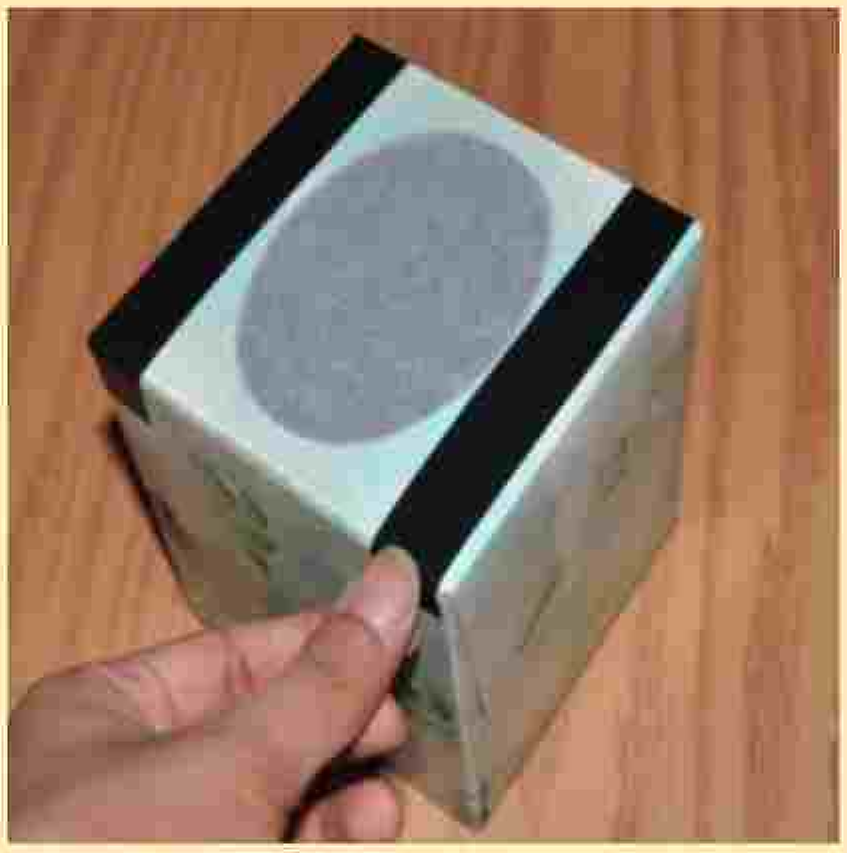


Figure 4.15

2. Hold the cardboard tube on the side of the box opposite the opening. Draw a circle on the box by tracing the cardboard tube.
3. Carefully cut the circle from the box by following the circle drawn in step 2. See Figure 4.16.

Figure 4.16

4. Cover the hole with a black sheet of paper and drill a pinhole at the centre of the black paper. This is a pinhole camera.

5. In a dark room, observe the image formed on the camera screen. An inverted image should be formed on the screen, as shown in Figure 4.17.

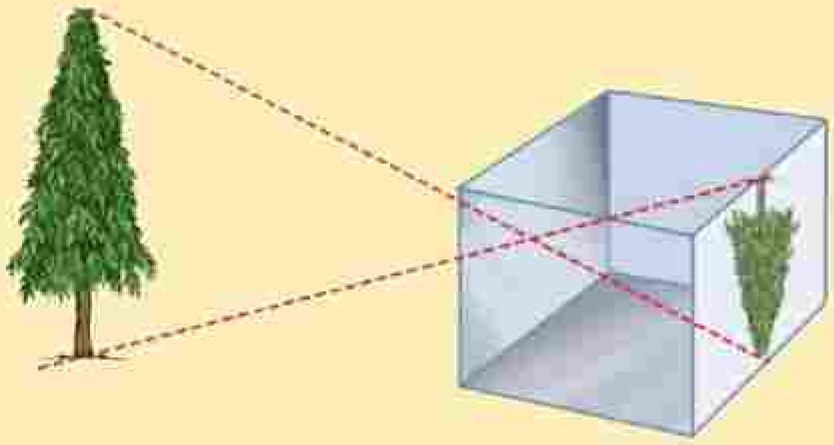


Figure 4.17

6. Drill a few more pinholes around the first pinhole and observe the images on the screen.

7. Now, tape the lens firmly to one open end of the cardboard tube as shown in Figure 4.18.

Figure 4.18

8. Remove the black sheet that covers the hole drilled in step 4 and slide the cardboard tube into it. Make sure that the end of the tube with the lens is pointing outwards as shown in Figure 4.19. You have now made a lens camera. Use your camera to view images of bright objects on the camera screen.

Figure 4.19

Questions

(a) What was the nature of the image formed on the screen of the pinhole camera?

(b) What happened when the number of pinholes was increased?

(c) What factors affect the image formed by a pinhole camera?

(d) What is the role of the lens in a lens camera?

(e) How is a lens camera advantageous over a pinhole camera?

A pinhole camera forms an inverted image on the screen as light passes through the pinhole. However, the image is blurred when more pinholes are drilled around the central pinhole. When a lens is in place, it receives more light and converges all the rays from a given point of an object. This allows a clear image to be formed on the screen of the camera.

Uses of cameras

The main function of a camera is to take photographs. However, specialised cameras have other additional functions as follows:

(a) Video cameras are used to take motion pictures. Figure 4.20 shows a video camera.

Figure 4.20: *A video camera*

(b) Closed-circuit television (CCTV) cameras are used for surveillance in high-security installations, banks, shopping malls, offices and residences. Figure 4.21 shows an example of a CCTV security camera.

Figure 4.21: *Closed-circuit television (CCTV) camera*

(c) Digital cameras are used to capture images that can be fed into computers. An example of digital camera is shown in Figure 4.22.

Figure 4.22: *A digital camera*

Exercise 4.5

1. A photographer used a camera fitted with a lens having a focal length of 50 mm to take a photograph of a flower that is 5 cm in diameter. Suppose the flower is placed 20 cm in front of the camera lens.
   (a) At what distance from the film should the lens be adjusted to obtain a sharp image?
   (b) What would be the diameter of the image of the flower on the film?

(c) What is the nature of the lens of this camera?

2. You are standing at a point 1.2 m in front of a plane mirror while holding a camera. If you wish to take a picture of your own image, at what distance should you focus your camera?

3. A pin-hole camera is used to photograph a person who is 2.8 m tall and standing 2.7 m in front of the camera. If the film is placed 10 cm behind the pinhole, calculate the height of the produced image.

The human eye

The human eye is a natural optical instrument that is exceptionally important for human life. It belongs to a general group of eyes found in nature called "camera-type eyes" since the optical behaviour of the eye is similar to that of a lens camera. The human eye can respond to a range of light frequencies. The human eye looks whitish with a central black spot as seen in Figure 4.23.

Figure 4.23: *The human eye*

Structure of the human eye

The eye is nearly spherical and is about 2.5 cm in diameter. The front portion is somehow more sharply curved and is covered by a tough, transparent membrane called the cornea. The region behind the cornea contains a liquid called aqueous humour. Next to the aqueous humour, there is a crystalline lens, which is a capsule containing a fibrous jelly, hard at the centre and progressively softer at the outer portions.

The crystalline lens is held in place by ligaments that attach it to the ciliary muscles which encircle the lens. In front of the lens, there is an aperture with a variable diameter known as the pupil. The pupil's size is controlled by the iris, which is attached to the ciliary muscles. The iris is responsible for the colour of the eye. It acts as the diaphragm in a lens camera. Behind the lens, the eye is filled with a thin watery jelly called the vitreous humour. This jelly helps to focus the rays of light and also maintains the shape of the eye.

After the vitreous humor, there is a lining in the rear inner surface of the eye called the retina. The retina hosts some photosensitive cells known as cones and rods which respond to the light falling on them. Cones and rods are connected to millions of nerves which are later joined together to form the optic nerve. The eye is protected by a tough whitish skin known as the sclera. Figure 4.24 shows parts of the human eye.

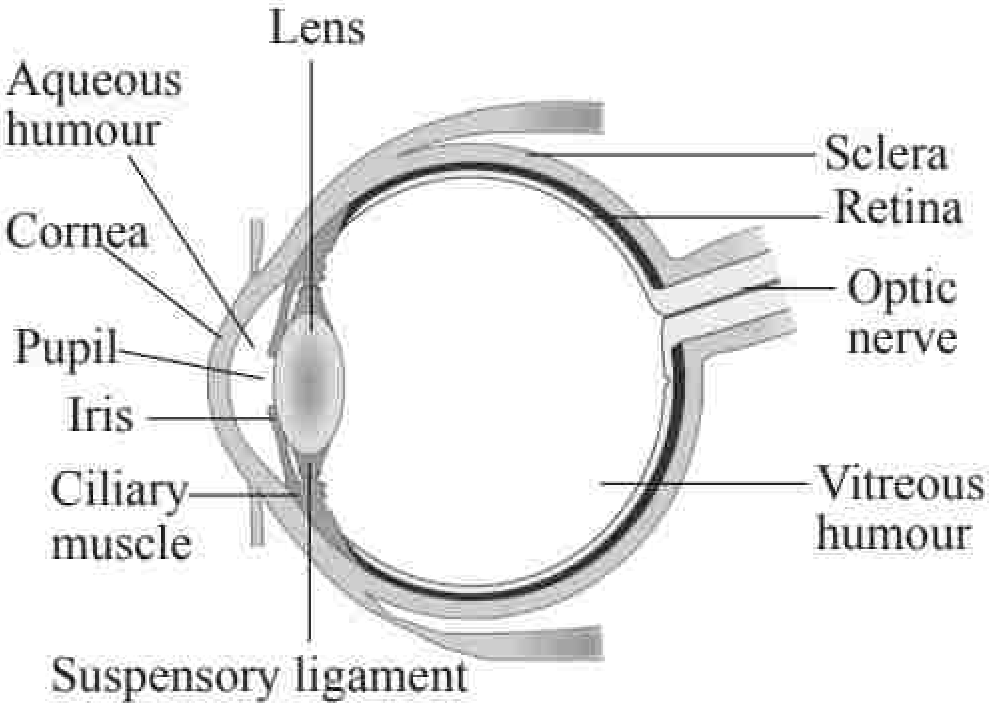


Figure 4.24: *Parts of a human eye*

Images formed by the human eye

When light strikes the eye, it passes through the cornea. As the cornea is shaped like a dome, it bends the light to help the eye focus. The light through the cornea and the aqueous humour enters the eye through the pupil. The iris controls the size of the pupil, which in turn determines the amount of light that enters the eye. Next, light passes through the lens. The lens works together with the cornea to focus light correctly on the retina. Light from the lens travels through the vitreous humor towards the retina. The image is formed on the retina as illustrated in Figure 4.25. When light hits the retina to form the image, the photoreceptors on the retina turn the light into electrical signals, which travel to the brain through the optic nerve.

The indices of refraction of both the aqueous humour and the vitreous humour are about 1.336, which is nearly equal to that of water. The crystalline lens, while not homogeneous, has an average index of refraction of 1.437. This is not very different from the indices of the aqueous and vitreous humour. As a result, most of the refraction of light entering the eye occurs at the outer surface of the cornea.

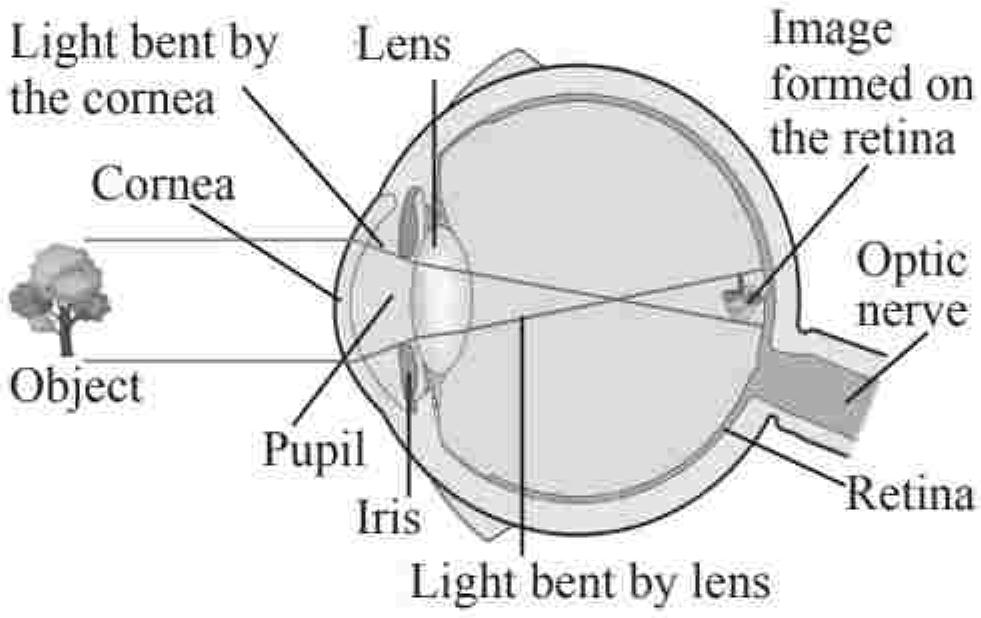


Figure 4.25: *Image formation by the human eye*

Accommodation of the human eye

For an object to be seen sharply, the image must be formed exactly at the location of the retina. The eye adjusts to different object distances u by changing the focal length f of its lens. This is because the lens-to-retina distance, corresponding to v, does not change. However, this is different from the lens camera in which the focal length is fixed and the lens-to-sensor distance is changed. For the normal eye, an object at infinity is sharply focused when the ciliary muscle is relaxed. This causes the curvature of the lens to decrease, thereby increasing its focal length (Figure 4.26(a)). To focus sharply on a closer object, the tension in the ciliary muscle increases causing the ciliary muscle to contract. This causes the lens to bulge, resulting into the increase of its curvature and decrease of its focal length (Figure 4.26(b)). The process by which the eye focuses on objects at different distances by varying the focal length of the lens is called accommodation.

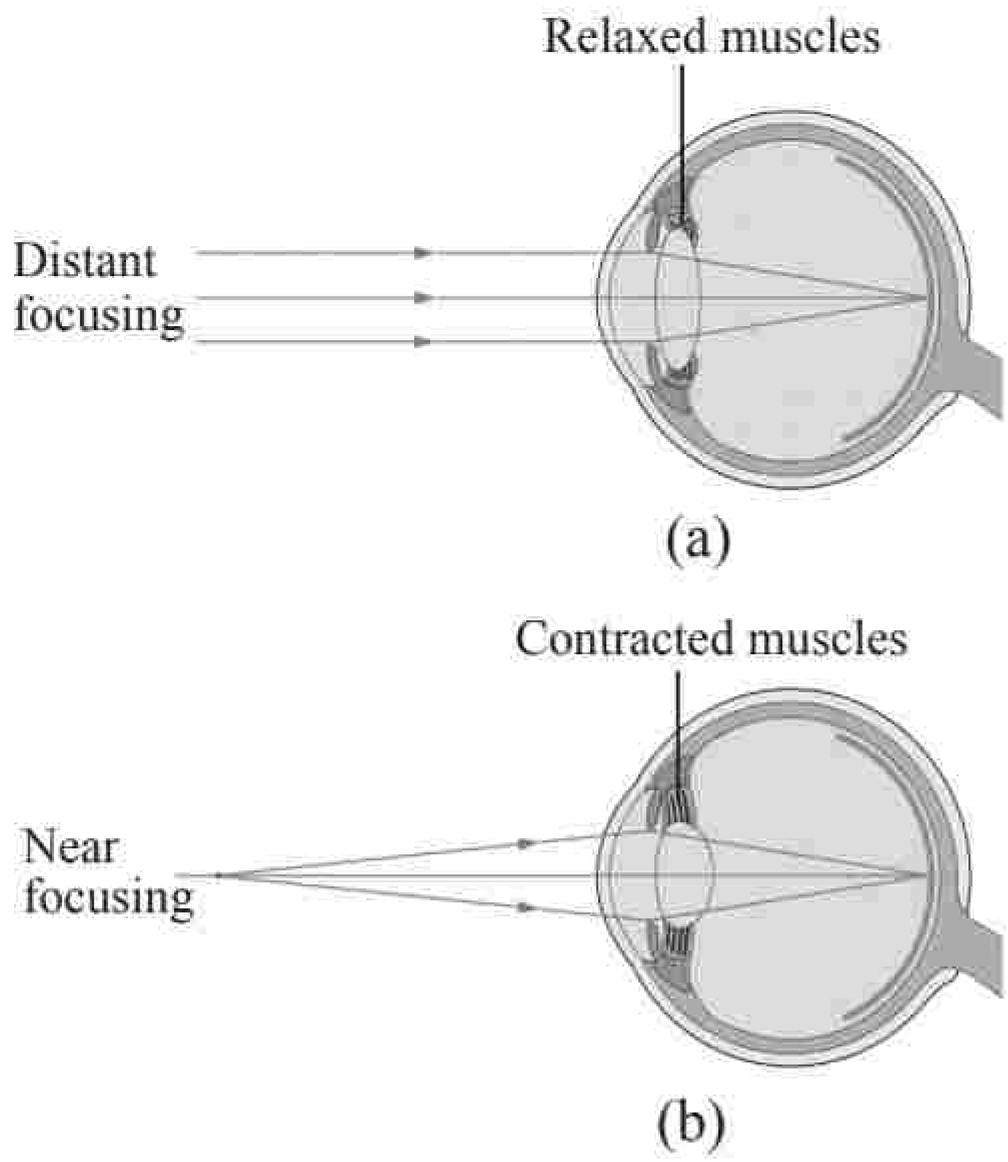


Figure 4.26: *Focusing distant and near objects*

The extremes of the range over which distinct vision is possible are known as the far point and the near point of the eye. The far point of a normal eye is the maximum distance at which the light from the object can still be focused onto the retina. The near point is the shortest distance from the eye at which the light from the object can be focused onto the retina. The eye tends to be blind when the object is closer to the eye than the near point. The position of the near point depends on the amount by which the ciliary muscle can increase or decrease the curvature of the crystalline lens. The range of accommodation gradually diminishes with age because the crystalline lens grows throughout a person's life and the ciliary muscles become less capable of distorting a larger lens. For this reason, the near point gradually recedes as one grows older. The recession of the near point is a condition called presbyopia.

Activity 4.3

Aim: To determine far and near point of the eye

Materials: manilla paper, sellotape

Procedure

1. Draw a vertical line about 50 cm long on a sheet of manila paper.
2. Stick the paper on the wall.
3. Stand about 2 m from the wall and cover one of your eyes with your hand.
4. Cover the line by bringing the index finger of your other hand close to your face.
5. Without moving your finger switch to the other eye.
6. Now move to about 4 m from the wall and repeat the process.
7. Move as far as possible from the wall and repeat steps 3-5. Record all your observations.

Question

Explain the appearance of the line at each of the distances.

Defects of the human eye

For an object to be seen by the human eye, light from the object must be focused on the retina. However, there are cases when light from the object cannot be focused on the retina. This arises mainly from incorrect distance relationships in the eye. A normal eye can form an image on the retina for an object at infinity when the ciliary muscles are relaxed, and for a nearby object when the ciliary muscles are contracted as illustrated in Figure 4.27. In some cases, the size of the eyeball is defective leading to conditions known as short-sightedness (myopia) and far-sightedness (hyperopia).

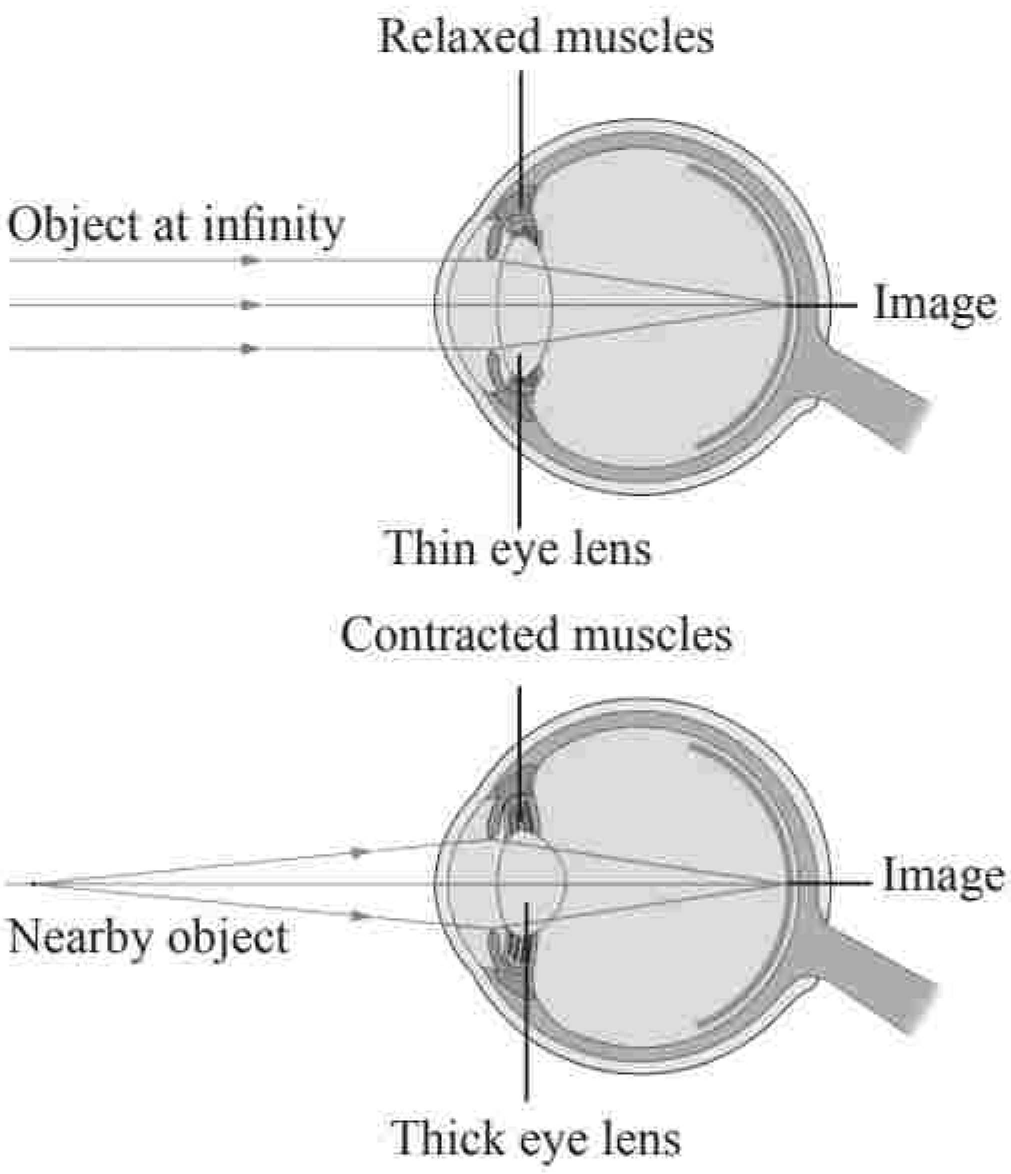


Figure 4.27: *Focusing of a distant and close object by a normal eye*

Short-sightedness (myopia)

In the myopic (short-sightedness or near-sightedness) eye, the eyeball is too long from front to back in comparison with the radius of curvature of the cornea (or the cornea is too sharply curved). This causes the rays of light from an object at infinity to be focused in front of the retina. The most distant object for which an image can be formed on the retina is then nearer than infinity. Therefore, a person suffering from short-sightedness can see nearby objects clearly but not distant objects. Figure 4.28 illustrates myopic eye.

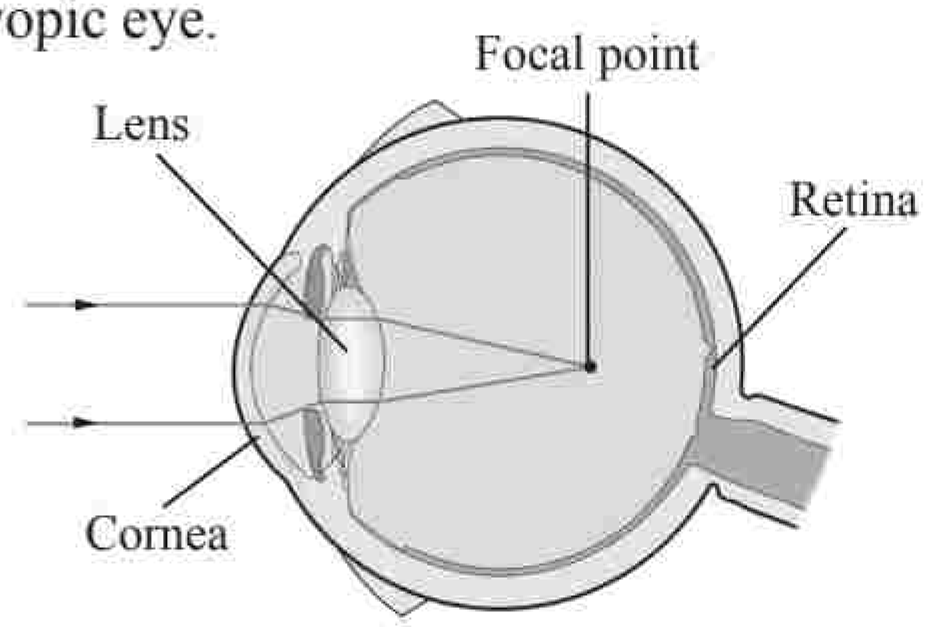
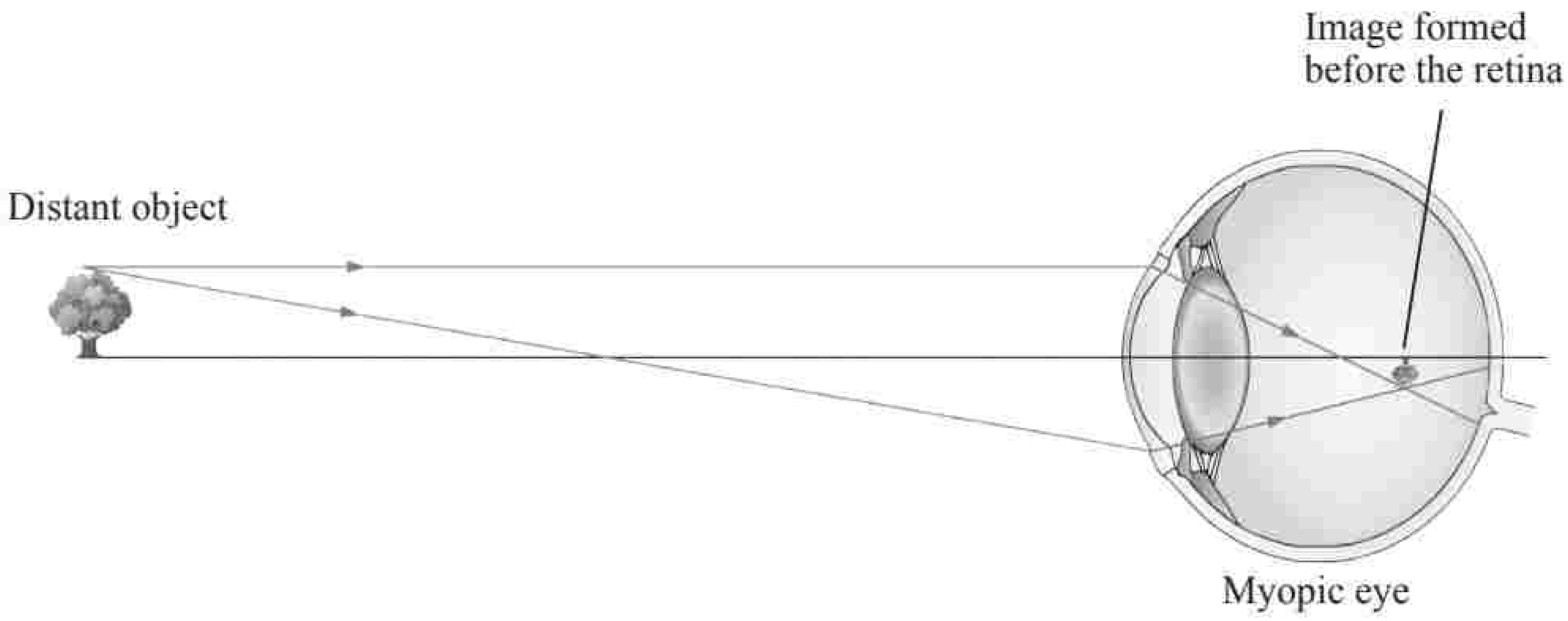


Figure 4.28: *Myopia*

Correction of myopia

To correct myopia, rays of light from a distant object should be diverged before entering the eye. Therefore, correcting the myopic eye involves the use of a concave (diverging) lens which forms the image of a distant object close to the eye lens. The image formed by the glass lens is then focused onto the retina by the eye lens. Figure 4.29 illustrates the correction of a myopic eye.



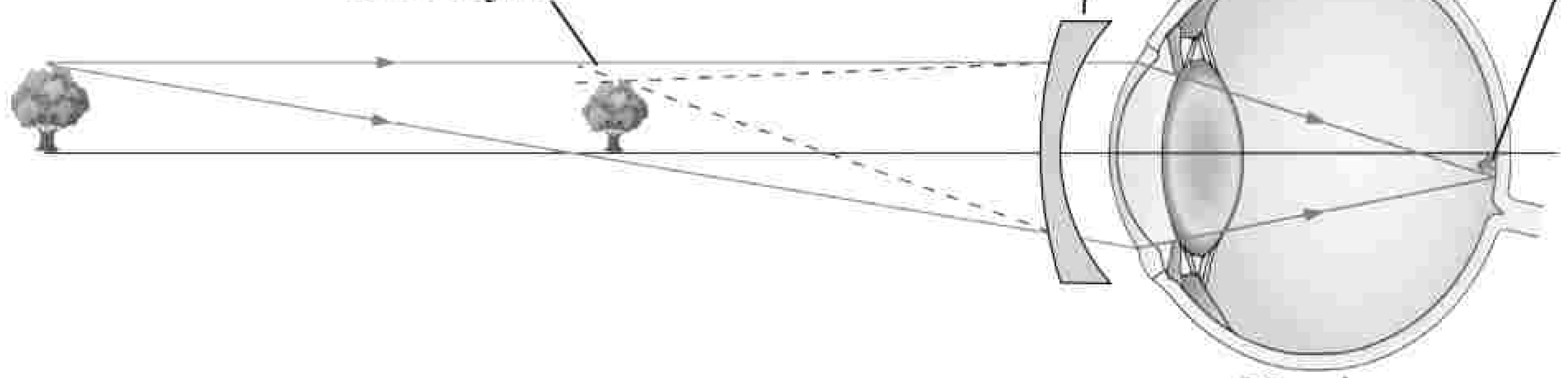


Figure 4.29: *Correcting a myopic eye*

Note that, lenses used for vision correction are usually described in terms of the lens power, *P*, which is given by the reciprocal of the focal length expressed in metres. Hence,

$$P = \frac{1}{f}$$

The unit of lens power is the dioptre, D. Thus, a lens with $f = 0.50$ m has a power of 2.0 D, $f = 0.25$ m corresponds to 4.0 D, and so on. The numbers used for the prescription of the glass lenses are usually powers expressed in dioptre.

Long-sightedness (hyperopia)

In the hyperopic (long-sightedness or far-sightedness) eye, the eyeball is too short or the cornea is not curved enough. This causes the rays of light from a near object to be focused behind the retina. A person suffering from long-sightedness can see distant objects clearly but cannot see nearby objects clearly. The near point of the eye may be more than a metre away making ordinary reading difficult. Figure 4.30 illustrates the hyperopic eyes.

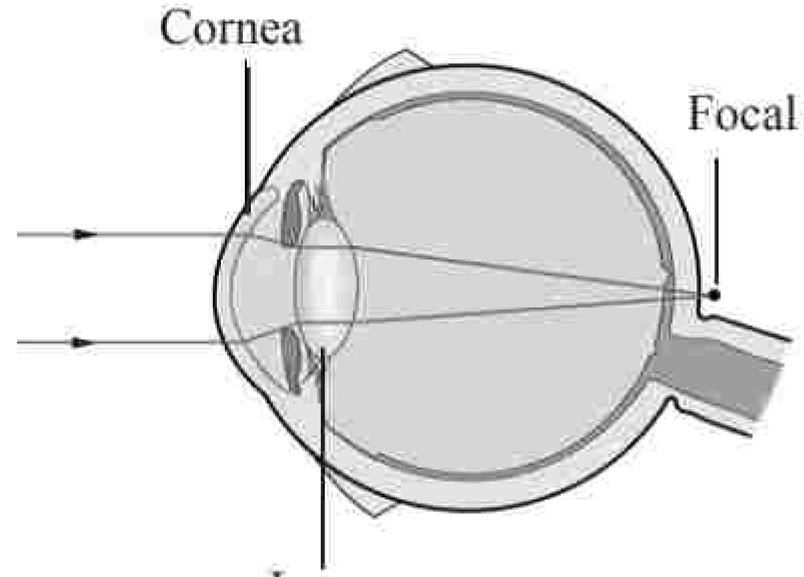


Figure 4.30: *Hyperopia*

Note that, the myopic eye produces too much convergence of light rays from a distant object while a hyperopic eye produces insufficient convergence of rays of light from the near object.

Presbyopia

Presbyopia is the gradual loss of the eye's ability to focus on nearby objects because of old age. The term "presbyopia" comes from a Greek word that means "old eye." This defect is normally observed after the age of 40 years. Presbyopia is caused by loss of eye lens' flexibility which is necessary to change its focal length in order to focus objects. This results in insufficient accommodation leading to image formation behind the retina as shown in Figure 4.31.

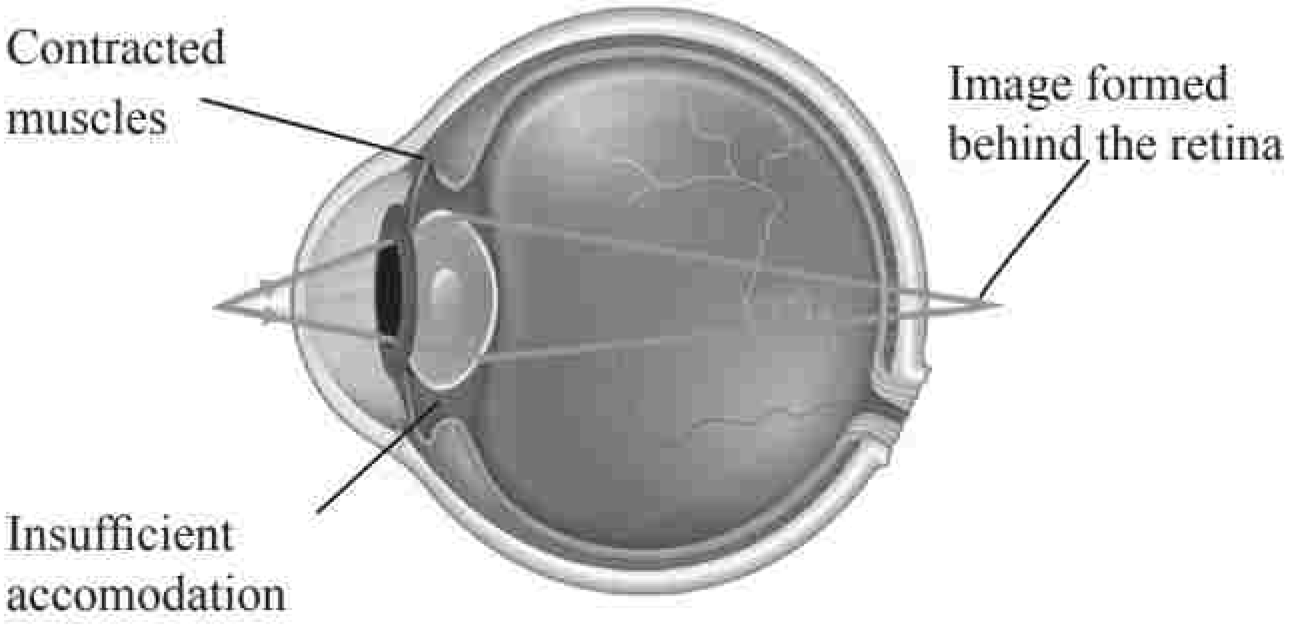


Figure 4.31: *Presbyopic eye*

Correction of hyperopia and presbyopia

In the case of either a presbyopic or a hyperopic eye, the near point is farther from the eye than normal. Therefore, to see clearly an object at a normal reading distance, which is about 25 cm, a convex lens is required to form a virtual image at or beyond the near point. The lens converges the rays of light from a near object to form the image at or beyond the near point. The eye lens then focuses the image of the glass lens onto the retina. Figure 4.32 shows the correction of the hyperopic or presbyopic eye.

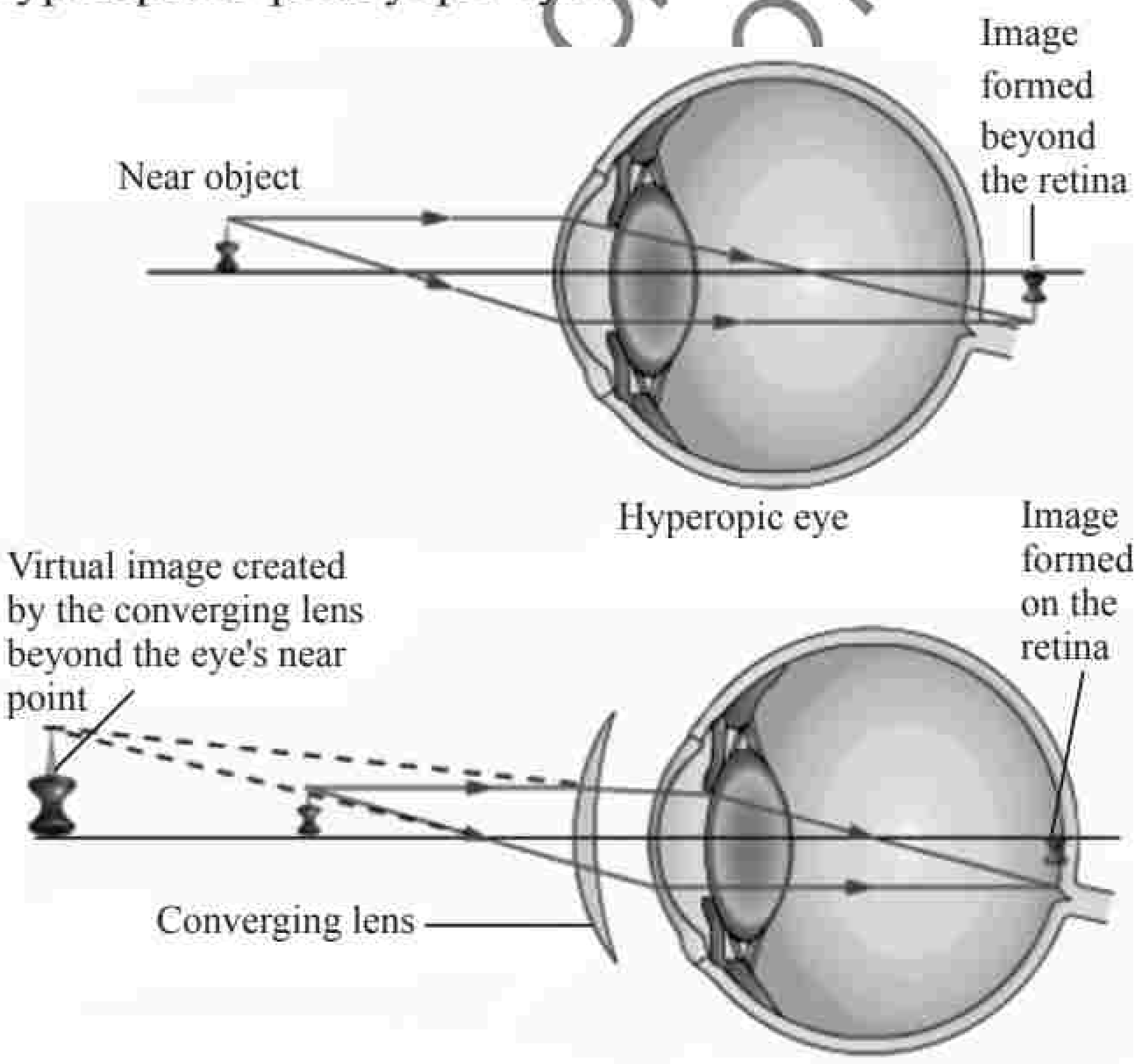


Figure 4.32: *Correction of the hyperopic eye*

Astigmatism

Astigmatism is the imperfection in the curvature of the eye that causes the formation of blurred images on the retina. It occurs when either the cornea or the lens has mismatched curves. That is, instead of having a curve like a round ball, the surface is egg-shaped. This causes the lens to have different focal points when looking at the same object. Consequently, rays of light from the object are focused at different points leading to image formation at different positions on the retina. The result is blurred vision. Figure 4.33 illustrates astigmatism. Astigmatism is often present at birth and may occur in combination with near-sightedness or far-sightedness.

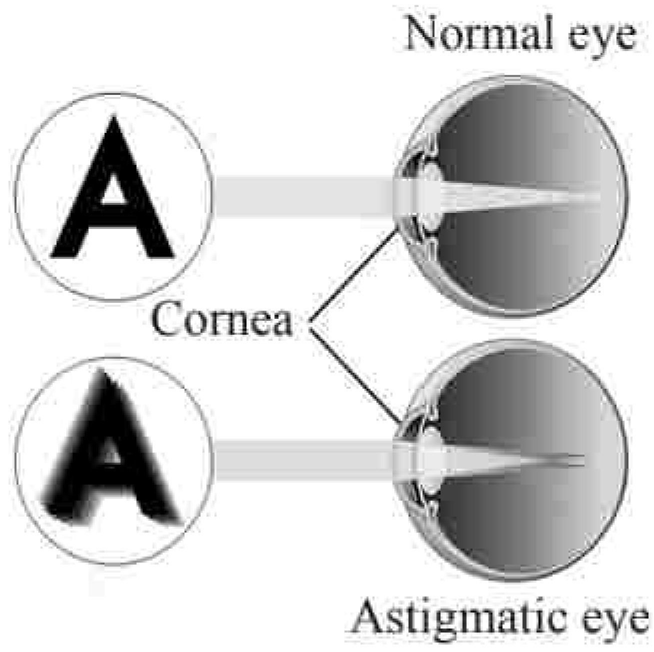


Figure 4.33: *Astigmatism*

Astigmatism can be corrected by either using corrective lenses or performing eye surgery.

Example 4.7

The far point of a myopic eye person is 50 cm in front of the eye. What is the nature and power of the lens required to correct the problem?

Solution

The concave lens should make the objects at infinity appear at the far point.

For object at infinity, $u = \infty$, far point of the defected eye, $v = -50$ cm

By lens formula:

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{-50\text{ cm}} + \frac{1}{\infty} = \frac{1}{-50\text{ cm}}$$

$$f = -50\text{ cm} = -0.5\text{ m}$$

Therefore,

$$\text{power} = \frac{1}{f} = \frac{1}{-0.5\text{ m}} = -2.0\ D$$

Comparison between the lens camera and the human eye

The lens camera has many similarities with the human eye. Table 4.1 shows the differences and similarities between lens camera and human eye.

Table 4.1: Similarities and differences between lens camera and human eye.

Criteria	Lens camera	Human eye
Lens	Uses convex lens	Uses convex lens
Image	The image is real, reduced and inverted	The image is real, reduced and inverted
Light control	The amount of light entering the camera is controlled by the diaphragm	The amount of light entering the eye is controlled by the pupil
Image point	The image is formed on the special film	The image is formed on the retina
Focusing	Focusing is done by altering the distance between the lens and the film	Focusing is done by altering the shape of the lens which alters its focal length
Image processing	The image is processed chemically in a process called "Developing"	The image is converted to electrical signals that travel through the optic nerve to the brain for processing

Exercise 4.6

1. The distance from the cornea to the retina of someone's eye is 2.1 cm. Determine the effective focal length of this eye if an object is

   (a) at infinity.

   (b) located 1 m away.

   (c) located 25 cm away.

2. A far-sighted eye can only focus on objects beyond 100 cm. What are the focal length and power of the contact lens needed to correct this problem?
3. A near-sighted eye can only focus on objects closer than 50.0 cm. What sort of contact lens will correct this problem? Determine its power.
4. (a) Differentiate between magnifying power and magnification of a lens.
   (b) A near point of a long-sighted patient is 90 cm. Determine:
   (i) the focal length of a lens that can be used to enable the patient see clearly an object which is at a distance of 25 cm from the patient's eye
   (ii) the power of the patient's eye
   (iii) the magnification of the patient's eye lens

Chapter summary

1. Optical instruments use refraction or reflection of light to form different types of images.
2. Microscopes make it possible to see very small objects by producing an enlarged image of an object.
3. Telescopes and binoculars allow us to see objects that are far away.
4. Cameras provide us with a recorded image of an object. The image can be a still image or a moving image (video).
5. Projection lanterns produce enlarged images of objects so that they can be projected onto a screen to allow a large audience to see the image.
6. The human eye can discern colour, judge distance and see great detail from both near and far positions by adjusting the focal length of its lens.
7. Myopia is a condition in which close objects appear clearly, but far ones do not. This is because the image is formed in front of the retina.
8. Hyperopia is a condition in which images of nearby objects are blurry, whereby the image is formed behind the retina.
9. Presbyopic is the gradual loss of the eye's ability to focus on nearby objects, and it occurs because of old age.
10. Astigmation is the imperfection in the curvature of the eye that causes the formation of blurred images. It occurs when either the cornea or lens has mismatched curves.

Revision exercise 4

1. Choose the correct answer from the given alternatives.

   (i) The part of the human eye that corresponds to the film in a camera is the ________.

   A iris
   B pupil
   C retina
   D cornea

   (ii) In the astronomical telescope, f_0 is the focal length of the objective lens and f_e is the focal length of the eyepiece lens. For greater magnification, f_0 should be ____.

   A much larger than f_e
   B equal to f_e
   C slightly smaller than f_e
   D much smaller than f_e

   (iii) Which of the following statements is not the reason on why a microscope has an objective lens of short focal length?

   A It allows more light to be collected.
   B It keeps the distance between the objective lens and the eyepiece lens small.
   C It gives the microscope more magnifying power.
   D It makes the image formed to be inverted.

   (iv) Which of the following combinations of lenses can be used on a compound microscope?

   (v)

	Objective lens	Eyepiece lens
A	Long-focus converging lens	Short-focus converging lens
B	Long-focus converging lens	Short-focus diverging lens
C	Long-focus converging lens	Long-focus converging lens
D	Short-focus converging lens	Long-focus converging lens

2. When using a magnifying glass, the image distance should be approximately 25 cm from the eye for the object to be viewed clearly. Explain.
3. What is the maximum magnification of a magnifying glass whose focal length is 2 cm?
4. An astronomer wants to order a large concave mirror for a telescope that is to produce high-quality images. With explanation, advise whether the astronomer should order a spherical mirror or a parabolic mirror?
5. An astronomical telescope is used to view an object at infinity and has an objective lens with a focal length of 15.0 cm. Where must the eyepiece of focal length 0.5 cm be placed to form an image at infinity? What is the total angular magnification?

6. In a particular compound microscope, the objective lens and eyepiece lens are 15 cm apart. The focal length of the objective lens is 2.0 cm, while that of the eyepiece lens is 6.25 cm.
   (a) How far from the objective lens should an object be placed in order to obtain the final image of a distant vision?
   (b) What is the total magnifying power of the microscope?

7. Explain the meaning of the following terms as used in optical instruments.
   (a) Magnifying power
   (b) Accommodation

8. Why are prism binoculars preferred over traditional ones?

9. (a) Explain how the image of an object is formed in the human eye.
   (b) Give two defects of the human eye and explain how they can be corrected.
   (c) Describe two ways in which a lens camera and the human eye are similar.
   (d) Describe two ways in which a lens camera and the human eye are different.

10. The objective and eyepiece of a compound microscope are 24 cm apart. Suppose the focal lengths of the objective and the eyepiece are 3 cm and 9 cm respectively.
   (a) Where must a specimen be located to produce a final virtual image at infinity?
   (b) What could be the total magnification of this microscope? (Assume a microscope is used by a person whose nearest distance for a distinct vision is 25 cm).

11. A far point of a certain myopic eye is 50 cm in front of the eye. An eyeglass lens is to be worn to correct this myopic eye. If the glass lens is worn 2 cm in front of the eye, find:
   (a) The focal length of the lens that will help the wearer to clearly see an object at infinity.
   (b) The power of the lens.

12. A 1.8 m tall student is standing 2.5 m in front of a digital camera that uses a converging lens whose focal length is 0.05 m.
   (a) Find the image distance and state whether the image is real or virtual.
   (b) Determine the magnification and height of the image.

13. Two students complain that they cannot see objects that are closer than 1 m from their eyes.
   (a) What problem do they have? Describe it using a neat diagram.
   (b) How can their problem be solved?
   (c) Determine the power of the device that can be used to solve their problem.

Chapter Five

Thermal expansion

Introduction

In daily life, we have experienced medical doctors measuring human body temperature using a thermometer, and welders joining pieces of metals together. We have also seen bridges in which metals are separated by gaps so that they have room to expand on hot days. For the same purpose, concrete roads are built with some gaps. All these are possible due to the application of thermal expansion which occurs as a result of temperature change. This signifies that, thermal expansion is important in human development. In this chapter, you will learn about thermal energy, thermal expansion of solids, liquids and gases. The competencies developed will enable you to use thermostats in different home appliances such as electric irons, heaters and refrigerators, and design fire alarms.

Thermal energy

All solids, liquids and gases are made up of small particles. The particles have kinetic energy and are constantly in motion. The movement of these particles inside the matter results into thermal energy. The thermal energy that can be transferred from one place to another is termed as heat. If you touch a cup of boiling water, you feel hot and when you touch cold object, you feel cold. Since the degree of coldness or hotness at a particular time defines an object's temperature, then heat is a function of temperature. The energy possessed by a body due to its temperature is called the thermal energy or the heat content of the body.

When an object is heated, its temperature may rise or its state may change at constant temperature. The object may also expand. Change of state and rise in temperature can be noticed easily. However, the increase in dimensions of a heated solid, volume of heated liquids and gases can be observed when heat is applied. For example, the diameter of unheated solid metal ball is smaller than the diameter of the same solid metal ball when heated. Also, if water is warmed in a glass flask fitted with a capillary tube dipped in the water and passing through a cork, the water is seen to rise more as heating continues. Similarly, an inflated balloon with a small amount of air shows an increase in volume if exposed outside in sunlight on a sunny day. These phenomena reveal that thermal expansion occurs when material expands due to an increase in its temperature.

There are three types of thermal expansion, namely; linear expansion, areal expansion and volume expansion. These types of expansion depend on the dimensions that undergo change. In linear expansion, the length of the object changes when the temperature increases. It mostly takes place in solids. The areal expansion occurs when the surface area of a heated object is altered. It is also called superficial expansion. On the other hand, when heating causes a change of volume of an object, it is termed as volume expansion. Volume expansion occurs in solids, liquids as well as gases.

Sources of thermal energy

An object which produces heat, is a source of thermal energy. Examples of sources of heat are the Sun, fuels, nuclear reactors, geothermal and electric current.

The Sun

Every day, the Sun radiates energy in the form of heat and light to the surrounding. The Sun's heat comes within the Sun itself. Since the Sun is a star, it is made up of hydrogen and helium atoms. At high pressure and temperature in the Sun's core, hydrogen atoms fuse to form helium. During the fusion process, a large amount of energy is released. This energy reaches the earth in form of radiation (Figure 5.1). On absorption, this energy can be converted to heat.

The energy from the Sun can be used in various ways. For example, passively, it is used in heating the windows, walls and floor of our homes. In passive heating, the heat from the Sun is absorbed by building materials during the day and released when the Sun is absent. Also, the solar heaters frequently used in our daily lives absorb Sun's energy and convert it to thermal energy. The energy from the sun can also generate electricity through concentrating solar power (CSP). In CSP, the energy from the sun is concentrated using mirrors so as to generate high temperature. The high temperature produces heat in a heat exchanger which produces steam. The steam drives a turbine to produce electricity.

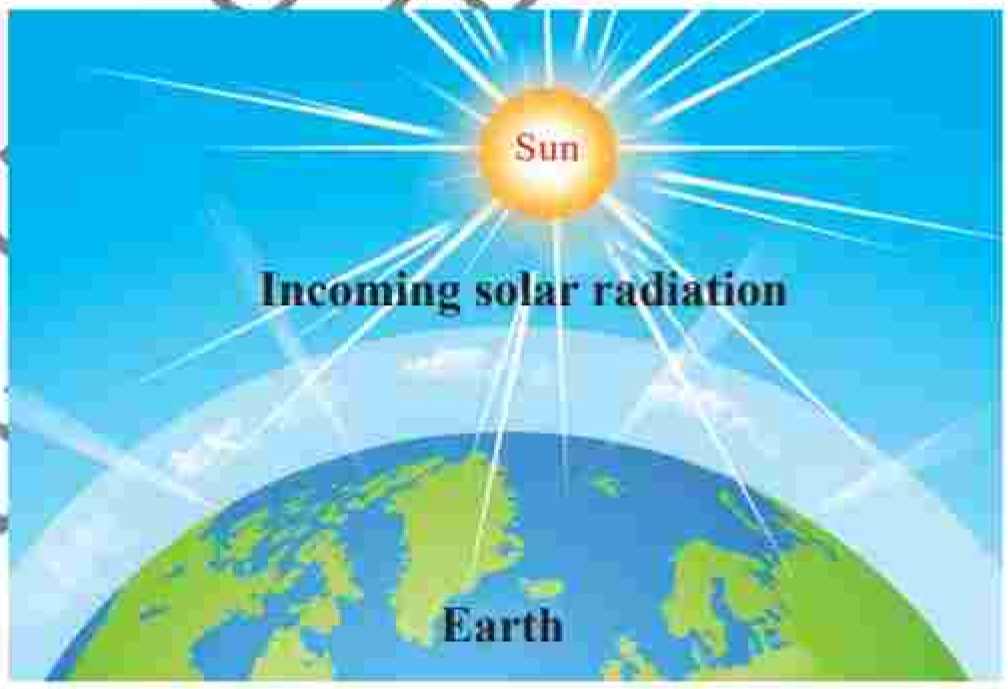


Figure 5.1: *The Sun as a source of thermal energy*

Fuels

When materials such as wood, oil, coal and gas are burnt, heat, light or power is generated. For example, when wood is burnt, the energy produced is used for cooking food and heating water in homes (Figure 5.2(a)). Cars use the mechanical energy (Figure 5.2(b)) produced from gasoline to travel from one place to another, and natural gas is used in cooking stoves in our daily life. Oil, gas and coal are formed from the remains of organisms called fossils. The fuel derived from these remains is called fossil fuels. However, fossil fuels emit carbon dioxide to the atmosphere, polluting the environment (Figure 5.2(b)).

(a) (b)

Figure 5.2: The use of fossil fuel to produce (a) heat energy, and (b) mechanical energy

Nuclear reaction

Nuclear energy is generated from nuclear reactions. In nuclear reactions, the nuclei of certain elements are split (nuclear fission) or combined (nuclear fusion). When nuclear fission or fusion occurs, energy is released. The disadvantage of producing heat through this process is that, it produces hazardous radioactive by-products which pose health risks.

Geothermal energy

The word geothermal comes from two Greek words: geo, meaning earth and therme, which means heat. The main source of this heat is radioactivity in the earth's core. Heat from the core is constantly radiating outwards and warming rocks. The high temperature developed, leading to rock melting, hence forming magma (molten rock). The heat contained in the magma can be released to the surface of the earth through hot springs, volcano eruption, and geysers and fumaroles. This heat can also be extracted and converted to electricity. Figure 5.3 shows Mount Oldonyo Lengai active volcano releasing lava (flowing magma).

Figure 5.3: *Oldonyo Lengai Volcanic eruption*

Electric currents

If an electrical bulb is plugged to its holder and a current is switched ON for about ten minutes, then, turned OFF, you will feel warm upon touching the bulb. The heat you feel is the result of the conversion of electrical energy into heat energy. Some other electrical appliances which produce heat include electric heaters, electric iron and electric cooker. The heating elements in these appliances have high resistance. When the current passes through these elements, electrical energy is converted into heat.

Task 5.1

Using books and the internet, identify and discuss other sources of heat. Write a report on your finding and present it in the class.

Difference between heat and temperature

Heat of an object is not the same as temperature of that object, so it is important not to confuse between these two terms. Temperature is a measure of how cold or hot something is. For example, a low temperature is needed for refrigerators to function properly, whereas to boil water or cook food, a high temperature is needed. Heat is a form of energy that can be transferred between objects and it changes the temperature of the objects. In our homes, heat can be supplied by solar energy, electricity, natural gas, kerosene and firewood. Temperature is measured by thermometers in different scales like Celsius, Fahrenheit or Kelvin, while heat is given in Joule or Calorie. The differences between heat and temperature are summarized in Table 5.1.

Table 5.1: Differences between heat and temperature

Heat	Temperature
Derived physical quantity	Fundamental physical quantity
A measure of the amount of energy possessed by particles in an object	Degree of hotness or coldness of an object
The SI unit is joules	SI unit is kelvin
Measured using joulemetre	Measured using thermometer
Heat flows from higher temperature to low temperature	Temperature does not flow

Exercise 5.1

1. What is the origin of the heat of an object?
2. Why do you feel cold when you touch cold objects?
3. Explain how the Sun produces heat.
4. Suppose you are sitting near burning charcoal. You immediately start feeling warm. Is the warmth you feel caused by heat or temperature? Explain.

Thermal expansion of solids

The dimensions of a solid object increase by a small amount when heated and decreases when cooled. This increase and decrease cannot be seen readily. The kinetic theory of matter explains better this phenomenon.

Thermal expansion of solids in terms of the kinetic theory

Molecules in solids constantly vibrate about their equilibrium positions. When a solid object is heated, molecules gain more kinetic energy, hence an increase in molecules' translation and vibration. This kinetic energy causes neighbouring atoms or molecules to collide more with each other. As the heating continues, the distance of separation between the molecules increases. As a result, the molecules occupy more space. As such, length, area and volume increase. This is expansion.

Activity 5.1

Aim: To demonstrate thermal expansion in solids

Materials: ball and ring apparatus, water, source of heat, freezing mixture

Procedure

1. Pass the ball through the ring at room temperature, as shown in Figure 5.4.

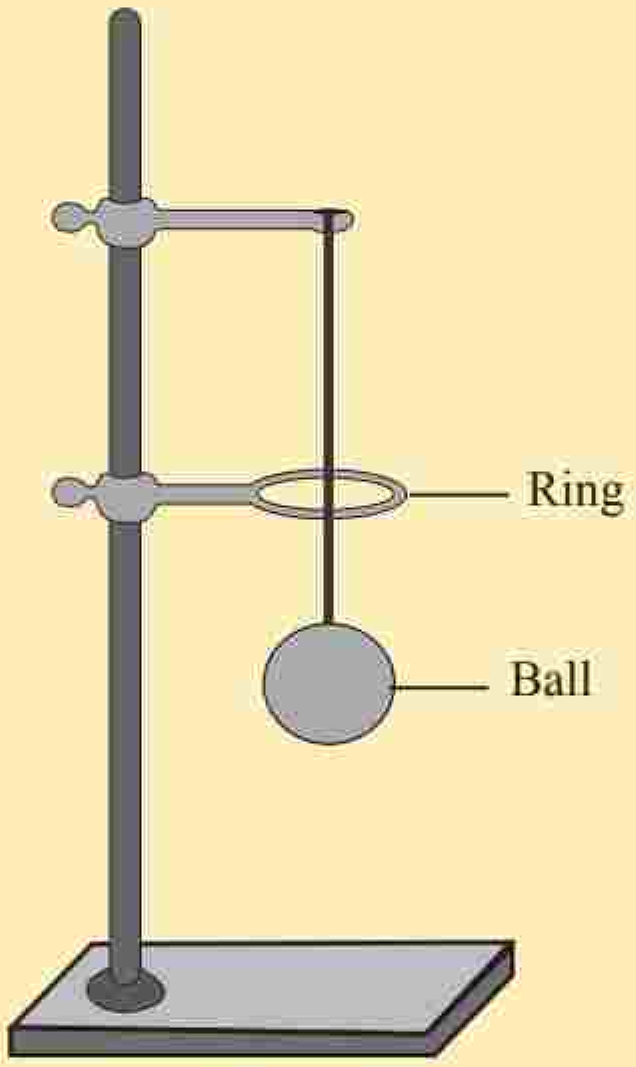


Figure 5.4

2. Observe what happens?
3. Remove the ball from the ring and immerse it into boiling water for about 1 minute.
4. Remove the ball from the hot water.
5. While the ball is still hot, try to pass it through the ring again, as shown in Figure 5.5.

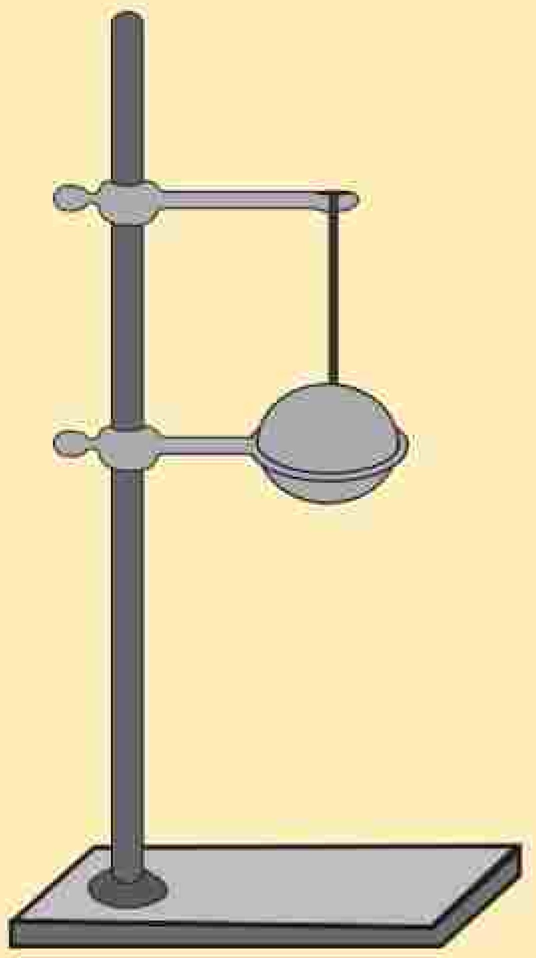


Figure 5.5

6. Record your observations.
7. Leave the ball on the ring for a few minutes.
8. Record your observations.
9. Remove the ball and immerse it in a freezing mixture for some time, then pass it through the ring.
10. Record your observations.

Questions

(a) Did the ball at room temperature pass through the ring?

(b) Did the heated ball pass through the ring?

(c) Did the heated ball pass through the ring after being immersed in the freezing mixture?

When the ball and ring are at the same temperature (room temperature), the ball fits into the ring and passes through. When the ball is heated and the ring is not heated, the ball does not fit in the ring since its volume increases and its diameter becomes larger. This makes it not to pass through the ring. This demonstration shows that, solids expand when heated. For the heated solid, the kinetic energy of the particles is increased. This makes the particles to vibrate faster and with greater amplitude. As a result, the particles move further apart, hence, the volume of the solid increases. See Figure 5.6.

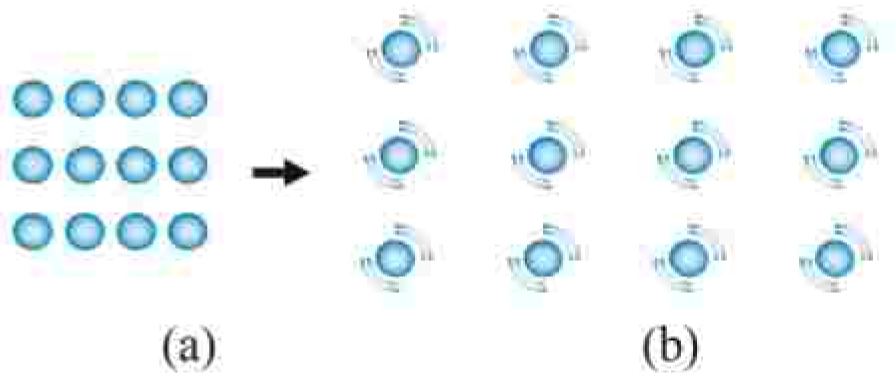


Figure 5.6: *Particles of a solid (a) at room temperature and (b) when heated*

If the ball is immersed in a freezing mixture, it will pass through the ring easily. This is because the amplitude of the vibrations of the solid particles is decreased and cause the distance of separation between neighbouring atoms to decrease. This results in a contraction of the length, area or volume of the solid.

Activity 5.2

Aim: To demonstrate the contraction in solids

Materials: source of heat, bar-breaking apparatus, cold water

Procedure

1. Loosen the wing nut and remove the cast-iron pin from the bar-breaker shown in Figure 5.7.

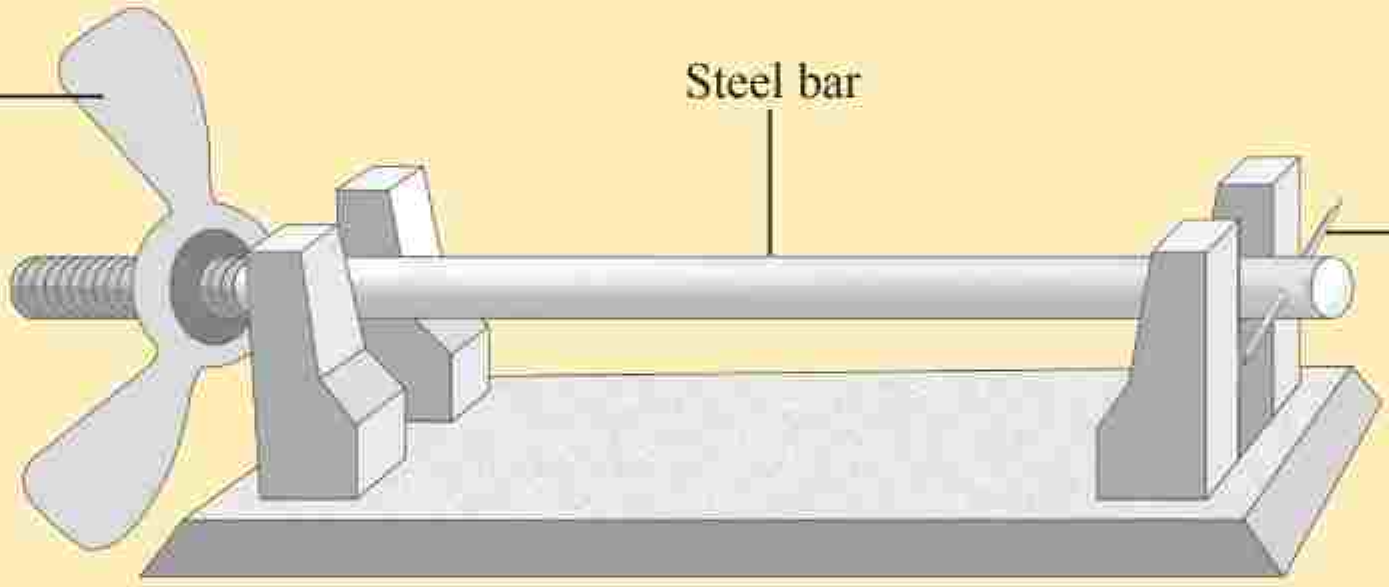


Figure 5.7

2. Try to break the pin with your hands.
3. Fit the cast-iron pin in the steel bar through the hole.
4. Tighten the nut, but not too tightly as to break the pin.
5. Using two Bunsen burners, heat the steel bar strongly and keep tightening the nut as you apply more heat.
6. Continue heating for about five minutes and then allow the bar to cool.
7. Pour some cold water on it so that it cools faster.
8. Observe what happens to the bar.

Questions

(a) Is it possible to break the pin using bare hands? Explain.

(b) What happened to the cast-iron pin on cooling the bar?

A bar-breaker is made up of a strong iron frame that holds a steel bar. One end of the bar is threaded and fitted with a wing nut, while the other end has a hole. A cast-iron pin is inserted in the hole as shown in Figure 5.7. When the bar is heated, it expands and the wing nut screwed to tighten the bar. About 30 seconds after the heating is stopped, the cast-iron pin snaps. The pin breaks due to contraction force which pulls it as the steel bar cools.

Linear expansion and expansivity of solids

When engineers construct metal bridges and railway lines, they must calculate the extent to which the length of metals changes when heated or cooled. This will enable them to set accurate length of the metal and avoid breakage and buckling of the metal. The buckling and breaking of the metals would damage the bridges or railway lines. The amount by which the length of the metal changes when heated or cooled is termed as the linear expansion of the metal.

The value of linear expansion depends on the length of the metal, the nature of the metal and the temperature through which the material is heated. Suppose a metal with original length, l_0, at a temperature, θ_0, is heated to a temperature θ_1. If the length changes to l_1 at temperature θ_1, then its increase in length is given by $\Delta l = l_1 - l_0$ as shown in Figure 5.8.

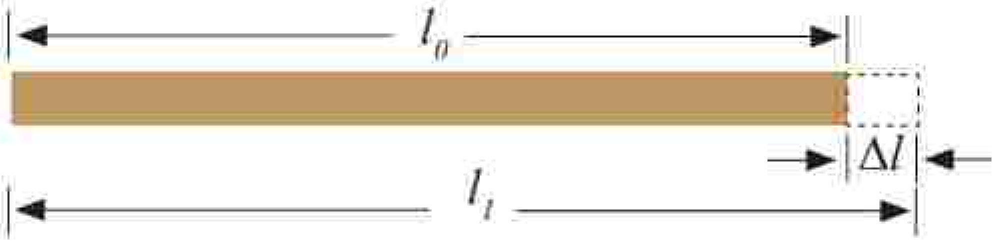


Figure 5.8: *An increase in length of a heated rod*

The temperature changes is given by:

$$\Delta\theta = \theta_1 - \theta_0$$

Note that, the change in length is directly proportional to the original length of the object and the change in temperature. Therefore, increase in length is given by,

$$(l_1 - l_o) \propto l_o \Delta\theta$$

$$(l_1 - l_o) = \alpha l_o \Delta\theta$$

where α is the constant of proportionality called coefficient of linear expansion or linear expansivity. The value of α is given by,

$$\alpha = \frac{l_1 - l_o}{l_o \Delta\theta}$$

$$\alpha = \frac{\Delta l}{l_o} \times \frac{1}{\Delta\theta}$$

Generally,

$$\alpha = \frac{\textit{increase in length}}{\textit{original length} \times \textit{rise in temperature}}$$

Therefore, coefficient of linear expansion or linear expansivity can be defined as the increase in length of a body per unit original length when its temperature rises by 1 °C or 1 K. The SI unit of linear expansivity is per kelvin (K^{-1}) or per degree centigrade $(°C^{-1})$.

Example 5.1

A block of concrete $5.0\ \text{m}$ long expands to $5.00412\ \text{m}$ when heated from $25\ °\text{C}$ to $100\ °\text{C}$. Determine the linear expansivity of concrete.

Solution

$$\textit{Linear expansivity}\ (\alpha) = \frac{\textit{increase in length}}{\textit{original length} \times \textit{rise in temperature}}$$

$$\alpha = \frac{5.00412\ \text{m} - 5.0\ \text{m}}{5.0\ \text{m} \times (100\ °\text{C} - 25\ °\text{C})}$$

$$= 0.000010986\ /°\text{C}\ \text{or}\ 1.1 \times 10^{-5}\ /°\text{C}$$

Therefore, the linear expansivity of concrete is $1.1 \times 10^{-5}\ /°\text{C}$.

Example 5.2

A brick $30\ \text{cm} \times 18\ \text{cm} \times 10\ \text{cm}$ is at $20\ °\text{C}$. If the brick is heated to a temperature of $150\ °\text{C}$, what will be its new dimensions? (The coefficient of linear expansion of concrete is $1.2 \times 10^{-5}\text{K}^{-1}$).

Solution

$$\alpha = \frac{l_2 - l_1}{l_1 \Delta\theta}$$

$$l_2 = l_1 + \alpha l_1 \Delta\theta \text{ or } l_2 = l_1(1 + \alpha\Delta\theta)$$

This expression can be used to determine each of the new dimensions of the brick. In each case, the temperature change, $\Delta\theta$ is given by:

$$\Delta\theta = \theta_1 - \theta_0$$

$$= 150\ °\text{C} - 20\ °\text{C}$$

$$= 130\ °\text{C}\ \text{or}\ 130\ \text{K}$$

Dimension of length is given by:

$$l = 0.3\ \text{m} + 1.2 \times 10^{-5}\text{K}^{-1} \times 0.3\ \text{m} \times 130\ \text{K}$$

$$= 0.3005\ \text{m or}\ 30.05\ \text{cm}$$

Dimension of width is given by:

$$w = 0.18\ \text{m} + 1.2 \times 10^{-5}\text{K}^{-1} \times 0.18\ \text{m} \times 130\ \text{K}$$

$$= 0.1803\ \text{m or}\ 18.03\ \text{cm}$$

Dimension of height is given by:

$$h = 0.1\ \text{m} + 1.2 \times 10^{-5}\text{K}^{-1} \times 0.1\ \text{m} \times 130\ \text{K}$$

$$= 0.1002\ \text{m or}\ 10.02\ \text{cm}$$

The new dimensions of the brick will be $30.05\ \text{cm} \times 18.03\ \text{cm} \times 10.02\ \text{cm}$.

Example 5.3

An iron plate at $20\ °\text{C}$ has a hole of radius $8.92\ \text{mm}$ in the centre. An iron rivet with a radius of 8.95 mm at $20\ °\text{C}$ is to be inserted into the hole. To what temperature must the plate be heated for the rivet to fit into the hole? (Linear expansivity of iron is $1.02 \times 10^{-5}\text{K}^{-1}$).

Solution

Given $\alpha = 1.02 \times 10^{-5} K^{-1}$,

$r_2 = 8.95$ mm, $r_1 = 8.92$ mm, required to find $\Delta\theta$.

From the expression for linear expansivity,

$$\Delta\theta = \frac{r_2 - r_1}{\alpha r_1}$$

$$= \frac{8.95 \text{ mm} - 8.92 \text{ mm}}{1.02 \times 10^{-5} K^{-1} \times 8.92 \text{ mm}}$$

$$= 329.7 \text{ K}$$

Final temperature is

$$\theta_f = (293 + 329.7) \text{K} = 622.7 \text{K} \text{ or}$$

$$\theta_f = 20°C + 329.7°C = 349.7°C.$$

This is the temperature at which the iron plate must be heated. Linear expansivities of solid materials range between 273 K to 373 K. Table 5.2 shows the values of linear expansivities of some substances.

Table 5.2: Linear expansivities of some substances

Substance	Linear expansivity, α, ($\times 10^{-6}$ K^{-1})
Aluminium	26
Brass	19
Concrete	11
Copper	17
Steel	11
Iron	10.2
Glass	8.5
Pyrex glass	3.0
Silica	0.42
Diamond	1
Invar	0.9

Activity 5.3

Aim: To compare the expansion of different solids

Materials: brass, steel, aluminium and copper rods (of the same dimension), scale, Bunsen burner, retort stands, clamps

Procedure

1. Set up the apparatus as shown in Figure 5.9.

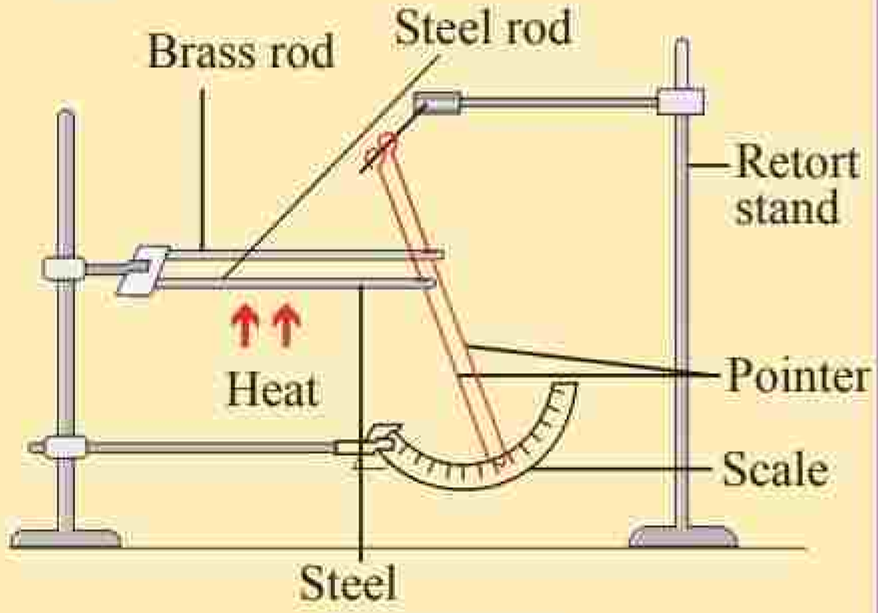


Figure 5.9

Make sure the pointers are tied to the rods using pieces of wire made from the same material and are initially at the same point on the scale.

2. Heat the brass and steel rods uniformly around the centre and note any change in the position of the pointers.
3. Record the value of the expansion produced in each rod on heating.
4. Repeat steps 1-3 using the copper and aluminium rods.

Questions

(a) How do different metals expand with time?

(b) Arrange the metals in order of increasing rate of expansion.

When equal lengths of different materials are heated through the same temperature change, they expand by different amounts. Brass, for instance, expands more than an equal length of steel rod. Expansion of different solids can be further demonstrated using materials like bimetallic strip.

Areal expansion and expansivity of solids

The area of a heated solid increases with an increase in temperature. The amount by which a unit surface area of a solid expands for a unit temperature increase is called the areal expansion or superficial expansion.

Consider a thin metal plate of area A_0 at temperature θ_o. When the temperature of the metal plate is raised to temperature θ_1, the area of the plate increases to A_1 as shown in Figure 5.10.

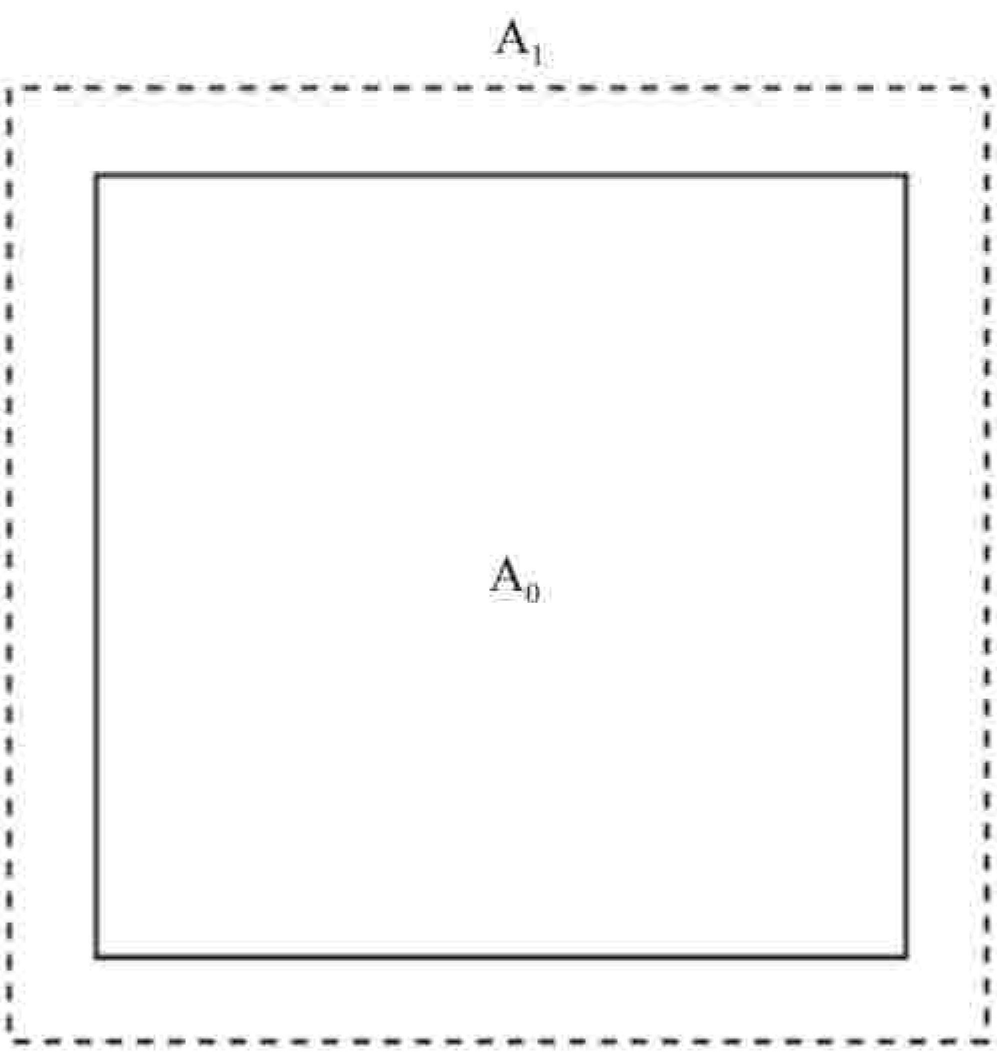


Figure 5.10: *Superficial expansion of a solid*

So, increase in temperature, $\Delta\theta = \theta_1 - \theta_0$

and the increase in area, $\Delta A = A_1 - A_0$

But the change in area is proportional to the original area of the object and the change in temperature thus,

$$\Delta A \propto A_0 \Delta\theta$$

$$A_1 - A_0 \propto A_0 \Delta\theta$$

$$A_1 - A_0 = \beta A_0 \Delta\theta$$

where β is a constant of proportionality called coefficient of superficial expansion or areal expansivity. The coefficient of superficial expansion can be expressed as:

$$\beta = \frac{A_1 - A_0}{A_0 \Delta\theta}$$

The SI unit of coefficient of superficial expansion is per kelvin (K^{-1}) or per degree centigrade $(°C^{-1})$.

There is a close relationship between coefficient of areal and linear expansion. Assume the steel square has sides of length l. The original area, $A_0 = l^2$ and the new area after a temperature increase,

$A_1 = A_0 + \Delta A = (l + \Delta l)^2$ then,

$$\Delta A = A_1 - A_0$$

$$= (l + \Delta l)^2 - l^2 = l^2 + 2l(\Delta l) + (\Delta l)^2 - l^2$$

$$\Delta A = 2l(\Delta l) + (\Delta l)^2$$

But, $\Delta l = l\alpha\Delta\theta$, so $(\Delta l)^2 = l^2\alpha^2(\Delta\theta)^2$

$$\Delta A = 2l^2\alpha\Delta\theta + \alpha^2 l^2 (\Delta\theta)^2$$

Since α is very small, a term containing α^2 is neglected, so that,

$$\Delta A = 2l^2 \alpha \Delta\theta$$

$$\Delta A = 2A_0 \alpha \Delta\theta$$

Remember $\Delta A = A_0 \beta \Delta\theta$ then $\beta = 2\alpha$.

Therefore, the coefficient of areal expansion is twice the coefficient of linear expansion.

Example 5.4

An engineer has one piece of metal plate only to be used in fixing a gap on a metal bridge. If the steel plate has area of $0.8\,\text{m}^2$ at 25 ºC, by how much must the engineer raise the temperature of the steel plate for it to fit a gap of $0.804\ \text{m}^2$? Use coefficient of linear expansion of steel as 0.000012 ºC^{-1}.

Solution

$$\beta = \frac{A_1 - A_0}{A_0 \Delta\theta}$$

$$\Delta\theta = \frac{A_1 - A_0}{\beta A_0}$$

Since $\beta = 2\alpha$,

$$\Delta\theta = \frac{0.804\ \text{m}^2 - 0.8\ \text{m}^2}{2 \times 0.000012\ ^\circ\text{C}^{-1} \times 0.8\ \text{m}^2} = 208.3\ ^\circ\text{C}$$

The temperature must be raised by $208.3\ ^\circ\text{C}$.

Volume expansion and expansivity of solids

When a solid is heated, its volume changes in response to the temperature change. The volume increases as the temperature of the solid increases. This means that, the particles in the solid are free to move in three dimensions (length, width and thickness), pushing each other, and increase the distance of separation from each other. The degree to which the volume of an object changes with the change in temperature is called the object's coefficient of volume expansion (volume expansivity). It is also called the coefficient of cubical expansion (cubical expansivity).

Let the initial volume of an object be V_0 at a temperature θ_0. If the object is heated to temperature θ_1, its volume will increase to V_1 as shown in Figure 5.11. The change in temperature of the object will be $\Delta\theta = \theta_1 - \theta_0$ and the change in volume will be $\Delta V = V_1 - V_0$.

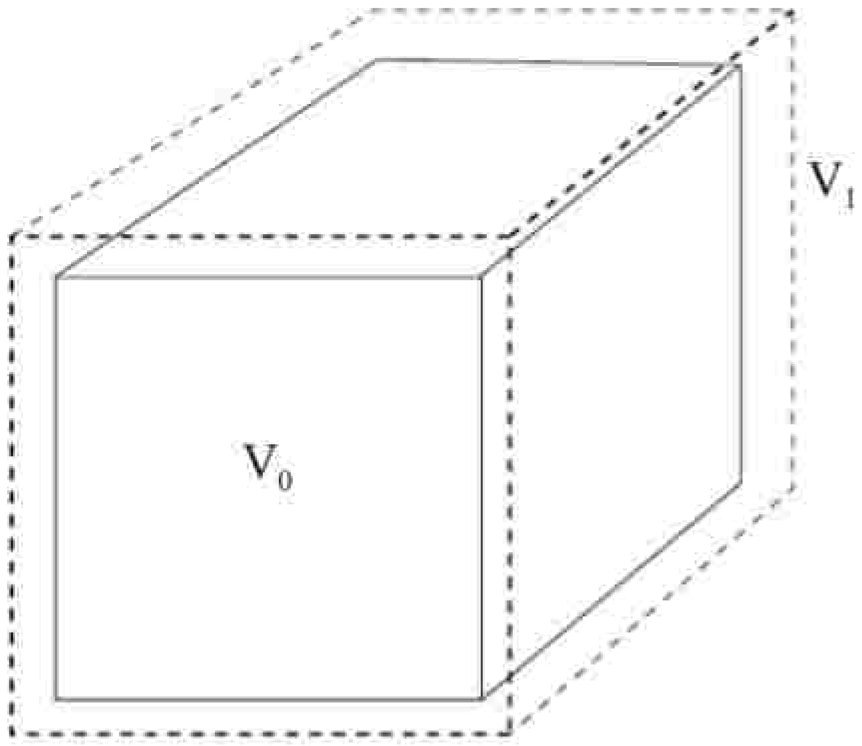


Figure 5.11: *Cubical expansion of a solid*

The change in volume is proportional to the original volume, $\Delta V \propto V_0$. Also, the change in volume is directly proportional to the change in temperature, $\Delta V \propto \Delta\theta$. Depending on the material of an object, the change in volume is given by,

$$V_1 - V_0 \propto V_o \Delta\theta \text{ or } V_1 - V_0 = \gamma V_o \Delta\theta$$

$$\gamma = \frac{V_1 - V_0}{V_o \Delta\theta}$$

where γ is the coefficient of volume expansion of a solid. If the object is in a cubic shape, then γ is the coefficient of cubical expansion. The coefficient of cubical expansion can also be related to coefficient of linear expansion.

Consider the volume change

$$\Delta V = V_1 - V_0$$

But $V_0 = l_0^3$

$$\Delta V = (l_0 + \Delta l)^3 - l_0^3$$

$$\Delta V = l_0^3 + 3l_0^2\Delta l + 3l_0\Delta l^2 + \Delta l^3 - l_0^3$$

But $\Delta l = \alpha l_0 \Delta\theta$

$$\Delta V = 3l_0^3\alpha\Delta\theta + 3l_0^3\alpha^2\left(\Delta\theta\right)^2 + \alpha^3 l_0^3\left(\Delta\theta\right)^3$$

Neglecting all terms containing α^2 and α^3.

$$\Delta V = 3l_0^3\alpha\Delta\theta$$

$$\Delta V = l_0^3(3\alpha)\Delta\theta$$

Also, $\Delta V = l_0^3\gamma\Delta\theta$.

Therefore, $\gamma = 3\alpha$.

Example 5.5

The diameter of an aluminium sphere at 20 ºC is 2.5 cm. If its diameter increases to 2.51 cm when heated, what is the final temperature? The coefficient of linear expansion of aluminium is $2.6 \times 10^{-5}\,{}^{\circ}\mathrm{C}^{-1}$.

Solution

$$V_0 = \frac{4}{3} \times 3.14 \times (1.25\text{ cm})^3 = 8.18\text{cm}^3$$

$$V_1 = \frac{4}{3} \times 3.14 \times (1.255\text{ cm})^3 = 8.28\text{cm}^3$$

$$\gamma = \frac{V_1 - V_0}{V_o\Delta\theta}$$

$$\Delta\theta = \frac{V_1 - V_0}{\gamma V_o}$$

$$\theta_1 - \theta_0 = \frac{V_1 - V_0}{\gamma V_o}$$

$$\theta_1 = \theta_0 + \frac{V_1 - V_0}{\gamma V_o}$$

But $\gamma = 3\alpha$

$$\theta_1 = 20\ {}^{\circ}\text{C} + \frac{8.28\text{ cm}^3 - 8.18\text{ cm}^3}{3 \times 2.6 \times 10^{-5}\ {}^{\circ}\text{C}^{-1} \times 8.18\text{ cm}^3}$$

$$= 176.7\,{}^{\circ}\text{C}$$

Therefore, the final temperature is 176.7 ºC.

Applications of expansion of solids

If an expanding or contracting object is restricted, it sets up a very large force. This force needs to be avoided as it can break or fasten an abject. The concept of expansion helps in creating expansion joints. An expansion joint allows an expanding object to settle in. The application of expansion in solids can be demonstrated in the bimetallic strips and riveting.

Bimetallic strip

A bimetallic strip consists of two different metals that expand at different rates when heated through the same temperature change. An example of such metals is brass and steel. The strips are bonded together throughout their length. When heated over the entire surface, the strips curve. The metal that expands faster forms the outside part of the curve (convex shape) while the one that expands more slowly is on the inside of the curve (concave shape). On cooling, brass contracts faster than steel. The bimetallic strip therefore bends towards the brass side. In this case brass will acquire concave shape and steel will acquire convex shape (Figure. 5.12).

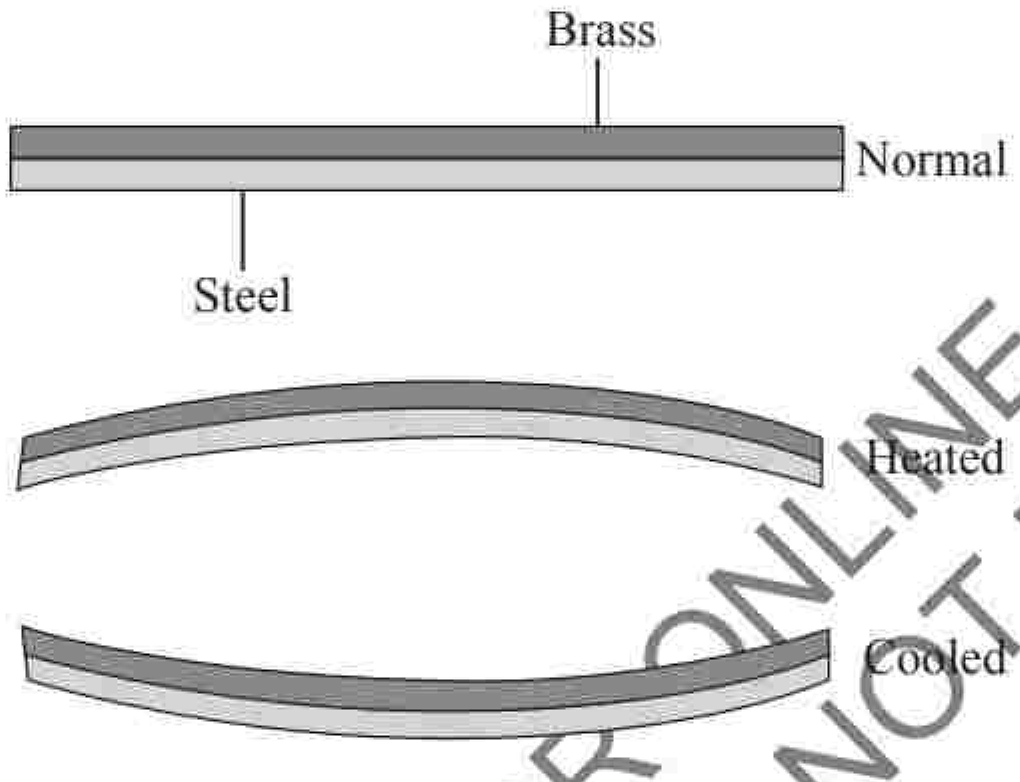


Figure 5.12: *Bimetallic strips*

The principle of bimetallic strip is applied in thermostats that are used in a variety of devices. A thermostat is a device used for maintaining a steady temperature range. It is basically a thermometer attached to a switch. It is used to turn a device ON or OFF when the thermometer reaches a certain temperature. Long bimetallic strips are coiled into spirals which makes them more sensitive to small temperature changes. Thermostats are used in many appliances such as electric irons, heaters, refrigerators, air conditioner, thermometers and fire alarms. Figure 5.13 illustrates fire alarm

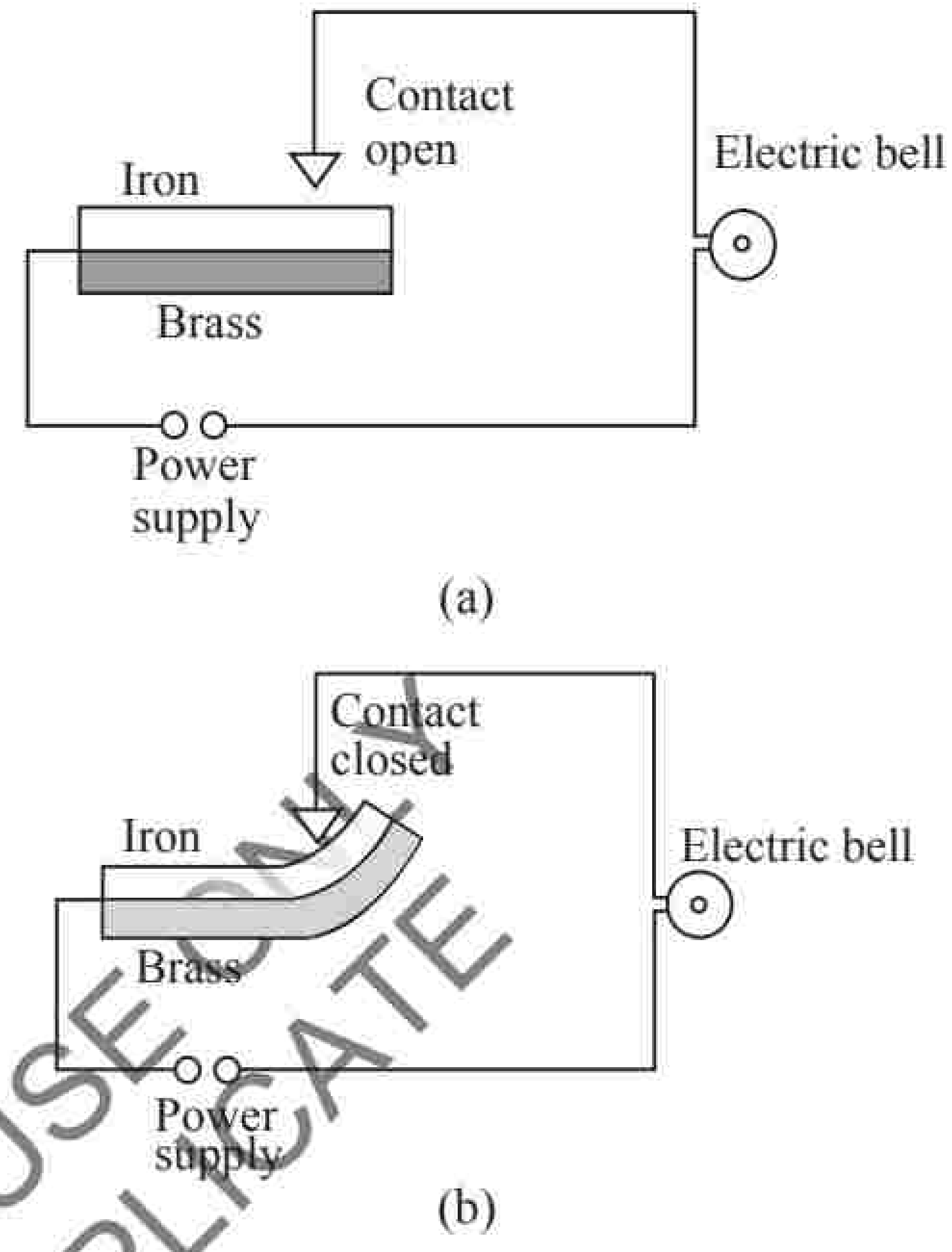


Figure 5.13: *A simple fire alarm circuit*

At room temperature, the iron and brass are of the same temperature, and the strip is not making contact to complete the circuit (Figure 5.13 (a)). At this temperature, no current is flowing through the circuit and the bell does not ring. When there is fire, the temperature rises and the strip expands. Since the brass expand faster than iron, the bimetallic strip bends towards the iron. The bending towards the contact completes the electric circuit (Figure 5.13(b)). This triggers the electric bell. On cooling, the brass contracts faster than iron.

An electric iron also uses a bimetallic strip as a mechanism of controlling the heat supplied to it. Figure 5.14 shows the principle of operation of the bimetallic strip in an electric iron.

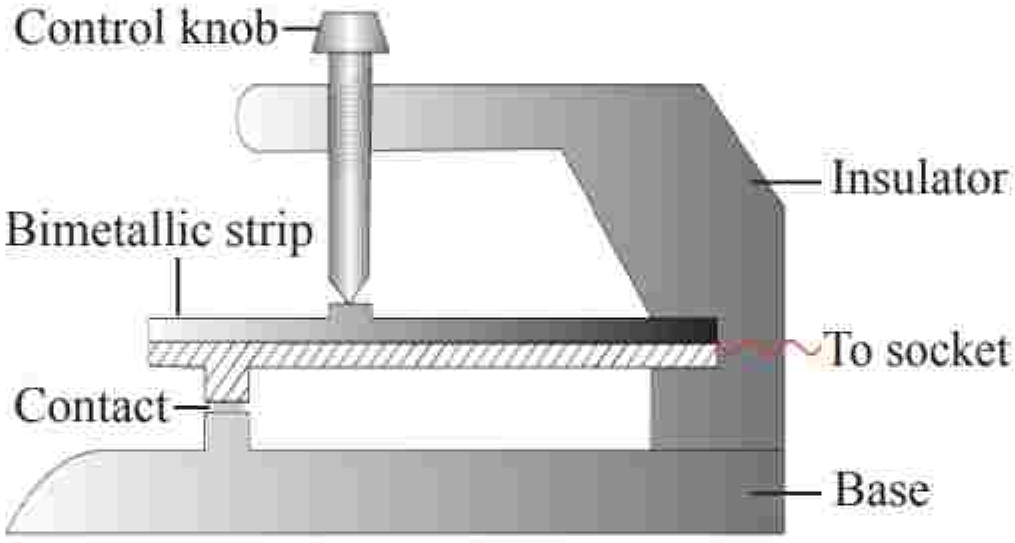


Figure 5.14: *An electric iron*

At moderate temperature, the contact point remains in physical contact with a bimetallic strip. When the temperature of iron rises, the strip begins to bend in the direction of the metal with the lower coefficient of expansion. As a result, the strip loses physical connection with the contact point, the circuit then opens and current stops flowing. As the circuit remains open for a certain time, the temperature falls, the strip assumes its original shape, and the current flows again.

In a bimetallic thermometer, the temperature is measured based on the different thermal expansions of the material of the strip. When the temperature increases, the metal strip bends due to different degrees of expansion. The extent to which the metal bends, is a measure of the temperature and can be read on a calibrated scale. The bimetallic strip used in thermometers is normally wound into a flat spiral. Figure 5.15 (a) and (b) show a bimetallic spiral and thermometer respectively.

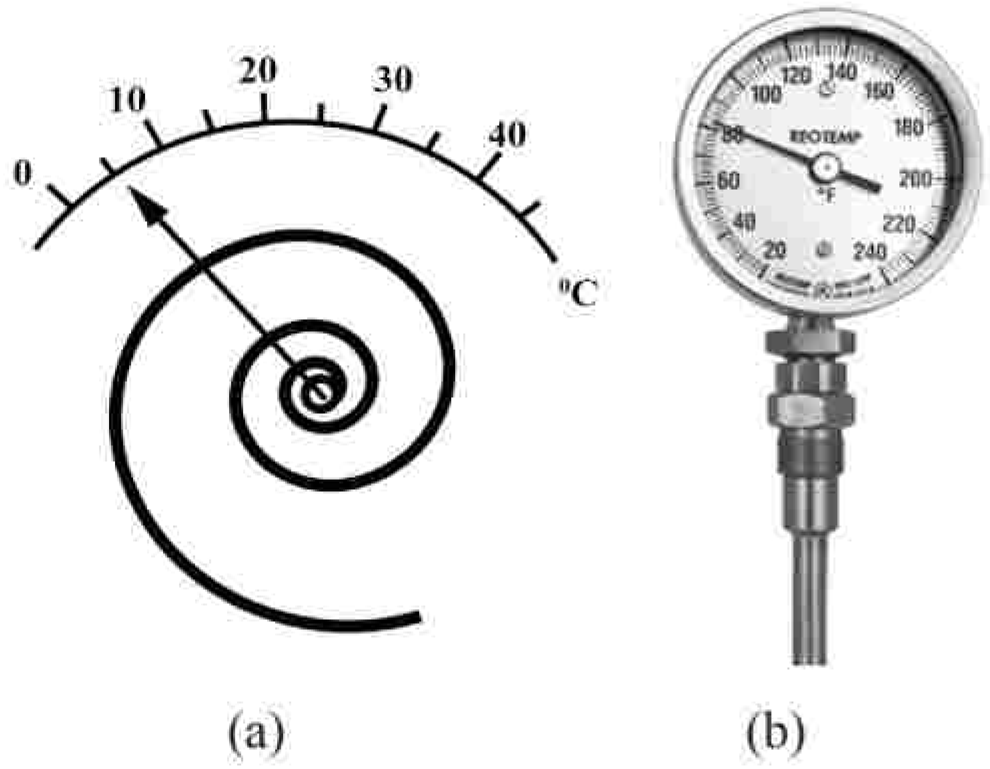


Figure 5.15: *Bimetallic spiral and its thermometer*

An increase in temperature makes the spiral curl in the clockwise direction, forcing the pointer to move over a calibrated scale. Bimetallic thermometers can be used as recording thermometers by attaching a pen to the pointer. The pen is positioned in such a way that, it marks on a revolving chart. One advantage of the bimetallic thermometer is that, it can be used to measure a much wider range of temperatures than the liquid-in-glass thermometer. Bimetallic thermometers are used to record temperature of inaccessible structures such as the temperature in marine ships, aeroplanes and engines of vehicles.

Construction of railway lines

One of the most common and oldest applications of the knowledge of expansion of solids is in the construction of railway lines. Gaps are left between successive rail bars to allow for expansion during hot weather. Two successive pieces of rail can also be connected with a fishplate to form an expansion joint as shown in Figure 5.16.

Figure 5.16: *Expansion joint and fishplate in a railway line*

Alternatively, the rail bars are left with slanting edges in order to form an overlapping joint. A small space is left between the two edges

Construction of bridges and house roofs

Bridges and house roofs made of steel girders also need allowance for expansion. A common method of allowing for the expansion in bridges is to have one end fixed while the other end rests on rollers (Figure 5.17). This allows movement in both directions. Roofing iron sheets on the other hand are corrugated (made of ridges) in order to allow for safe expansion and contraction. If roofing iron sheets were flat, they would twist and tear at the nails due to expansion and contraction.

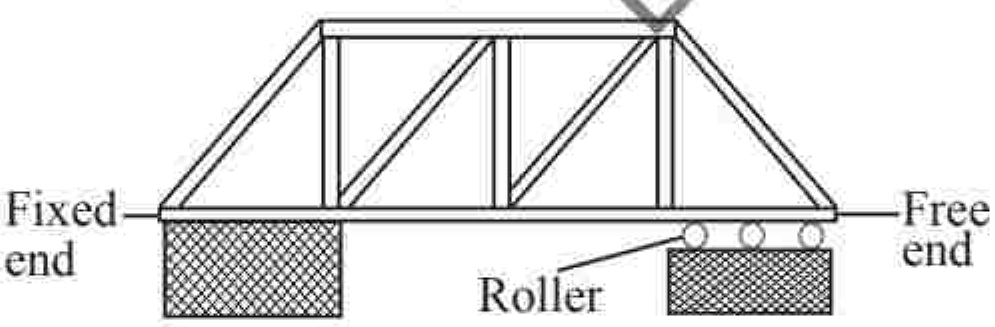


Figure 5.17: *A steel bridge with rollers to allow for expansion*

Installation of telephone cables and electrical transmission wires

Overhead telephone cables and electrical transmission wires are loosely fixed on the poles during installation to allow free contraction and expansion. The cables therefore appear stretched during cold morning and loose during hot afternoon as shown in Figure 5.18 (a) and (b) respectively.

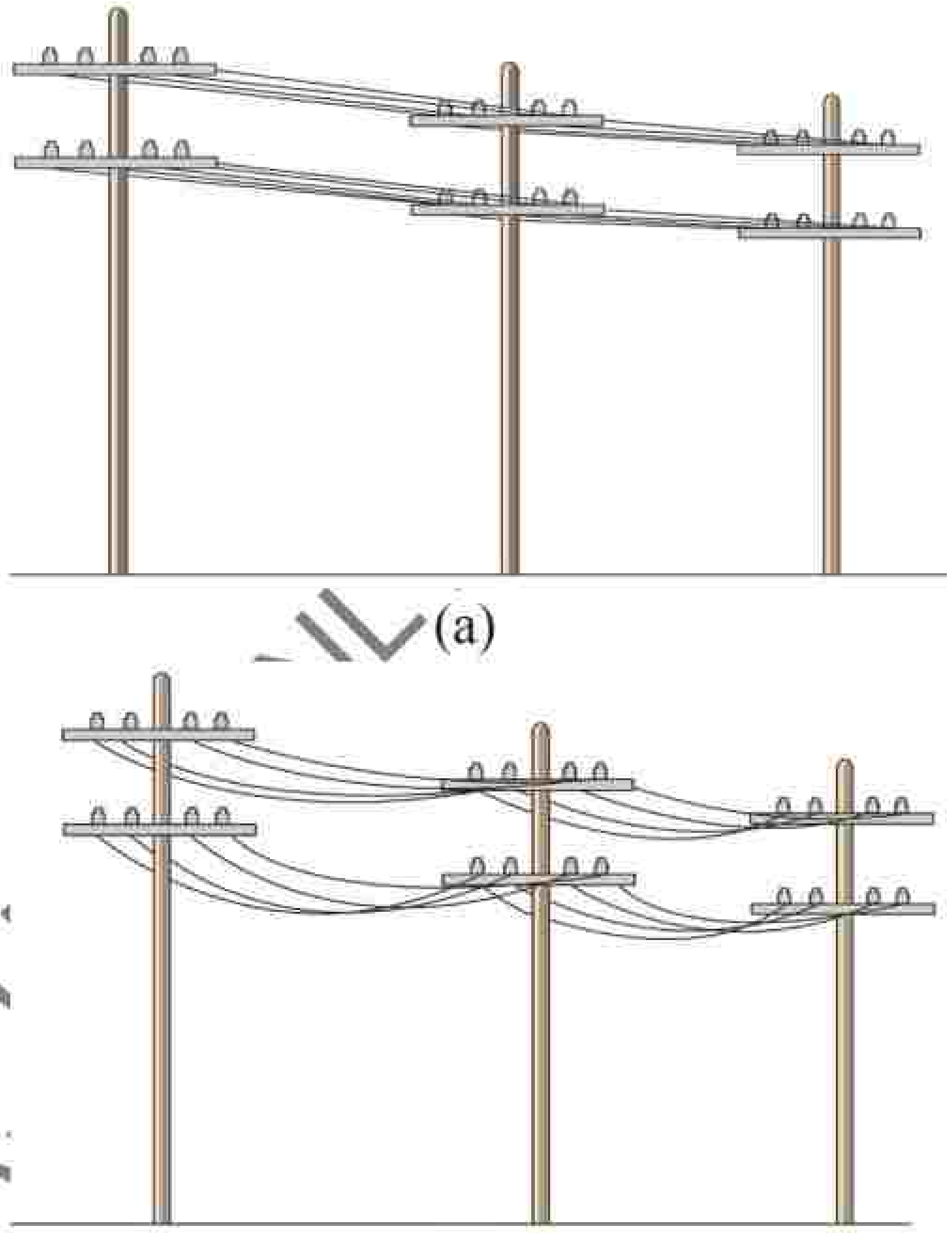


Figure 5.18: *Appearance of an overhead telephone cables on (a) cold morning and (b) hot afternoon*

Riveting

Another application of the knowledge of expansion is in the riveting of metals. The rivet is fitted when it is hot and is then hammered flat. On cooling, it contracts pulling the two plates firmly together. See Figure 5.19. Furthermore, train wheels are fitted with steel tyres which must be changed from time to time due to wearing out. To fit them tightly, the tyres are fitted when red-hot. On cooling, they contract and fit very tightly and therefore do not require screws and nuts.

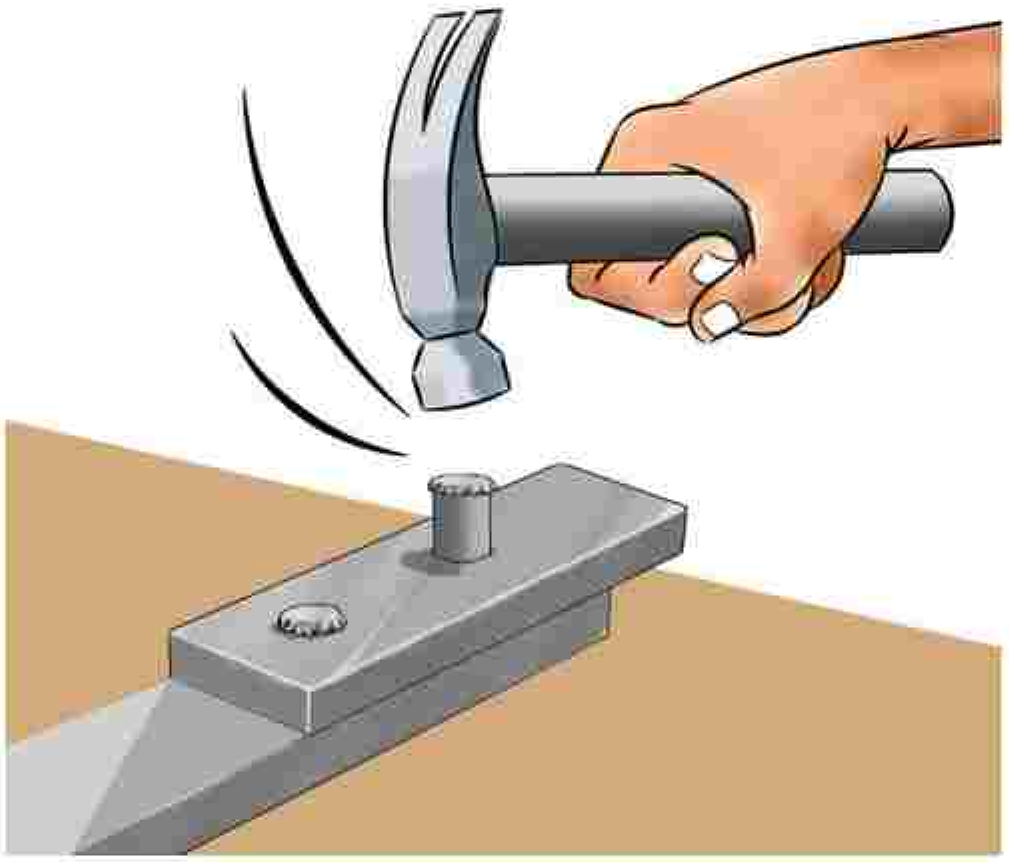


Figure 5.19: *Riveting*

Task 5.2

Identify other devices that apply the principle of bimetallic strip, then describe their mode of action. Present your work in the class.

Exercise 5.2

In the following questions, use constants provided in Table 5.2 where necessary.

1. Use kinetic theory to explain why solids expand when heated.
2. Heat originates from the collision of particles. Justify.
3. Explain why a glass container with thick walls is more likely to crack than one with thin walls when a very hot liquid is poured into each of the glasses.
4. A compound metal bar made of two strips, one of steel and another of brass bend when heated. Explain.
5. Explain the working principle of a fire alarm and bimetallic thermometer.
6. A brass plate has a hole whose radius is too small for an iron rivet to fit in. Explain two ways the rivet can be made to fit in the hole.
7. An aluminium lid on an ordinary glass jar fits so tightly that it cannot be unscrewed. Should the jar and lid be immersed in hot water or cold water to loosen the lid? Explain your answer.
8. Figure 5.20 shows a bimetallic thermostat used to regulate a cooler and a heater in a classroom. The metal on top expands less than the metal below. To keep the temperature in the room constant, which of the two devices, A or B, should be the heater? Explain your answer.

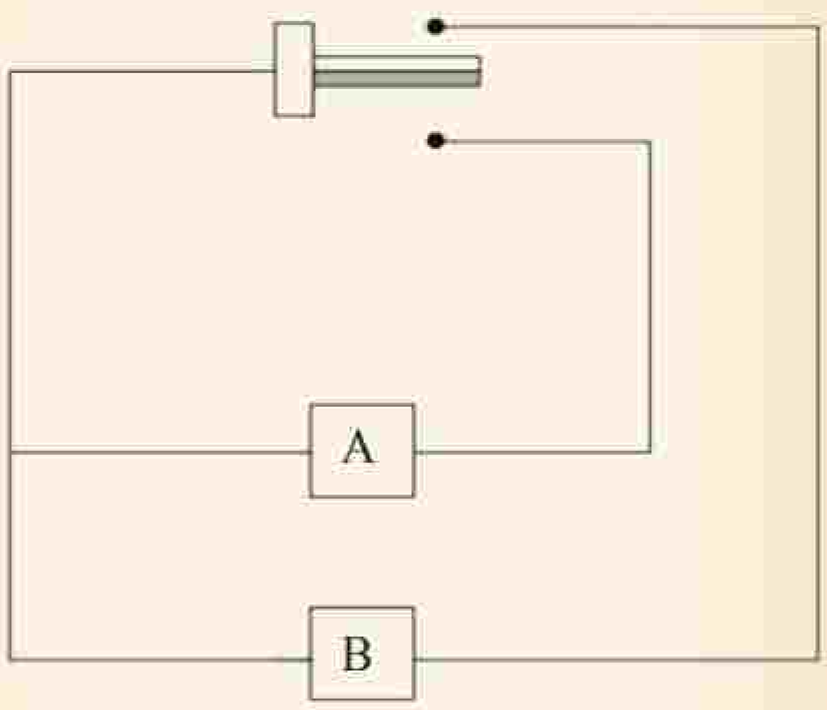


Figure 5.20

9. A steel tower has a height of 324.00 m at a temperature of 18 °C. How tall is the tower on a day when the temperature is 35 °C?

10. A road is made of concrete slabs separated by expansion joints. Each slab is 25 m long. If the temperature of the road varies from −15 ºC to 45 ºC, how wide should the expansion joints be?

11. A copper pipe of length 100 cm at 15 °C increases its length by 0.15% when steam at 100 °C passes through. Find the coefficient of linear expansivity of copper.

12. Two poles are 50 m apart. An aluminium cable of length 50.1 m and at a temperature of 25 °C is clamped between them. At what temperature would there be no sag in the wire?

13. A brass rod of length 0.997 m at 20 °C is hung from a steel frame with a height of 1.00 m at 20 °C as shown in Figure 5.21. At what temperature would the brass rod just touch the floor?

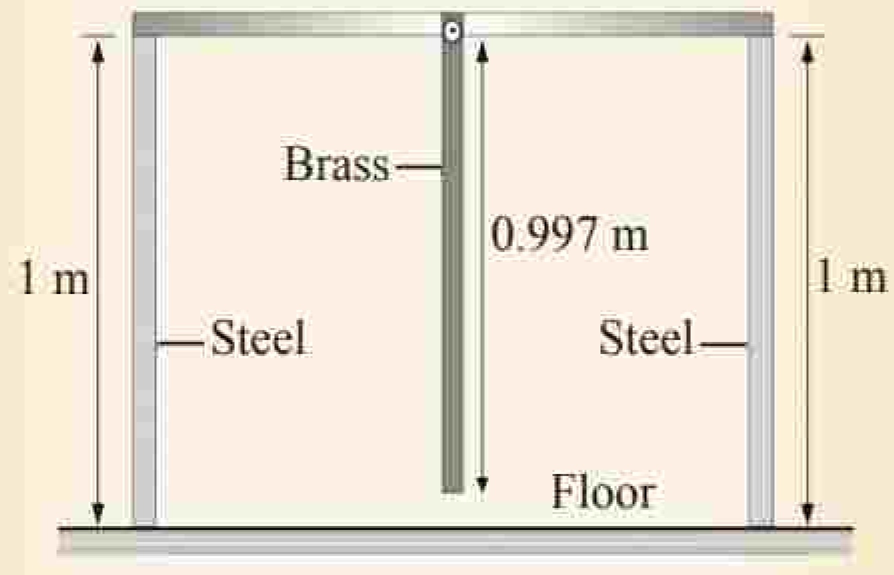


Figure 5.21

14. The linear expansivity of brass is 0.000019 K^{-1}. Explain the meaning of this statement.

Thermal expansion of liquids

Unlike solids, heating liquids requires the liquid to be poured into a vessel. The liquid takes the shape of the container. For this reason, a liquid has no length or area but has only volume. The increase in volume of the heated liquid is called thermal expansion. When the liquid is heated, the average kinetic energy of the particles increases. The heating causes liquid molecules to move faster and further apart hence occupying more space. This results into volume increase, and a liquid is said to expand. When liquid is cooled, the particles reduce their kinetic energy and they start moving closer together. The liquid contracts, occupying less space and so the volume decreases.

Activity 5.4

Aim: To observe the expansion of water

Materials: round-bottomed flask ($500\ cm^3$), potassium permanganate, glass tubing, rubber cork, hot waterbath, retort stand, marking pen, water, source of heat

Procedure

1. Pour water in a waterbath.
2. Fill the flask with coloured water.
3. Push the rubber cork with the glass tubing fixed through it until the water rises to some height in the glass tubing as shown in Figure 5.22.

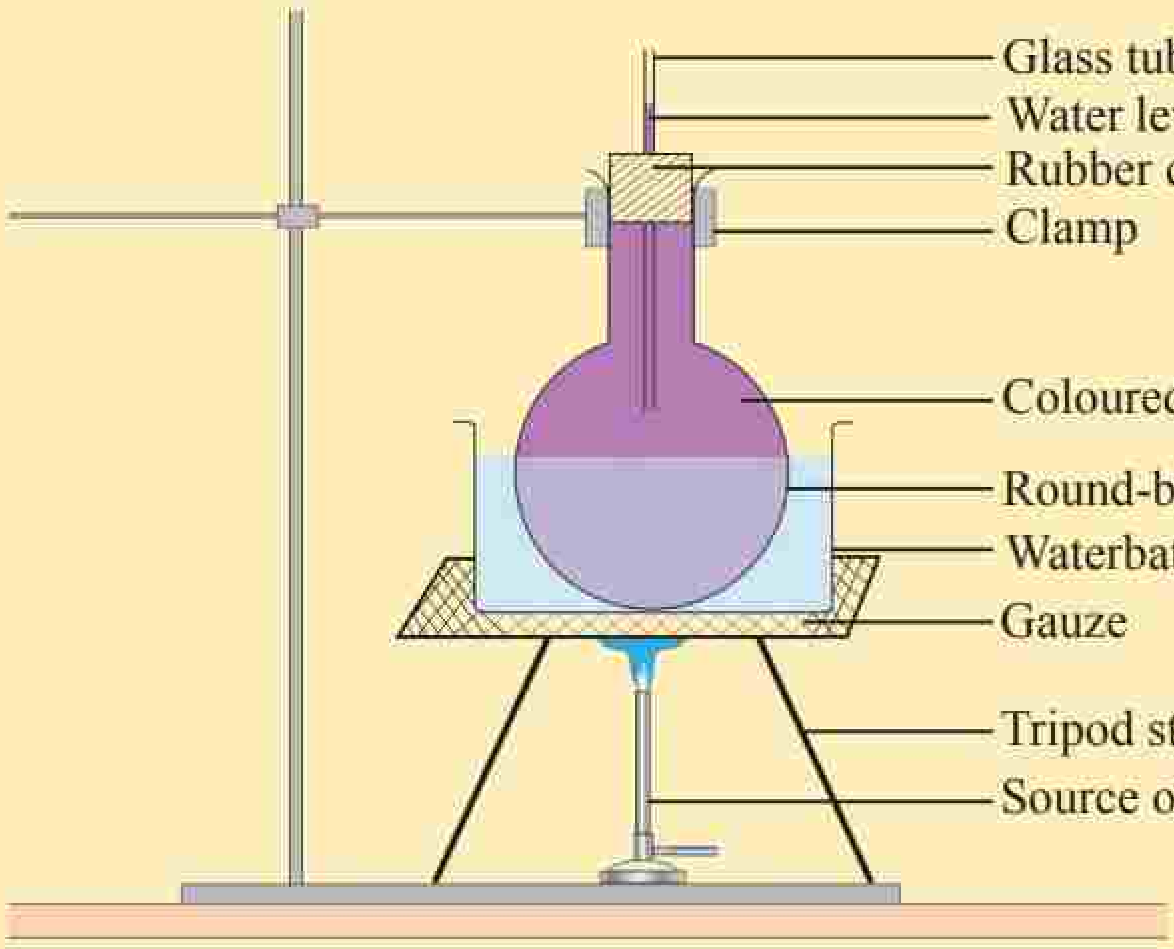


Figure 5.22

4. Mark the level of water in the glass tubing.
5. Place the flask in the waterbath and heat the waterbath for some time.
6. Carefully observe the level of heated water in the glass tube.

Questions

1. Explain your observation on the level of water in the glass tubing as the water in the flask was heated.
2. How can the level of heated water be compared to that of water that is not heated?

The water level will first drop due to an increase in the volume of the flask on being heated. The increase in volume is due to the expansion of the glass. The level of the water will then rise as the flask remains in the hot waterbath. This is because the molecules of water acquire more kinetic energy. The kinetic energy moves the molecules far apart inside the flask, resulting in increased water volume. The expansion of the flask is insignificant compared to that of the water. Since the water increases in volume, its density decreases.

Activity 5.5

Aim: To compare the expansion of different liquids

Materials: three round-bottomed flasks and their stoppers, three glass tubes, source of heat, waterbath, thermometer, ruler, water, cooking oil and kerosene (enough to fill one flask each), retort stands, clamps, marking pen

Procedure

1. At room temperature, fill the three flasks with equal amount of water,

cooking oil and kerosene, each in a separate flask.

2. Insert the glass tubes through the stoppers and push the stoppers such that each liquid forms the same initial length in the glass tube (Figure 5.23).

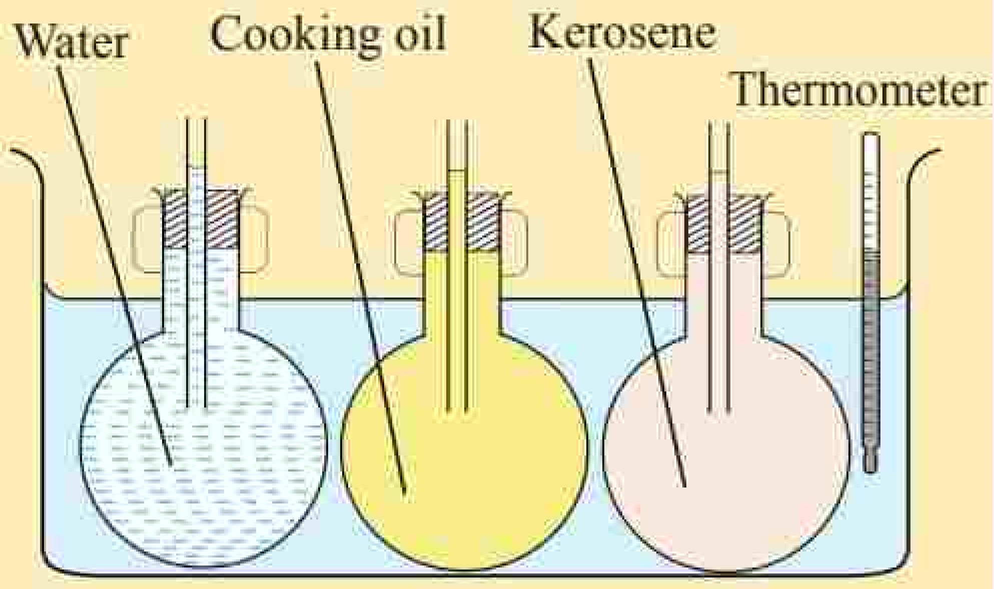


Figure 5.23

3. Mark the level of the liquid in each glass tube.
4. Place the flasks in the waterbath and insert the thermometer in the water bath.
5. Hold the flasks in an upright position using clamps. The waterbath should be large enough to hold the three flasks.
6. Heat the waterbath to a constant temperature.
7. Allow enough time for the three flasks to attain thermal equilibrium with the waterbath.
8. Measure and record the height of each liquid in the glass tube against the corresponding temperature.

Questions

(a) What happens to the level of each tube after heating?

(b) Which liquid level was the highest?

(c) Which liquid level was the lowest?

The levels of the liquids in the tubes rose. The level rose for different liquids in the order of kerosene, cooking oil and water. The different levels show that, some liquids expand more than others for a given rise in temperature.

Apparent volume expansion of a liquid

Liquids are always heated in a vessel. To find the volume expansion of a liquid, it is important to consider the expansion of the vessel. In this case, the heat reaches the vessel before it reaches the liquid. The expansion which is observed here is the difference between the increase in the volume of the liquid and the increase in the volume of the vessel. However, the expansion of the vessel is usually smaller than that of the liquid, so it is not considered. The volume expansion observed without considering the expansion of the vessel is known as *apparent expansion.* It is always less than the true expansion of the liquid.

The real expansion of the liquid must consider both the expansion of the vessel and the expansion of the liquid. This is obtained by adding the expansion of the liquid and the expansion of the part of the vessel containing liquid.

Activity 5.6

Aim: To demonstrate the apparent expansion of a liquid

Materials: a thick glass flask, heat source, potassium permanganate, capillary glass tube

Procedure

1. Fill a thick glass flask with coloured water.
2. Insert a capillary glass tube until the level of water is well above the flask.
3. Mark the level of water as y.
4. Heat the water while observing what happens to the level, y, initially as you start heating.
5. Mark the new level as, x.
6. While heating is continuing, observe the liquid level in comparison with levels x and y.
7. Mark the final level.

Question

Briefly explain the observation made during heating of water.

When the container is heated, it expands, so its volume increases, causing the water level to fall slightly. As heating continues, the level of the water rises. This is because the expansion of the water is much more than that of the container. In measuring expansion of liquids, we are only concerned with the change in the volume of the liquid and not the volume of the vessel containing the liquid. The apparent expansion of a liquid is equal to the actual expansion of the liquid without considering the increase in volume of the containing vessel up to the liquid level.

Volume expansion and expansivity of liquids

Consider the change of volume of liquid caused by the increase in temperature such as the expansion of alcohol in a thermometer. If the initial volume is V_0, final volume is V_1 and temperature change is $\Delta\theta$, then the change in volume is,

$$\Delta V = V_1 - V_0$$

The volume expansivity (γ) of liquid is the fractional change in volume per unit temperature change expressed as:

$$\gamma = \frac{\Delta V}{V_o \Delta\theta}$$

The SI unit for volume expansivity is K^{-1}. Table 5.3 shows the values of volume expansivity for different liquids.

Table 5.3: Coefficients of volume expansion for various liquids

Liquid	$\gamma\ (1 \times 10^{-5}\ °C^{-1})$
Benzene	124
Gasoline	95
Glycerine	53
Kerosene	99
Mercury	18
Methanol	122
Water at 20 ° C	21
Water at 35 ° C	35
Water at 90 ° C	70

Effect of heat on density of liquids

When a liquid is heated, the kinetic energy of its particles increases. These particles move far apart occupying more space. This means that the volume occupied by the particles increases. If evaporation is ignored, the mass of the liquid remains constant. By definition,

$$density = \frac{mass}{volume},$$

thus, density decreases as the volume increases. Therefore, as the liquid is heated, its density decreases.

Anomalous expansion of water

Liquids expand linearly with the rise of temperature, except for water where expansion is not linear. Water can exist in three states; ice, liquid water and steam. When ice is heated from $-4\,°C$ to $0\,°C$, it expands like other liquids and start to melt to form water at $0\,°C$. If the heating continues from $0\,°C$ to $4\,°C$, the melting continues accompanied by decrease in volume of water. The contraction continues until the water reaches its minimum volume at the temperature of $4\,°C$. Above $4\,°C$, the water expands. In other words, if water is cooled from $4\,°C$ to $0\,°C$, it expands. This unusual behaviour of water to contract as temperature increases from $0\,°C$ to $4\,°C$ instead of expanding, is commonly referred to as anomalous expansion of water. This unusual behaviour of water can be demonstrated well in Activity 5.7.

Activity 5.7

Aim: To investigate the effect of heat on the density of water

Materials: boiling tube, a fitting cork with two holes, two thermometers, freezing mixture (mixture of ice and salt), a glass tube, a millimetre scale, a gas jar

Procedure

1. Fill the boiling tube with pure water.
2. Insert one thermometer and a glass tube with a millimetre (mm) scale through the cork.
3. Place the set-up in the gas jar as in Figure 5.24.
4. Fill the jar with a freezing mixture, and insert a thermometer in the mixture.

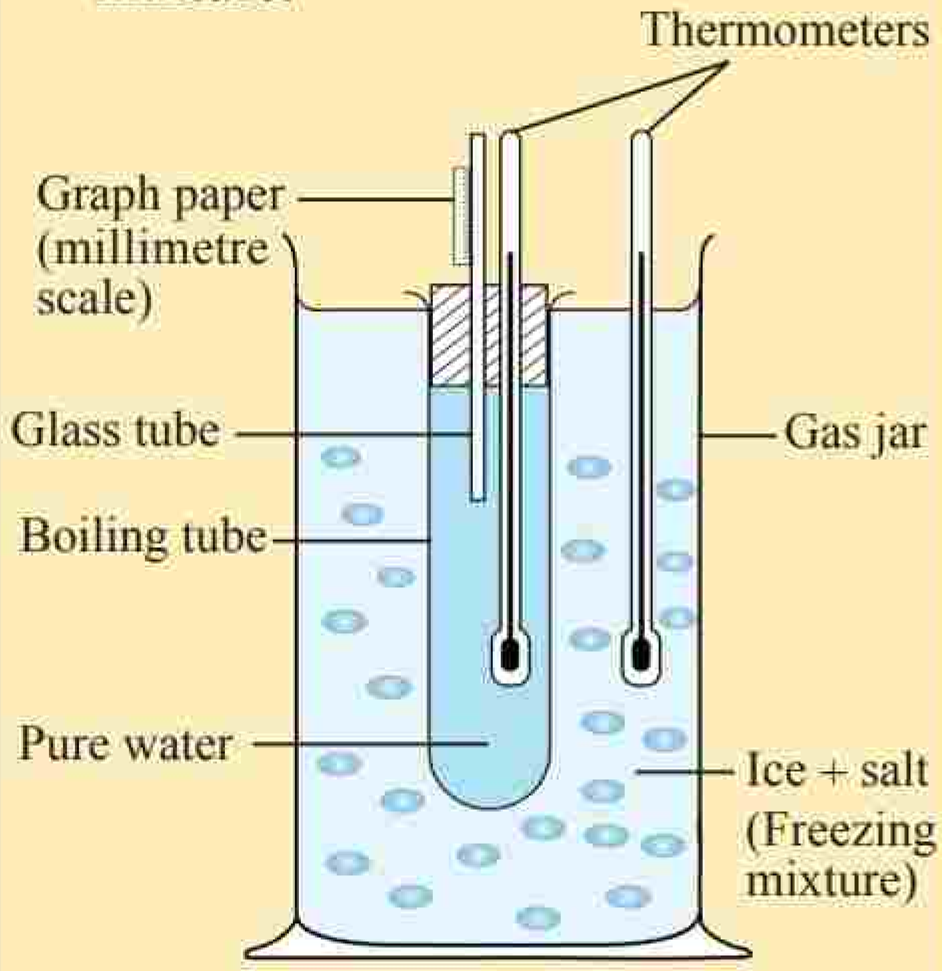


Figure 5.24

5. Wait until the temperature of the water inside the tube falls to $0\,°C$.

6. Remove the boiling tube from the gas jar when the reading of the thermometer in the tube falls to 0 °C.
7. Record the temperature reading against the water level.
8. Continue recording the corresponding temperature reading against the corresponding water level at temperature intervals of 1 °C as the water warms to room temperature.

Questions

(a) Plot a graph of the millimetre scale reading against the temperature.

(b) From the graph, explain the trend in the volume of water as its temperature rises from 0 °C to room temperature.

As the temperature of water increases from 0 °C, the reading of the millimetre scale decreases. The reading of the scale reaches a minimum at 4 °C and then starts increasing gradually. A sketch of graph of the millimetre scale reading against temperature is shown in Figure 5.25.

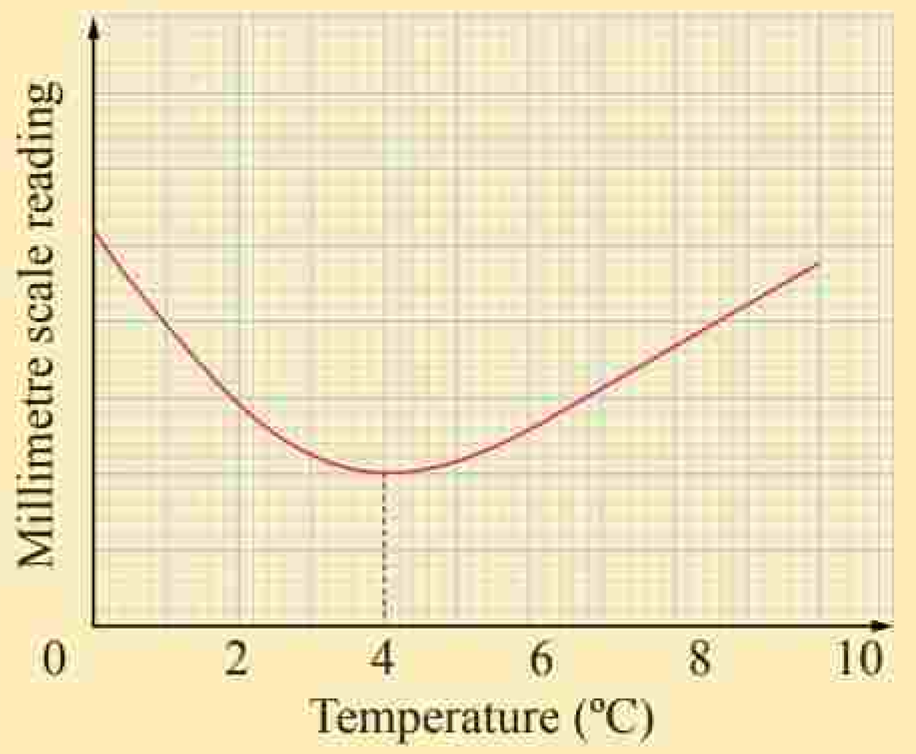


Figure 5.25: *A graph of millimetre scale reading against temperature*

Since the volume is proportional to the millimetre scale reading, the volume decreases and reaches a minimum at 4 °C. Moreover, mass is constant while the volume is decreasing, the density of the water increases to a maximum value at 4 °C as shown in Figure 5.26.

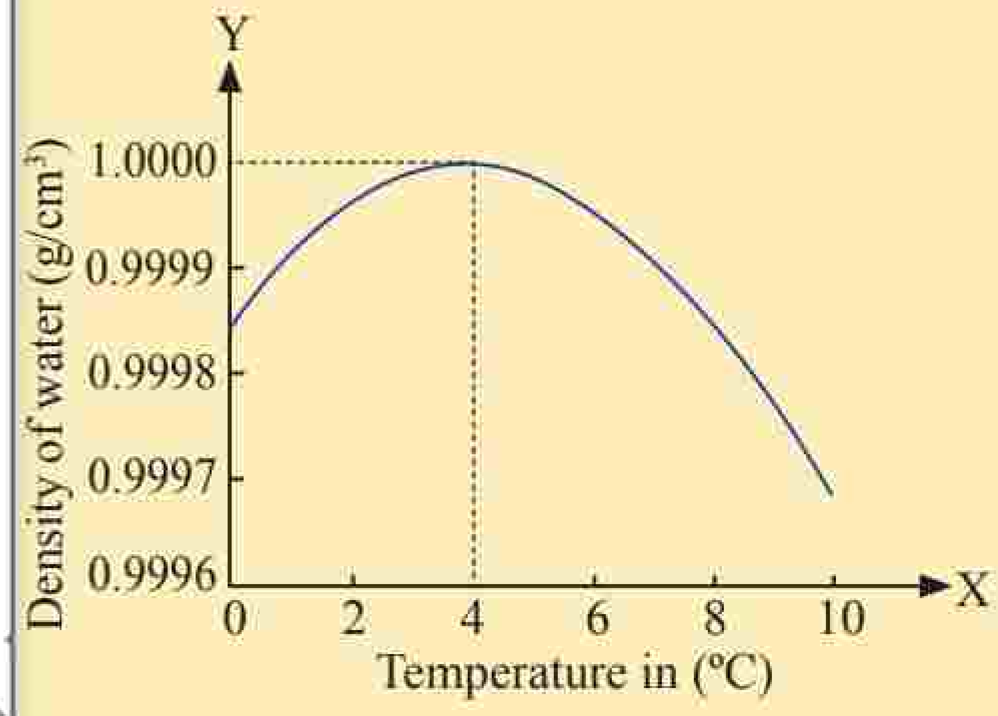


Figure 5.26: *Variation of the density of water with temperature*

The density versus temperature curves for water as compared with other liquids is shown in Figure 5.27.

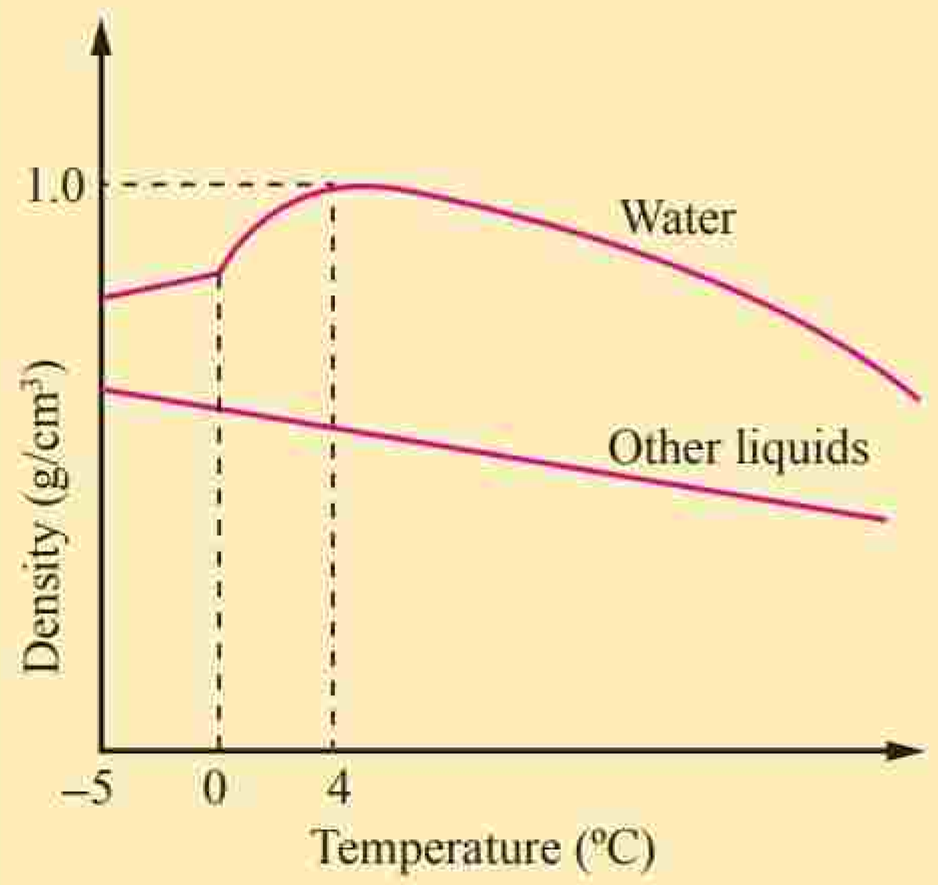


Figure 5.27: *Density-temperature curves for water and other liquids*

Note that, the density of water is maximum at 4 °C and does not vary uniformly while the densities of other liquids vary uniformly with the change in temperature. This unusual thermal expansion of water is what is referred to as anomalous expansion of water.

Activity 5.8

Aim: To investigate the anomalous expansion of water using Hope's Apparatus

Materials: Hope's apparatus, two thermometers, freezing mixture (ice + salt), pure water, stopwatch, two corks

Procedure

1. Insert the thermometers into the Hope's apparatus, one at the bottom opening and the other at the top opening as shown in Figure 5.28; use corks to support them.

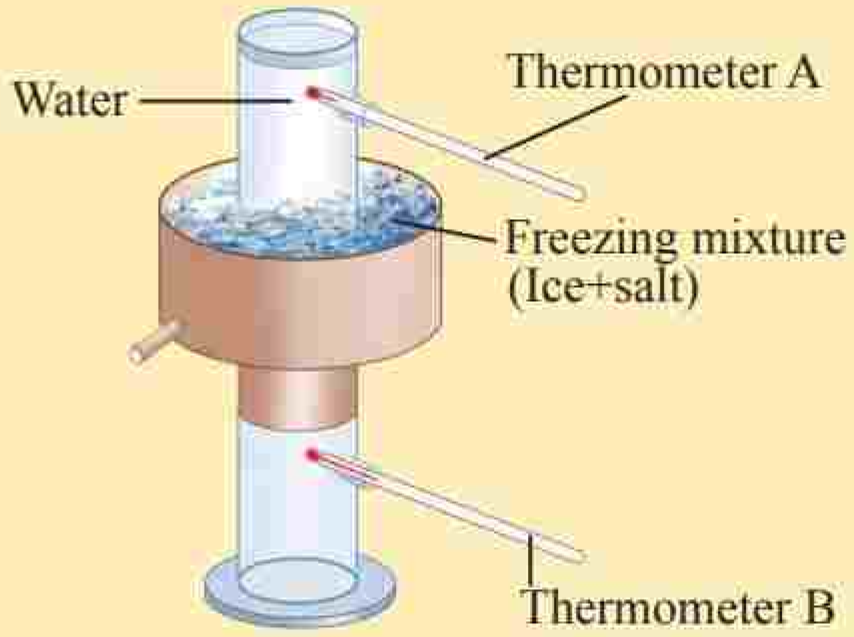


Figure 5.28

2. Fill the Hope's apparatus with pure water.
3. Note the readings in each of the thermometers.
4. Prepare a freezing mixture by mixing ice and salt.
5. Place the freezing mixture around the middle of the Hope's apparatus and start noting the readings of the thermometers at an interval of 30 seconds. Record your readings in a table.

Questions

(a) Describe the trend in the thermometers reading with time.

(b) Using the obtained readings, plot a graph of temperature against time for both thermometers on the same axes.

The reading of the lower thermometer starts falling while that of the upper one remains constant. When the temperature reading of the lower thermometer reaches 4 °C, it stops decreasing. On the other hand, the reading of the upper thermometer starts to fall until it goes below 4 °C. Figure 5.29 shows the expected graphs for the temperature recorded for both thermometers against time.

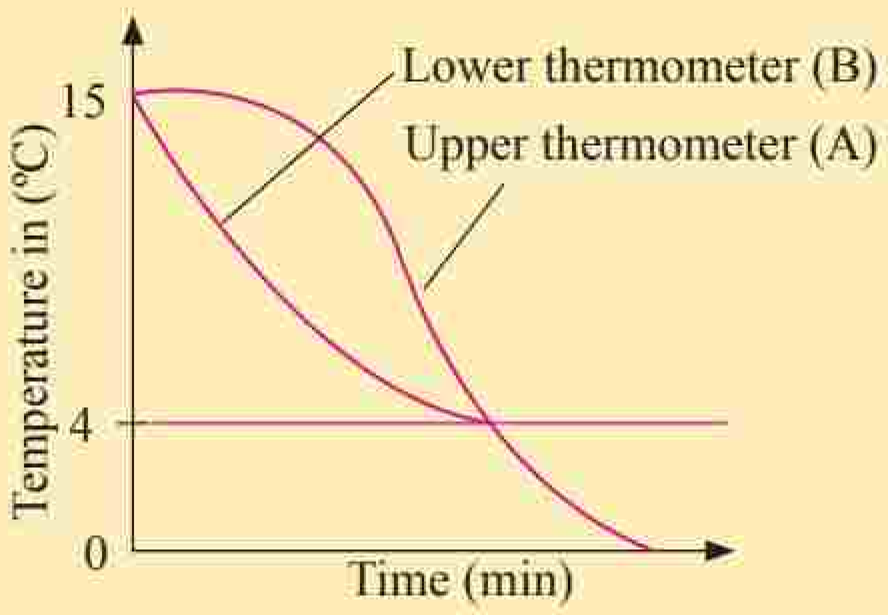


Figure 5.29: *A graph of temperature against time*

When the freezing mixture is put around the middle of the Hope's apparatus, it causes the temperature of the water around there to drop. This causes the water around to be denser and sink to the bottom as the warm water from the bottom rises to take its place. Because of this movement, the temperature reading of the lower thermometer drops steadily, while the upper thermometer reads a constant temperature. When the reading of the lower thermometer reaches $4\ °C$, the density of water at the lower part reaches a maximum value. Further cooling produces water of less density which moves up and therefore, the reading of the upper thermometer starts to drop.

Applications of anomalous behaviour of water

The peculiar expansion of water has an important bearing on the preservation of aquatic life during extreme cold weather. Water in lakes and ponds usually freezes in winter. Ice, being less dense than water, floats on water. This insulates the water below against heat loss to the cold air above. Water at $4\ °C$, being most dense, remains at the bottom of the lake, while ice, being less dense than water, floats on the layers of water at different temperature, as shown in Figure 5.30.

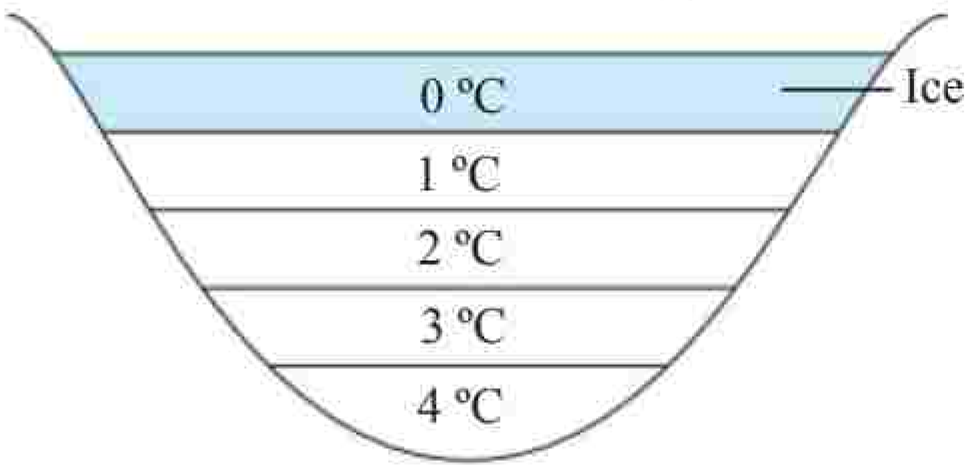


Figure 5.30: *Variations of temperature in a frozen lake*

This enables fish and other aquatic organisms to survive in the water below the ice. However, the anomalous expansion of water has undesirable effects. These include cracking of glass bottle when cooled in deep freezer after being filled with water and sealed. Also, when water freezes in a pipe; it may cause the pipe to burst due to expansion. Similarly, icebergs, being less denser than water, float in oceans thus posing a danger to ships. This is because navigators cannot see the submerged part of the iceberg which sometimes is very huge and can wreck a ship.

Applications of expansion of liquids

1. The expansion of liquids is used in measuring temperature in our everyday life activities. This is done by using liquid thermometers like mercury and alcohol thermometers. The liquid in the thermometers expands with an increase in temperature.
2. Industrial bottling of liquids provides empty space to allow for expansion during freezing of the liquids.

Example 5.6

The increase in volume of 10 cm³ of mercury when the temperature rises by 100 ºC is 0.182 cm³. What is the volume expansivity of mercury?

Solution

$$\gamma = \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{0.182\ \text{cm}^3}{10\ \text{cm}^3 \times 100\ °\text{C}}$$

$$= 1.82 \times 10^{-4}\ °\text{C}^{-1}$$

Exercise 5.3

1. Explain how fish and other aquatic organisms are able to survive in a frozen lake.
2. An ice block floats when immersed in water. Explain.
3. Draw a labelled diagram for a set up of an experiment to show that alcohol expands more than an equal volume of water for the same rise in temperature.
4. As an expert in thermal expansion, you are consulted by a company to construct a thermometer. The company has provided you with mercury, alcohol and gasoline. With justification, explain which liquid you will use in constructing the thermometer.
5. Briefly, explain how mass, volume and density vary when water is heated from $0\,^{\circ}\mathrm{C}$ to $90\,^{\circ}\mathrm{C}$ temperature?
6. A hollow glass sphere has a density of $1.30\ \mathrm{g/cm^3}$ at $20\ ^{\circ}\mathrm{C}$. Glycerine has a density of $1.26\ \mathrm{g/cm^3}$ at $20\ ^{\circ}\mathrm{C}$. At what temperature would the sphere begin to float in glycerine? (Volume expansivity of glycerine $= 53 \times 10^{-5}\ /\ \mathrm{K}$).
7. A $500\ \mathrm{cm^3}$ Pyrex beaker is 95% full of methanol at 15 °C. At what temperature will it be 100% full with the methanol? (Volume expansivity of methanol $= 122 \times 10^{-5}\ /\ \mathrm{K}$).
8. An ice sheet $5\ \mathrm{m}$ thick covers a lake that is $20\ \mathrm{m}$ deep. What is the temperature of the water at the bottom of the lake? Explain your answer.

Thermal expansion of gases

A gas, like a liquid, takes up the shape of the vessel in which it is contained. The volume of a gas remains constant if its temperature is kept constant, but when its temperature is changed, its volume also changes. Like solids and liquids, gases are sensitive to heating; its particles gain kinetic energy and move faster and far apart. A small amount of energy makes gas to expand much more than liquids and solids.

Activity 5.9

Aim: To demonstrate the behaviour of gases when heated

Materials: test tube, water, beaker, narrow glass tube, rubber band, source of heat, cork

Procedure

1. Arrange the apparatus as shown in Figure 5.31.

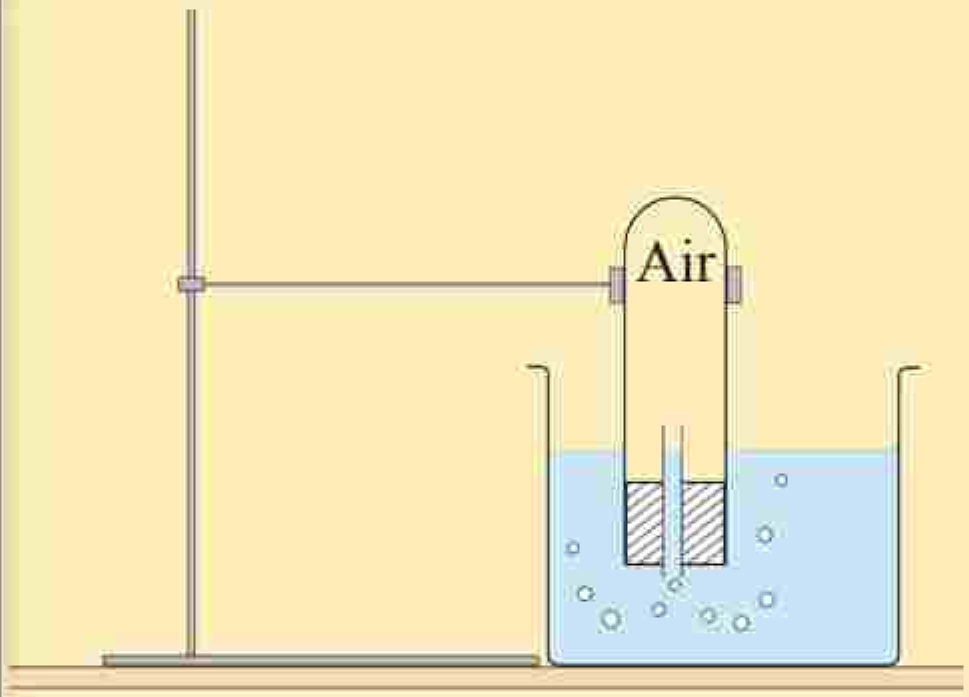


Figure 5.31

2. Count and record the number of bubbles coming out of the tube for 60 seconds.
3. Hold the test tube with your hand so as to warm it or place the test tube near a source of heat.

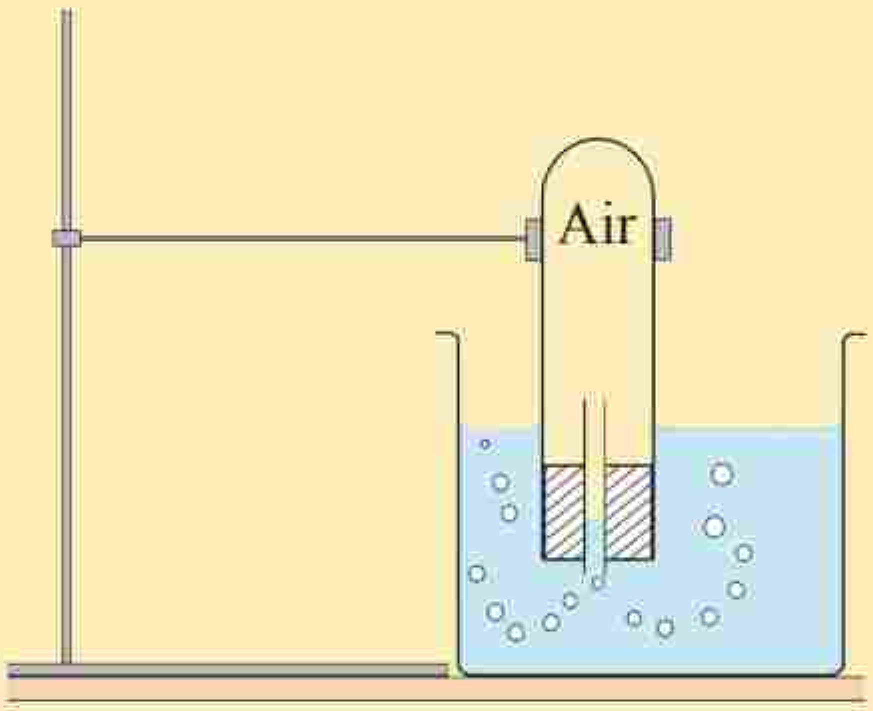


Figure 5.32

4. While holding the test tube with the hand or near the source of heat, count and record the number of bubbles coming out in ten seconds (Figure 5.32).
5. Take your hand or the source of heat away, then allow the test tube to cool for 5 minutes (Figure 5.33).

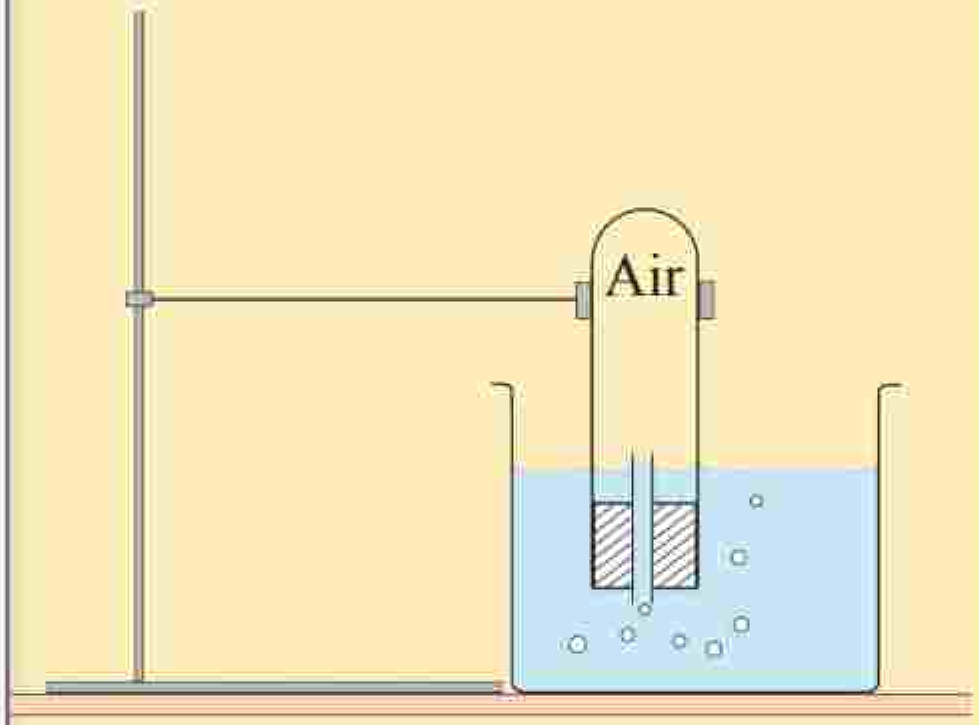


Figure 5.33

6. Carefully observe what happens in the test tube

Questions

(a) How many bubbles did you count before the test tube was warmed?
(b) How many bubbles did you count for a warmed test tube?
(c) What happened when the hand was removed from the test tube?

Few bubbles are seen coming out before the test tube was warmed. Many bubbles come out in the warmed test tube. But water enters the test tube when the hand is removed from the test tube. The number of bubbles varies because the warming hand heats the air in the tube. As the air expands, it requires more space and it comes out as bubbles in water. Removing the hand from the test tube results into cooling of the air in the tube. The particles move closer together and create space in the tube. The water in the beaker is pushed back through the tube by atmospheric pressure to take the empty space created by the escaping air.

Variation of pressure, temperature and volume of a gas

The variation of the volume of air occurs because of changes in temperature or pressure. When the molecules of air move randomly in a container, they hit the walls of the container, thus exerting force on the walls of the container. This force is equal to the rate of change of momentum of the molecules. Since the force acting per unit area defines the pressure, then pressure is said to be exerted on the wall by molecules. If the temperature is increased, the kinetic energy of the gases increases and causes the velocity of the molecules to increase. The increased velocity of the molecules increases the rate of change of momentum; as a result, greater force is exerted on the wall. Therefore, the gas pressure increases.

The behaviour of a fixed mass of gas is explained better by considering the three variables: pressure (P), volume (V) and temperature (T). If one variable is kept constant and the other two are varied, three relations can be established. These are:

(a) The relation between volume and temperature of a fixed mass of a gas at constant pressure.

(b) The relation between volume and pressure of a fixed mass of a gas at constant temperature.

(c) The relation between pressure and temperature of a fixed mass of a gas at constant volume.

The relationship between volume and temperature at constant pressure

The empirical relationship between the volume and the temperature of a fixed mass of a gas at constant pressure was first suggested by a French Physicist, Jacques Alexander Charles. This relation is also termed as Charles' law.

Activity 5.10

Aim: To verify the relationship between volume and temperature of a fixed amount of gas at constant pressure

Materials: thermometer, tall glass jar, water, uniform capillary tube, beakers, concentrated sulphuric acid, brine, ruler, boiling water, masking tape or rubber bands, crushed ice

Procedure

1. Push the entire length of the capillary tube into a deep beaker containing strong brine.
2. Heat the brine until it boils.
3. Remove the tube from the brine.
4. While the tube is still hot, dip the open end into a small beaker containing concentrated sulphuric acid so that a small length (about 5 mm) of the acid is drawn into the tube as it cools.

5. Leave the tube to cool further until the acid bead is drawn to a suitable position (just over halfway along the length of the tube).
6. Fix the capillary tube and the thermometer to the ruler with rubber bands or masking tape at both ends.
7. Put the set-up in a tall glass jar as shown in Figure 5. 34.

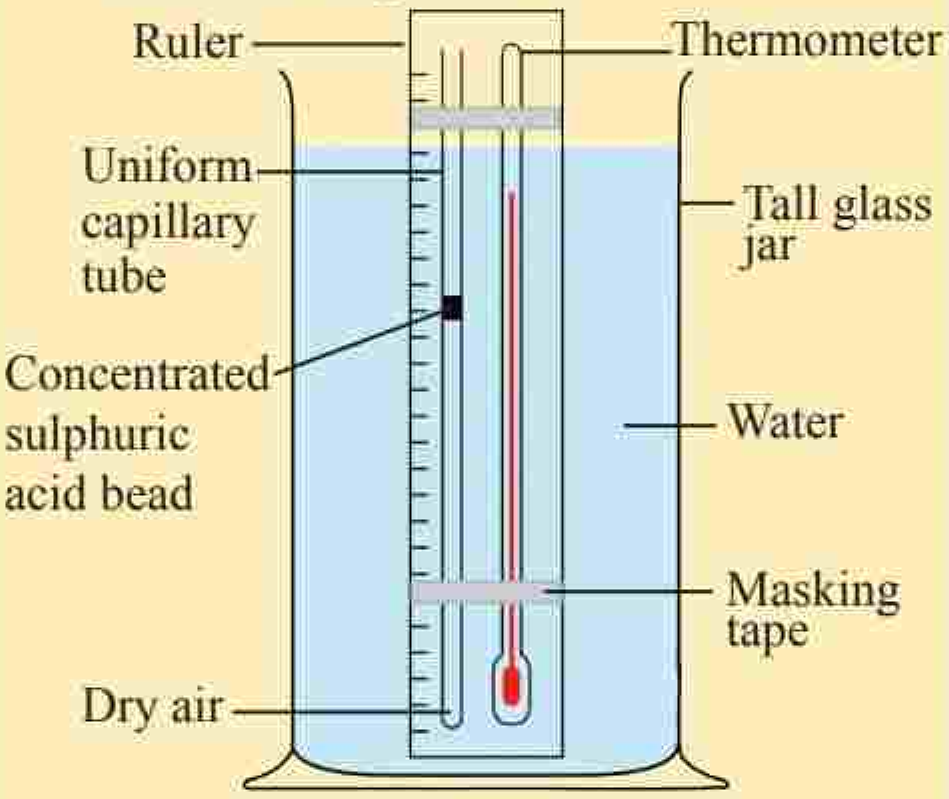


Figure 5.34

8. Put some crushed ice in the glass jar. Record the temperature reading and the corresponding length of the air column trapped by the acid.
9. Raise the temperature of the ice by adding some hot water and stir until the temperature of the water remains constant. Record the temperature reading and the corresponding length of the air column.
10. Repeat step 9 five more times to obtain more sets of readings. Record your results in a table. **Note:** Always ensure that, the trapped air column is under the water surface in the glass jar.

Questions

(a) Plot a graph of volume against temperature.

(b) From the graph, describe the relationship between the volume and the temperature of a fixed mass of gas.

Putting the capillary tube in boiling brine ensures that the tube is dry and hot. Besides trapping the air column, the acid acts as a drying agent for the air. Inside the tube, the air can expand or contract freely with temperature change. However, the pressure is constant and equal to the sum of atmospheric pressure and pressure due to the weight of the acid bead. The tube has a uniform cross section, so the volume of the air is proportional to the length. A plot of the graph of volume against temperature is a straight-line as shown in Figure 5.35. The y-intercept represents the volume of the gas at temperature of 0 °C .

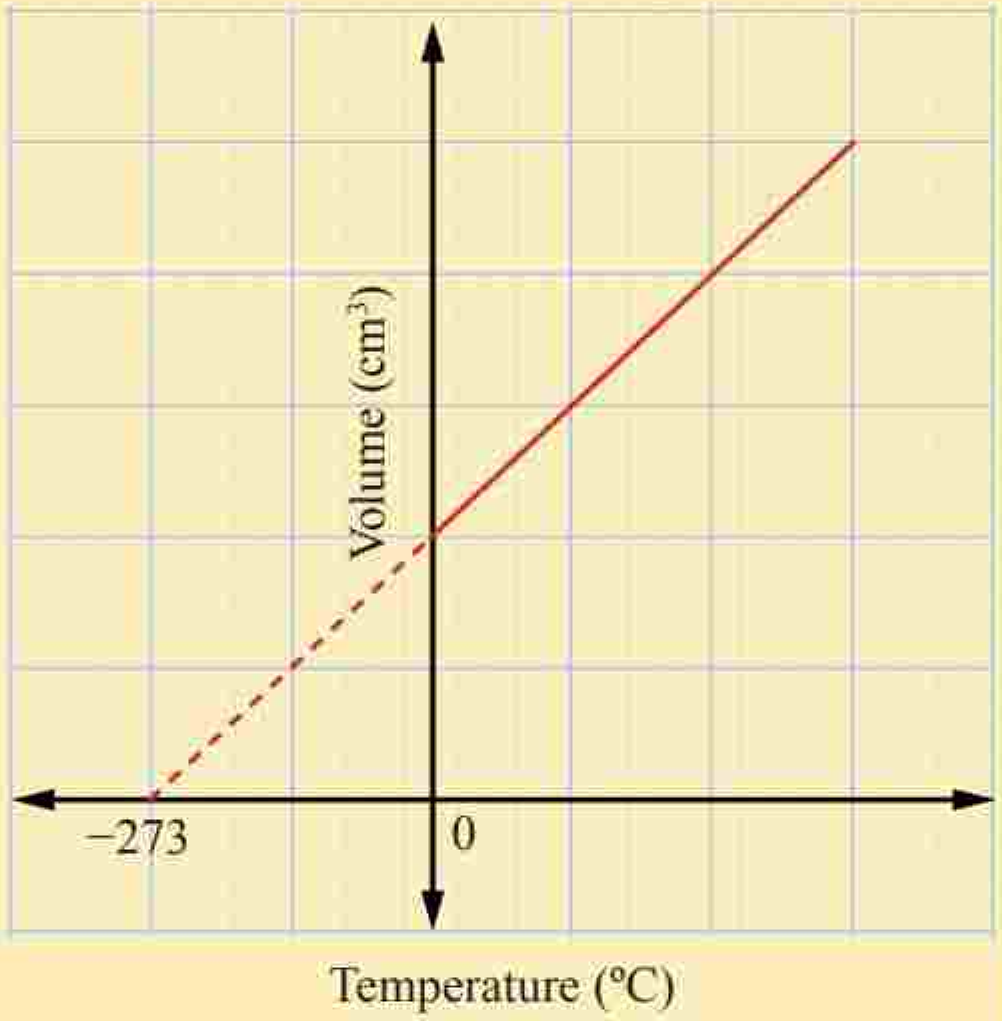


Figure 5.35: *Variation of volume of a gas with temperature*

If the line is extrapolated, it meets the horizontal axis at $-273\ °C$. This mark the temperature at which the volume of the gas is 0. At this temperature, the particles are no more in motion. Suppose the volume axis is shifted to the point $-273\ °C$, another scale is established with $-273\ °C$ corresponding to 0 K and $0\ °C$ corresponds to $273\ K$ as shown in Figure 5.36.

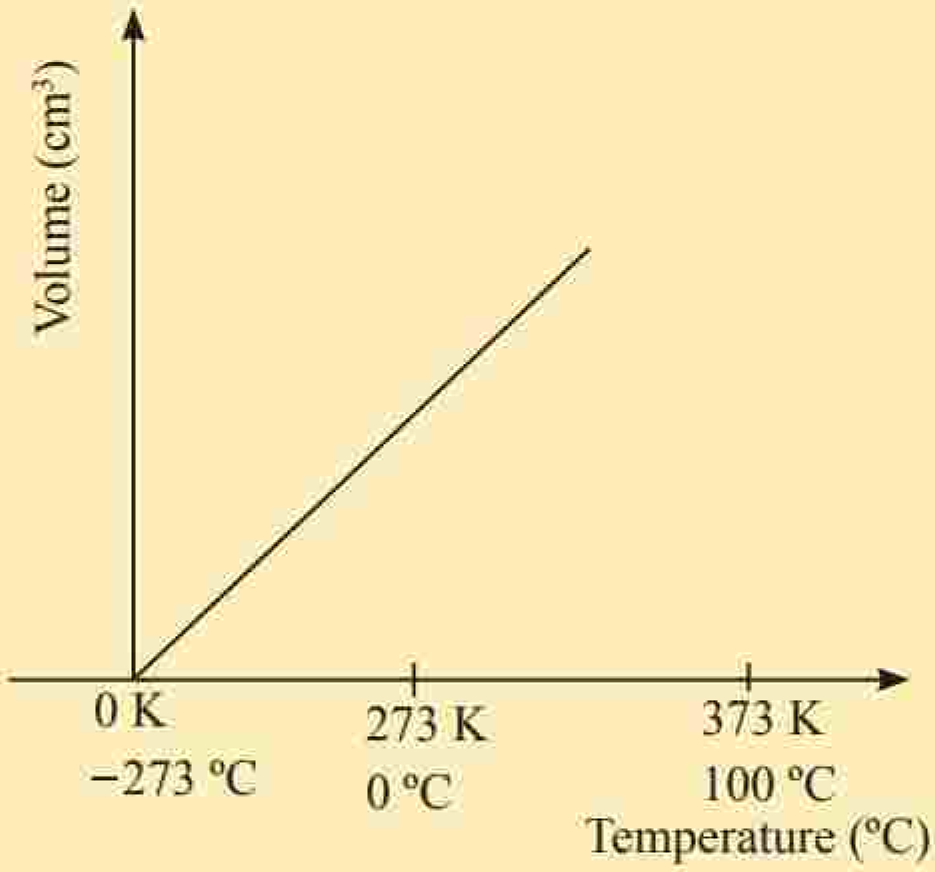


Figure 5.36: *Graph of volume against temperature*

Using the new origin in Figure 5.36, it can be seen that, volume is direct proportional to absolute temperature. That is

$$V \propto T \text{ or } V = kT$$

where k is the gradient of the graph, and it represents the constant pressure. The relationship is named as Charles' law which states that, "*The volume of a fixed mass of a gas is directly proportional to the absolute (Kelvin) temperature provided the pressure remains constant*".

If the temperature of the gas changes from T_1 to T_2, the volume also changes from V_1 to V_2. Therefore,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where V_1, T_1 and V_2, T_2 are the initial and the final values of volume and temperature, respectively.

Note that, the temperature has to be converted from Celsius scale to Kelvin scale. When temperature is expressed in kelvin, it is said to be in absolute scale of temperature. The absolute zero temperature is the lowest temperature of about 0 K or $-273\ °C$ that can be attained theoretically. Practically, it is not possible to attain this temperature because all gases liquefy before attaining it.

Example 5.7

A $0.20\ m^3$ container with a movable piston holds nitrogen gas at a temperature of 20 °C. What will be the volume of the gas if the temperature is increased to $50\ °C$?

Solution

Since the container is fitted with a movable piston, we can assume that the pressure will remain in equilibrium with the surroundings and therefore constant. We can, therefore, use Charles' law to solve the problem. The initial and final temperatures must be converted to Kelvin scale.

Given

$T_1 = 20\ °C + 273 = 293\ K$,

$T_2 = 50\ °C + 273 = 323\ K$, $V_1 = 0.20\ m^3$

From Charles' law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{V_1 T_2}{T_1}$$

$$= \frac{0.20 \text{ m}^3 \times 323 \text{ K}}{293 \text{ K}} = 0.22 \text{ m}^3$$

Example 5.8

A gas occupies a volume of 20 cm^3 at $27\ °\text{C}$ and at normal atmospheric pressure. Calculate the final temperature if the gas is heated resulting in a change of volume of 1.88 cm^3.

Solution

$V_1 = 20 \text{ cm}^3$,

$V_2 = 20 \text{ cm}^3 + 1.88 \text{ cm}^3 = 21.88 \text{ cm}^3$,

$T_1 = 300 \text{ K}$, $T_2 = ?$

From Charles' law,

$$T_2 = \frac{V_2 T_1}{V_1}$$

$$= \frac{21.88 \text{ cm}^3 \times 300 \text{ K}}{20 \text{ cm}^3}$$

$$= 328 \text{ K}$$

Therefore, the temperature is 328 K or 55 °C.

The relationship between volume and pressure at a constant temperature

The relationship between the volume of a fixed mass of a gas and its pressure when the temperature is held constant was first derived by Robert Boyle in 1660. The relationship is also known as Boyle's law.

Activity 5.11

Aim: To verify the relationship between the volume and pressure of a fixed mass of a gas at constant temperature

Materials: scale, oil, glass tube, oil reservoir, air pump, Bourdon gauge

Procedure

1. Trap some air above oil in a strong glass tube.
2. Set up the apparatus as shown in Figure 5.37.

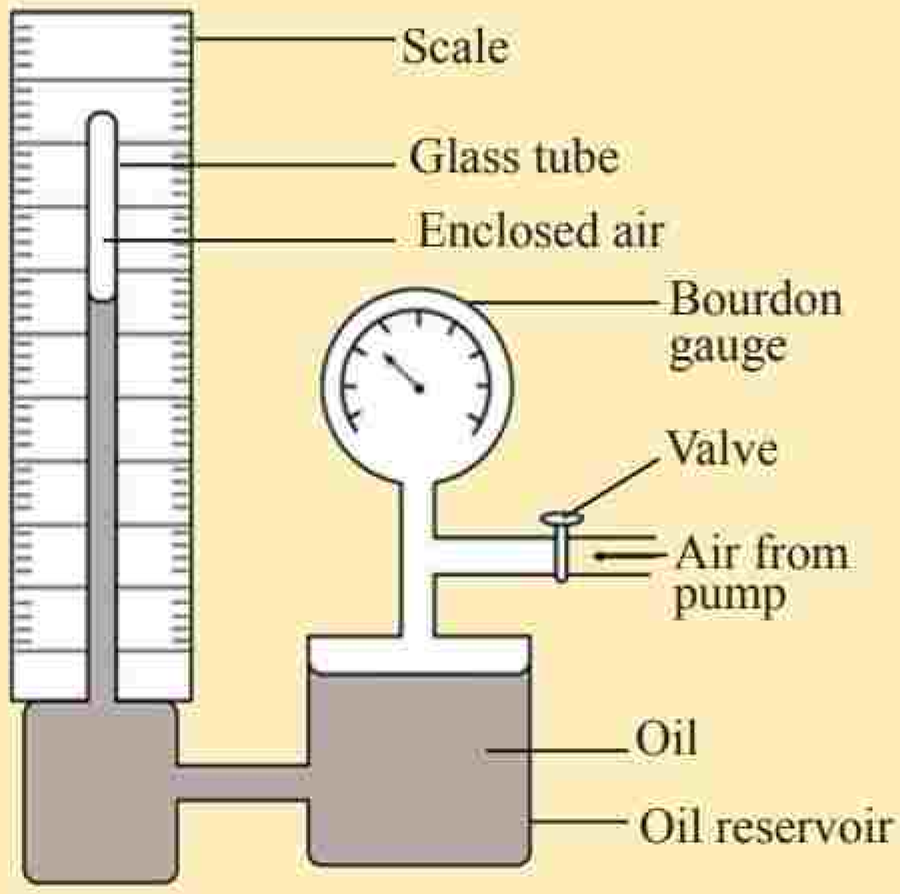


Figure 5.37

3. Increase the pressure on the trapped air by pumping more air from outside into the oil reservoir, as illustrated in Figure 5.37.
4. Read and record the volume on the scale for various values of pressure on the Bourdon gauge. The temperature of the air is

constant and is equal to the room temperature. However, for the temperature to remain constant, pressure must be varied slowly and enough time is provided before reading the volume.

5. Disconect the pump, then lower the pressure slowly in stages by opening the valve of the pump slightly.
6. Read and record the values of pressure and volume.

Questions

(a) Plot a graph of volume against pressure.

(b) Plot a graph of the reciprocal of volume (V) against pressure (P).

(c) From your graph, describe how the volume of a fixed mass of a gas varies with pressure.

Figure 5.38 shows the expected shape of the graph of volume against pressure while Figure 5.39 shows the graph of reciprocal of volume against pressure.

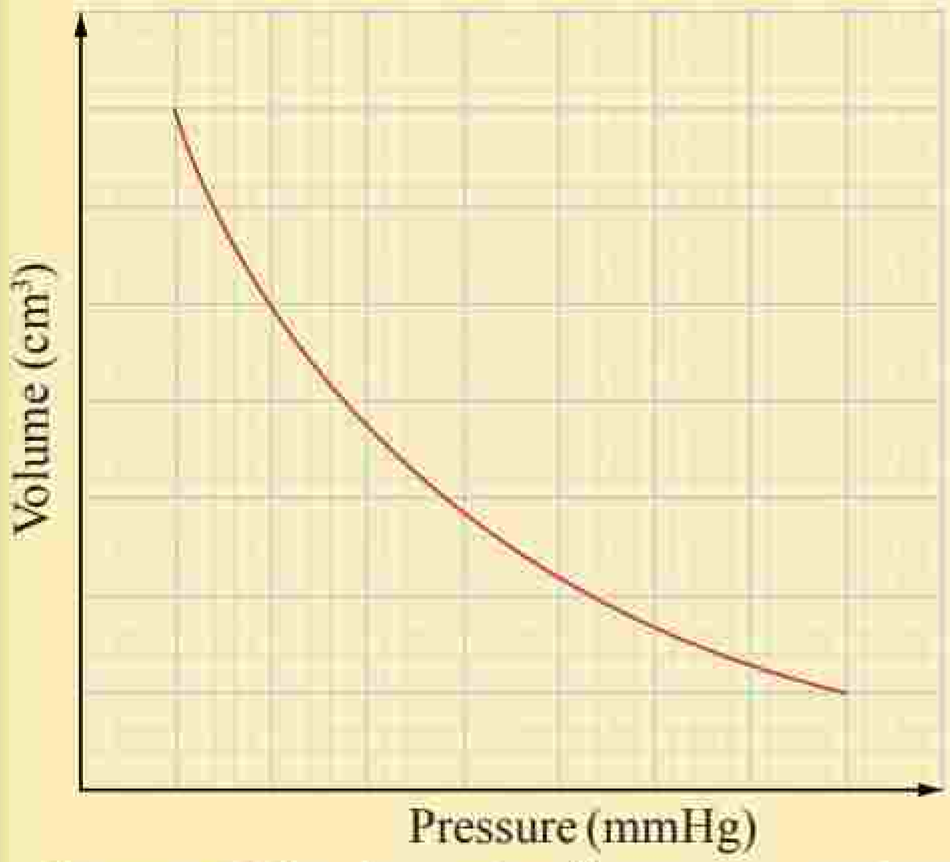


Figure 5.38: *A graph of pressure against volume*

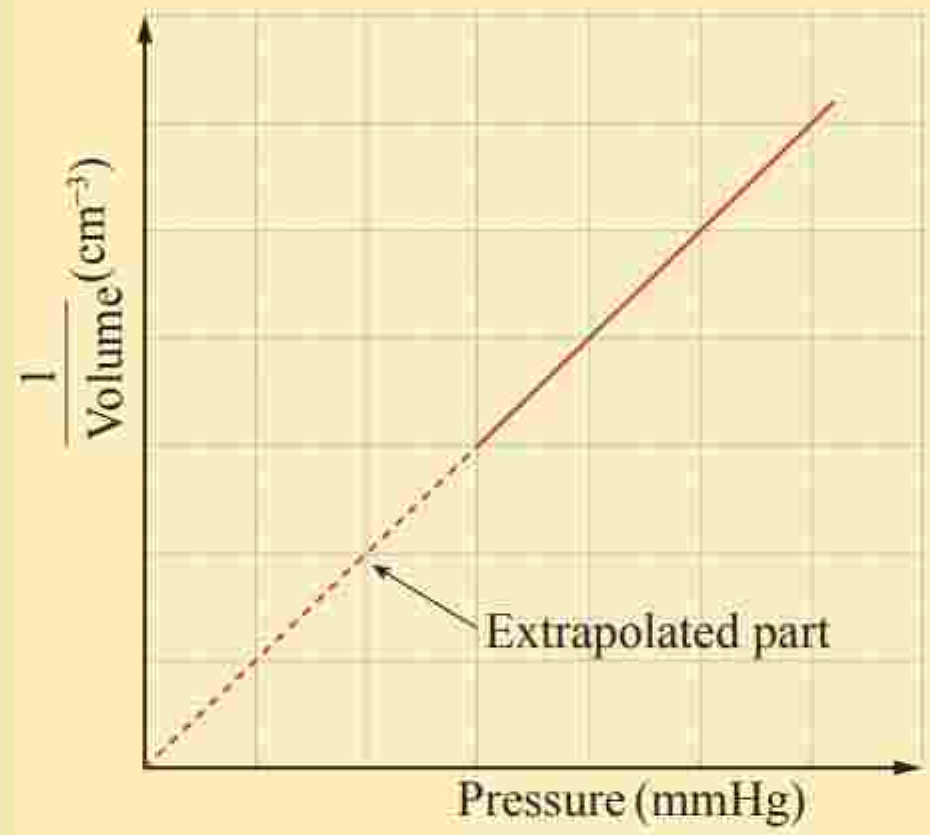


Figure 5.39: *A graph of the reciprocal of volume, $\frac{1}{V}$, against pressure, P*

The results of this experiment can be summarised using Boyle's law of gases which states that, "*The volume of fixed mass of a gas is inversely proportional to its pressure if the temperature is kept constant*". That is,

$$pressure \propto \frac{1}{volume} \text{ or}$$

$PV = constant$ (Constant temperature)

If the pressure of a gas changes from P_1 to P_2, the volume changes from V_1 to V_2. Then,

$$P_1V_1 = P_2V_2$$

where P_1, V_1 and P_2, V_2 are the initial and final values of pressure and volume, respectively.

Example 5.9

A gas in a cylinder occupies a volume of 465 ml when the pressure on it is equivalent to 725 mm of mercury. What will be the volume of the gas when the pressure on it is raised to 825 mm of mercury while the temperature is held constant?

Solution

$P_1 = 725$ mmHg, $P_2 = 825$ mmHg , $V_1 = 465$ ml

Using Boyle's law

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1V_1}{P_2}$$

$$= \frac{725 \text{ mmHg} \times 465 \text{ ml}}{825 \text{ mmHg}} = 408.6 \text{ ml}$$

Therefore, the volume of the gas will be 408.6 ml.

Example 5.10

An empty barometer tube, 1 m long, is lowered vertically downwards, into a tank of water. What will be the depth at the top of the tube when the water has risen 20 cm inside the tube? (Atmospheric pressure may be assumed equal to 10. 4 m column of water).

Solution

Let h = depth in m of water-level in tube below surface, then,

$P_1 = 10.4$ m of water

$P_2 = (10.4 + h)$ m of water

$V_2 = (0.8 \times A)\,\text{m}^3$, where A is the cross-sectional area of the tube

$V_1 = (1 \times A)\,\text{m}^3$

Using Boyle's law,

$$P_1V_1 = P_2V_2$$

$$10.4 \times A = (10.4 + h) \times 0.8A$$

$$h = 2.6 \text{ m}$$

Therefore, the depth at the top of tube is $2.6 \text{ m} - 0.8 \text{ m} = 1.8 \text{ m}$.

The relationship between pressure and temperature at constant volume

To investigate the relationship between the pressure and the temperature of a fixed mass of a gas, when the volume of the gas is kept constant. The pressure is then measured as the temperature is varied. The relationship obtained when the volume of a gas is held constant while the pressure and temperature are varied is known as Pressure law.

Activity 5.12

Aim: To verify the relationship between the pressure and the temperature of a fixed mass of a gas at constant volume

Materials: source of heat, waterbath, Bourdon gauge, thermometer, round-bottomed flask, water, rubber tubing, rubber stopper, ice, stopwatch, glass tube

Procedure

1. Collect some dry air in a flask then close the flask tightly with a rubber stopper with two holes.

2. Insert a thermometer through one of the holes so that it fits tightly. Insert a tightfitting glass tube through the other hole.
3. Using a short, tight-fitting rubber tubing, connect the glass tube to the Bourdon gauge.
4. Place the flask in a waterbath and hold it in such a way that it does not touch the sides or the bottom of the container, as shown in Figure 5.40.

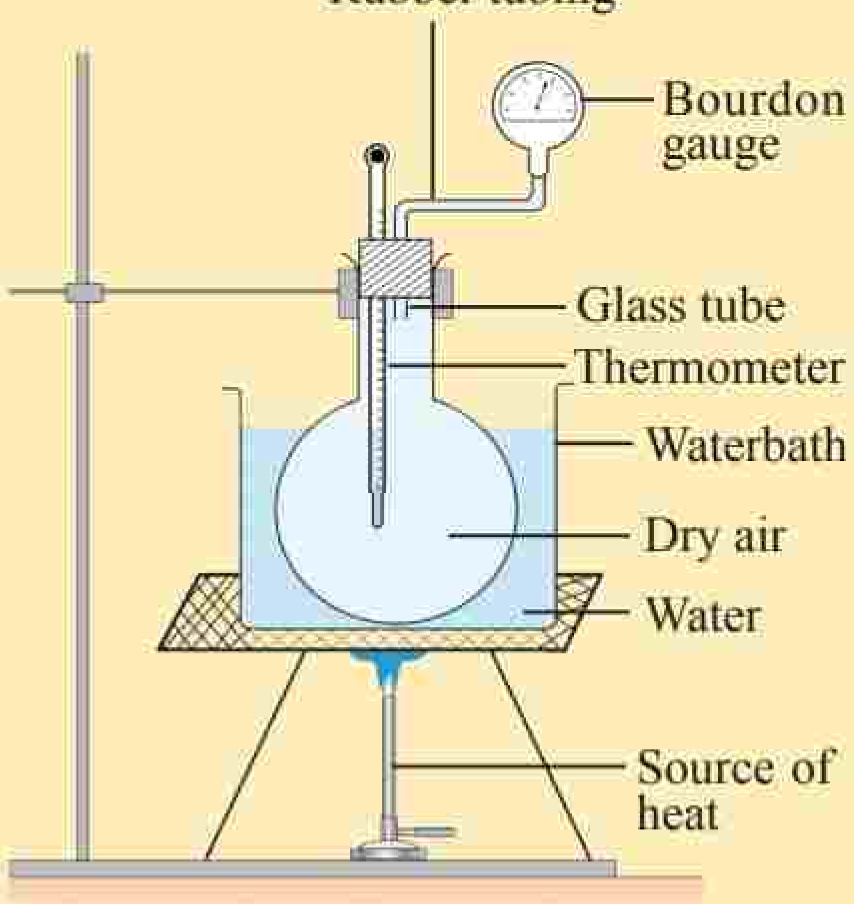


Figure 5. 40

5. Lower the temperature of the water in the waterbath by adding ice in stages while stirring.
6. Record the pressure of the air in the flask at regular temperature intervals (5°C).
7. Increase the temperature of the air in the flask in stages by heating the water and stirring it.
8. Measure and record the pressure of the gas using the Bourdon gauge at intervals of 5°C until the water boils.

Questions

(a) From your results, plot a graph of pressure against temperature.

(b) Explain the relationship between the pressure and the temperature of a fixed mass of a gas at constant volume.

The expected shape of the graph of pressure against temperature is a straight line as shown in Figure 5.41. The gradient of a straight line is constant. This means that, at constant volume;

$P \propto T$, $P = kT$, thus, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$.

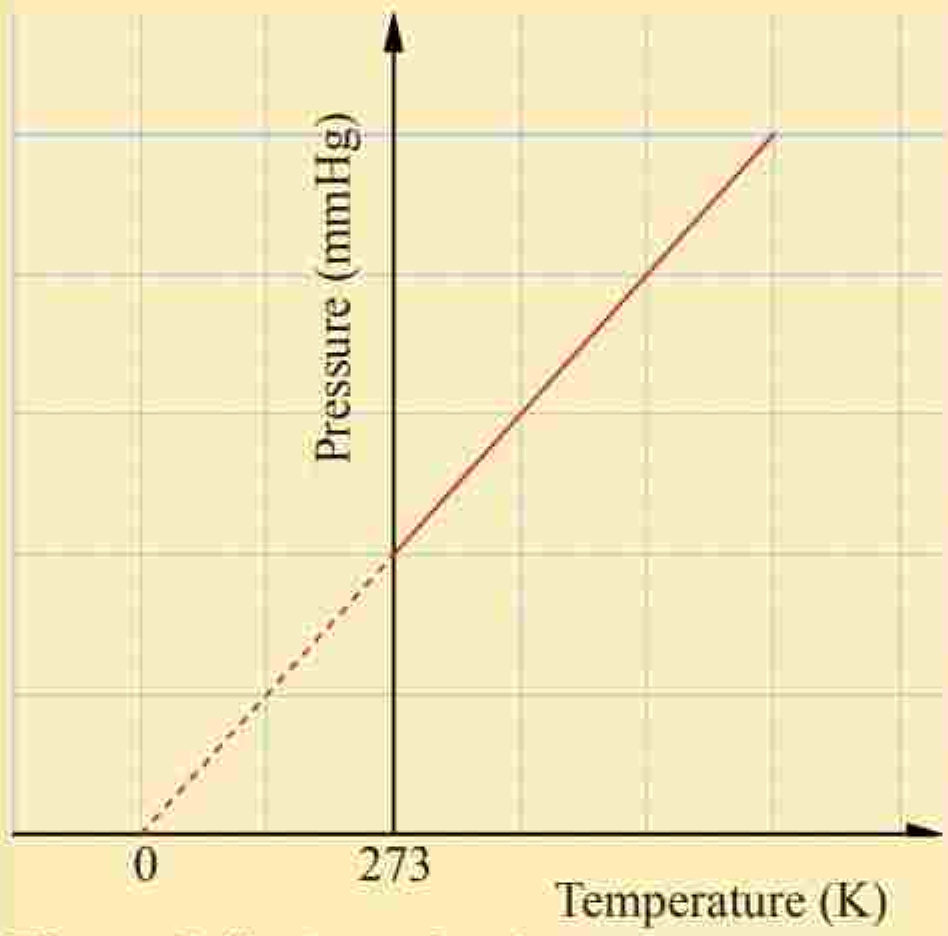


Figure 5.41: *A graph of pressure against temperature*

This relationship gives a statement of the Pressure law, which states that, "*The pressure of a fixed mass of a gas is directly proportional to the absolute temperature if the volume is kept constant*".

When a gas is heated, the molecules gain kinetic energy and move about fast. The number of impacts per second for each molecule increases since the speed of the molecules increases. If the pressure of the gas in this way increases, the volume is held constant.

Example 5.11

A rigid metal container holds carbon dioxide gas at a pressure of 2×10^5 Pa and a temperature of $30\ °C$. To what temperature must the gas be lowered for the pressure to reduce to 1×10^5 Pa?

Solution

Since the gas is held in a rigid container, we can assume that the volume is constant.

Given

$T_1 = 30\ °C + 273 = 303\ K$,

$P_1 = 2 \times 10^5\ Pa$, $P_2 = 1 \times 10^5\ Pa$

Using Pressure law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = \frac{T_1 P_2}{P_1}$$

$$= \frac{303\ K \times 1 \times 10^5\ Pa}{2 \times 10^5\ Pa} = 151.5\ K$$

The temperature that the gas must be lowered to is 151.5 K or $-121.5\ °C$.

The general gas equation from the gas laws

Changes in pressure, temperature and volume of a fixed mass of a gas can take place simultaneously. In this situation, the three gas laws (Boyle's law, Charles' law and the Pressure law) are combined to form one gas equation. Consider a fixed mass of a gas under the three gas laws: Charles' law, $\frac{V}{T} = constant$; Boyle's law, $PV = constant$; and Pressure law, $\frac{P}{T} = constant$. Combining any two of the three equations you get,

$$\frac{PV}{T} = constant$$

If the pressure, volume and temperature of the gas change then,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Therefore, $\frac{PV}{T} = constant$ is a general gas equation that includes all three gas laws.

Standard temperature and pressure

The temperature and pressure, which are important variables for the expansion of gases, vary from place to place. In order to compare the behaviours of different set of gases, the pressure and temperature are standardized. There is a standard reference point for the physical properties of gases, such as temperature and pressure. These standard conditions are: temperature ($0\ °C$ or $273.15\ K$) and pressure (1 atmosphere = 760 mmHg $= 10^5$ Pa). At this standard temperature and pressure (STP), the volume occupied by 1 mole of any gas is 22.4 litres. An equal volume of gases at the same temperature and pressure contains an equal number of molecules.

Example 5.12

A sample of oxygen gas has a volume of $0.11\ m^3$ at a temperature of $12\ °C$ and a pressure of $8.1 \times 10^4\ Pa$ while a sample of nitrogen gas has a volume of $0.18\ m^3$ at a temperature of $22\ °C$ and a pressure of $1.03 \times 10^5\ Pa$. Which gas will have the larger volume at standard temperature and pressure (STP)?

Solution

For oxygen:

$T_1 = 12\ °C + 273 = 285\ K$, $T_2 = 273\ K$,

$P_1 = 8.1 \times 10^4\ Pa$, $P_2 = 1.013 \times 10^5\ Pa$,

$V_1 = 0.11\ m^3$

At STP,

$T = 273\ K$, $P = 1.013 \times 10^5 Pa$,

Using general gas equation,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$= \frac{8.1 \times 10^4\ Pa \times 0.11\ m^3 \times 273\ K}{285\ K \times 1.013 \times 10^5\ Pa}$$

$$= 0.084\ m^3$$

For nitrogen:

$T_1 = 22\ °C + 273 = 295\ K$

$T_2 = 273\ K$

$P_1 = 1.03 \times 10^5\ Pa$, $P_2 = 1.013 \times 10^5\ Pa$,

$V_1 = 0.18\ m^3$, $V_2 = ?$

Using general gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$= \frac{1.03 \times 10^5\ Pa \times 0.18\ m^3 \times 273\ K}{1.013 \times 10^5\ Pa \times 295\ K}$$

$$= 0.17\ m^3$$

At standard temperature and pressure (STP), the nitrogen gas would have a volume that is more than twice the volume of oxygen gas.

Applications of the expansion of gases in daily life

The following are some applications of the expansion of gases in daily life:

1. Land and sea breezes

Land and sea breezes are a result of the expansion of air caused by unequal heating and cooling of adjacent land and sea surfaces. The detail will be presented in chapter 6.

2. The piston engine

The internal combustion engines used in vehicles have four basic parts: a carburetor (fuel injector), pistons, cylinders and spark plugs as shown in Figure 5.42. Fuel mixes with air in the carburetor or is sprayed through the fuel injector. The mixture goes into a cylinder, which is a long air pocket like a steel can with one end open. The mixture is ignited by a spark from the spark plug in a spark-ignition engine, releasing heat. This heat increases the pressure inside the cylinder. This moves the piston, a metal plug that fits easily into the cylinder but is loose enough to slide up and down.

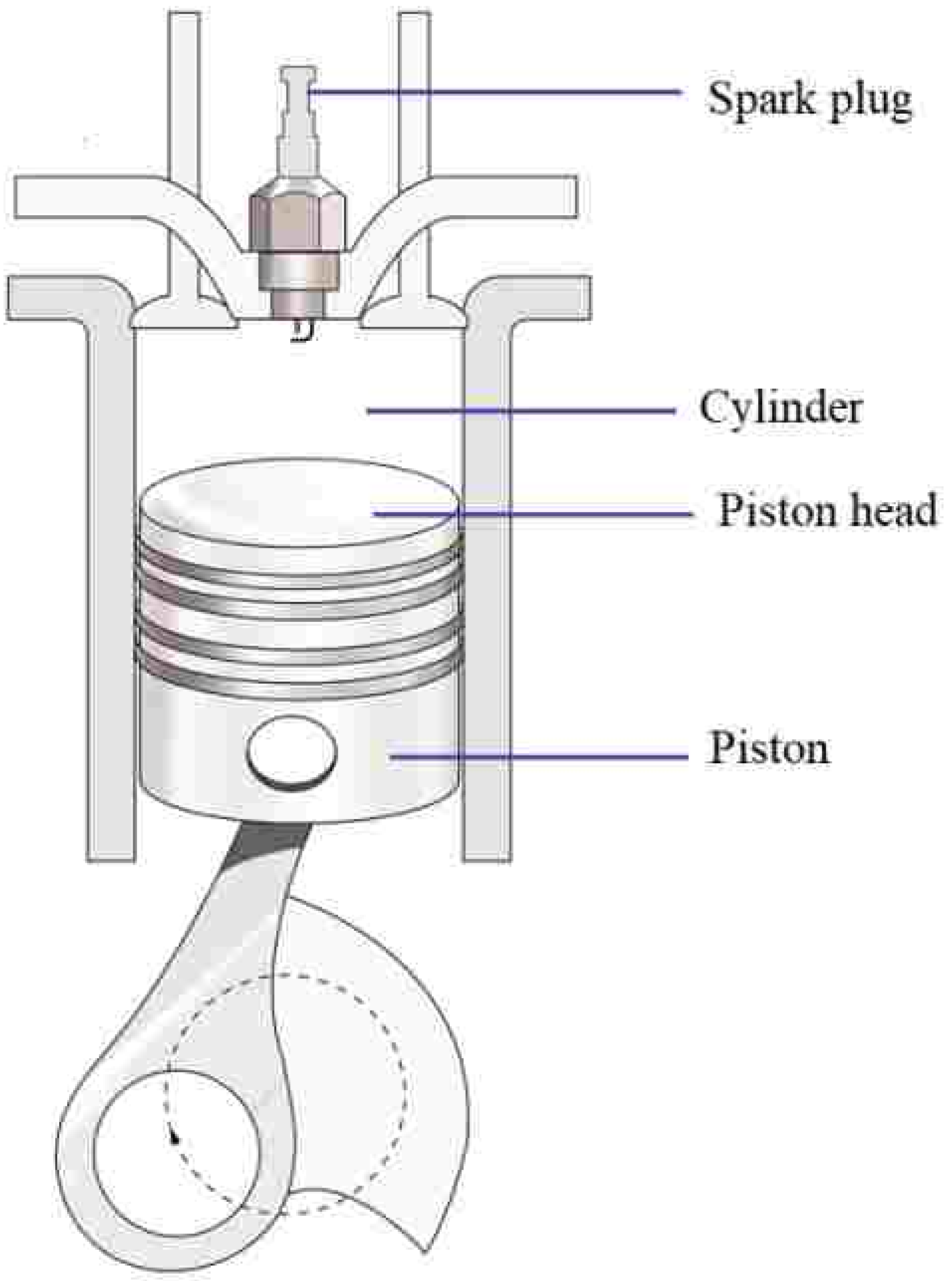


Figure 5.42: *A cylinder and a piston*

When a piston in an engine is pushed up into its cylinder, the upward movement traps the mixture of air and fuel above it and compresses it to a fraction of its original volume. This brings the molecules closer together, so they hit the piston head far more often. Thus, the pressure goes up. When the mixture is ignited, a controlled explosion occurs that makes the molecules move even faster and thus they bombard the piston head harder. This exerts very high pressure on the piston.

As the piston is pushed down in the cylinder, other engine parts keep it from blowing out of the cylinder. The piston is then pushed back up into the cylinder. The downward movement of the piston pushes a rod that turns a crankshaft. The turning crankshaft provides the motion to turn the wheels.

3. Firing bullets from guns

The mechanism of expanding gases is also used in firing bullets from guns. The expansion takes place in the cartridge. A cartridge or round of ammunition is made up of several components as shown in Figure 5.43.

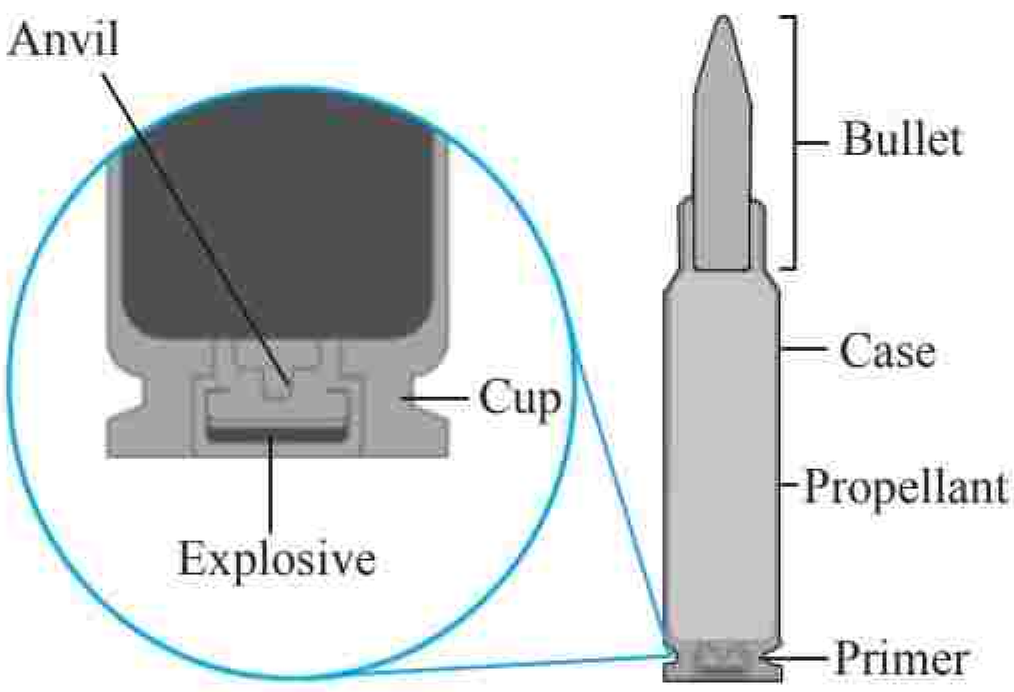


Figure 5.43: *A cartridge*

The case holds the primer which is the explosive portion of a round of ammunition. Propellants can vary from black gunpowder to a more modern smokeless powder that contains nitrocellulose. Propellants are carefully formulated to ignite and create an expanding gas that accelerates the bullet down the barrel.

When the trigger is pulled, the firing pin or striker hits the primer resulting in a minor explosion. The flame from this explosion then ignites the powder, which is held within the main compartment of the case. The gun powder does not explode, instead, it burns very rapidly releasing a lot of heat in the process. The rapid combustion of the powder causes a rapid expansion of the gases released, resulting in increased pressure within the casing. The increase in pressure causes the case to be forced outward against the walls of the chamber. With increasing pressure, the bullet is eventually forced out of the casing and starts to travel down the barrel of the gun.

Task 5.3

Brainstorm with your classmates on other applications of the expansion of gases. Present your applications in class.

Exercise 5.4

1. How does the kinetic theory of matter account for the pressure of gas?
2. What will be the value of a temperature of 70 °C on the absolute scale?
3. A bicycle tyre is pumped to a pressure of 2.2×10^5 Pa at 23 °C. After a certain time, the pressure is found to be 2.6×10^5 Pa. Assuming the volume of the tyre did not change, what is the temperature of the air in the tyre?
4. A gas occupies a volume of 4 m^3 at 37 °C and normal atmospheric pressure. What will the volume of the gas be if the pressure is held constant and the temperature is changed to,

   (a) 67 °C? (b) −50 °C?
5. In the manufacture of oxygen gas, each cubic metre of gas produced at atmospheric pressure and a temperature of −73 °C is compressed into cylinders of volume 0.05 m^3 and stored at 17 °C. What is the pressure of the gas in the cylinders in atmospheres?
6. A car tyre is inflated to a pressure of 2×10^5 Nm^{-2} at a temperature of 17 °C. During a journey, the tyre heats up to 47 °C. Assuming the tyre did not expand, what is the new pressure in the tyre?
7. Hydrogen gas occupying a volume of 1.6 m^3 at a pressure of 1.0×10^5 Pa is compressed to a volume of 0.8 m^3. If the temperature does not change, what is the final pressure of the gas?
8. The pressure in a metal gas cylinder at 20 ºC is 2 atmospheres. At what temperature will the pressure be doubled?

Chapter summary

1. Thermal energy is the energy possessed by a body due to its temperature.
2. Heat is a form of energy transferred from one body to another due to difference in temperature between the two bodies.
3. Temperature is the degree of coldness or hotness of a body. It is a measure of the average kinetic energy of the particles in matter.
4. The sources of thermal energy include the Sun, fuels, nuclear reactors and geothermal.

5. Thermal expansion is the increase of size of matter in response to an increase in temperature.
6. Matter in all of its states (solids, liquids and gases) expands when heated.
7. Linear expansivity of a substance is the increase in length per unit length of the substance when its temperature rises by 1 °C or 1 K.
8. The bimetallic strip consists of two bonded different metals that expand at different rates when heated through the same temperature change.
9. Expansion of solids is applied in the design of bridges, railway lines, concrete roads, hot-water pipes, pendulum clocks and the installation of overhead cables.
10. The apparent expansion of a liquid is given by the actual expansion of the liquid without considering the increase in volume of the containing vessel.
11. Volume expansivity is the fractional change in volume per unit temperature change.
12. The anomalous expansion of water is the unusual behaviour of water to expand instead of contracting when temperature changes from 4 °C to 0 °C, and the density decreases.
13. The anomalous expansion of water causes ice to float on water in cold regions. This allows water creatures to survive under the frozen surface water.
14. The anomalous expansion of water also has some undesirable effects. These include cracking of glass bottles filled with water and bursting of water pipes at freezing point. Floating icebergs also cause accidents on the seas.
15. The principle of expansion of liquids is used in liquid thermometers.
16. In all situations describing the behaviour of gases, temperature must be measured in kelvin.
17. Charles' law of ideal gases states that, the volume of a fixed mass of a gas is proportional to the temperature provided the pressure remains constant.
18. Boyle's law of gases states that, the volume of a fixed mass of a gas is inversely proportional to its pressure if the temperature is kept constant.
19. Pressure law states that, the pressure of a fixed mass of a gas is directly proportional to the absolute temperature if the volume is kept constant.
20. The standard temperature and pressure (STP) is a set condition for experimental measurements to enable comparisons between data sets.

Revision exercise 5

Choose the most correct answer in question 1 and 2.

1. You are provided with three metal rods X, Y and Z of equal length L, at the same temperature. Their coefficients of linear expansion are α_X, α_Y and α_Z such that $\alpha_X < \alpha_Y < \alpha_Z$. If the temperature of all rods increases by $\theta\,^{\circ}\mathrm{C}$, how will the final order of length of rods be?

   (a) $L_X > L_Y > L_Z$ (b) $L_Y > L_X > L_Z$

   (c) $L_Y > L_Z > L_X$ (d) $L_Z > L_Y > L_X$

2. Table 5.4 shows the pressure, volume and temperature of a fixed mass of gas before and after its pressure is increased.

   Table 5.4

Pressure	Volume	Temperature
P	V_1	T_1
$2P$	$\frac{3}{4}V_1$	T_2

   The value of T_2 in terms of T_1 is:

   (a) $\frac{2}{3}T_1$ (b) $\frac{8}{3}T_1$

   (c) $\frac{3}{2}T_1$ (d) $\frac{3}{8}T_1$

3. (a) What is thermal expansivity of a material?

   (b) How are linear, areal and volume expansivities of solid related?

   (c) Two metals have different linear expansivities. Describe the practical use of such metal combinations.

4. Explain each of the following observations:

   (a) A lid on a metal container can be unscrewed easily if the can is immersed in hot water for a few minutes.

   (b) Corrugated iron-sheet roofs make cracking noises on a cold night after a hot day.

   (c) It is difficult to unscrew wheel nuts in the morning, while it is relatively easy to unscrew them on a hot day.

5. Figure 5.44 shows a brass-invar bimetallic strip at room temperature.

Figure 5.44

   Given that brass expands more than invar when both are heated equally, sketch the appearance of the strip after being cooled to several degrees below room temperature.

6. A glass test tube was heated over a bunsen burner flame. Cold water was then quickly poured into the test tube. Explain why the test tube would break.

7. A rally car tyre is at an air pressure of 3×10^5 Pa and a temperature of 27 °C at start of the rally. The temperature rises to 57 °C when the car is racing. Assuming the tyre does not expand, what is the new pressure in the tyre?

8. (a) Define the term 'linear expansivity' of a solid.

   (b) The original length of a metal bar is 101.5 cm at 15 °C. Determine the linear expansivity of the metal if the bar increases in length by 1.40 mm when the temperature is raised to 100 °C.

9. A metal rod 80 cm long increased in length by 0.09 cm when the temperature was raised by 93.6 °C. Determine the linear expansivity of the metal.

10. A compound strip of brass and iron, 10 cm long at 20 °C, is held horizontally with iron uppermost. When heated from below with a Bunsen burner, the temperature of the brass is 820 °C and that of the iron is 770 °C. Calculate the difference in lengths of the iron and brass (Coefficient of linear expansion of brass = 0.000019/°C and Coefficient of linear expansion of iron = 0.000012/°C).

11. An iron tyre of diameter 50 cm at 15 ºC is to be shrunk on to a wheel of diameter 50.35 cm. To what temperature must the tyre be heated so that it will slip over the wheel with radial gap of 0.5 mm? (Linear expansivity of iron is 0.000012/K).

12. The difference in length between a brass and an iron rod is 14 cm at 10 °C. What must be the length of the iron for this difference to remain 14 cm, when both rods are heated to 100 °C? (Linear expansivity of brass $= 19 \times 10^{-6}$ / K and for iron $= 12 \times 10^{-6}$ / K).

13. A steel tape of correct length at 15 °C is used to measure a distance on a day when the temperature is 10 °C. Is the result too large or too small? If the linear expansivity of steel is 12×10^{-6} / K, what is the error in measuring a length of 20 m?

14. A brick at 20 °C has a dimension of 30 cm, 18 cm and 10 cm for length, width and height respectively. If the brick is heated to a new temperature of 150 °C, calculate the new dimensions. (Linear expansivity of brick is 11×10^{-6} K^{-1}).

15. An iron rod is 100 cm long at 0 °C. What must be the length of an aluminium rod at 0 °C if the differences between the lengths of the two rods remain the same at all temperatures? (Linear expansivities of iron and aluminium are 1.2×10^{-5} K^{-1} and 2.4×10^{-5} K^{-1}, respectively).

16. Figure 5.45 shows a circuit diagram for controlling the temperature of a room.

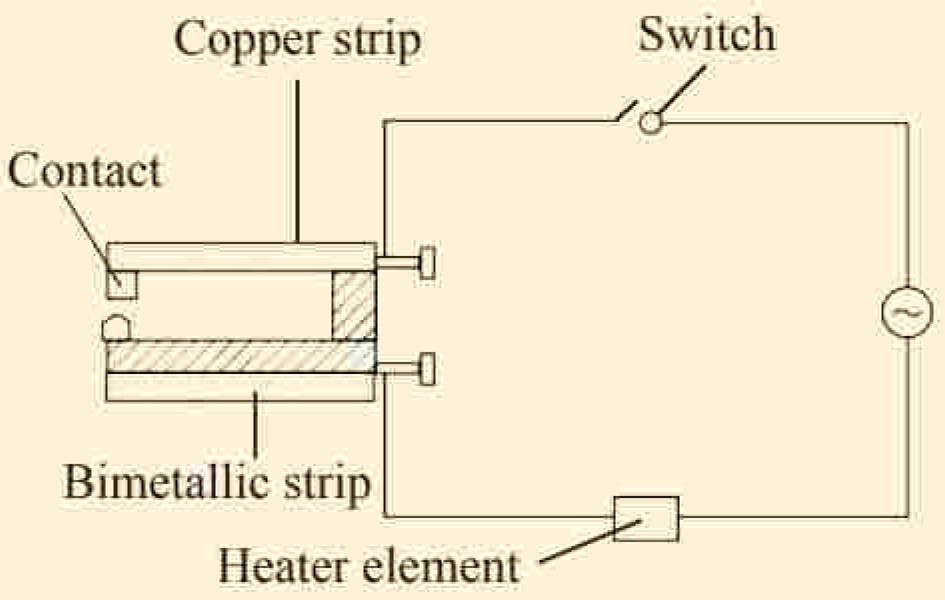


Figure 5.45

(a) Explain the function of the bimetallic strip.

(b) Describe how the circuit controls the temperature when the switch is closed.

17. The temperature of the air in contact with a thin layer of ice formed on the surface of a pond is just below 0 °C as shown in the Figure 5.46.

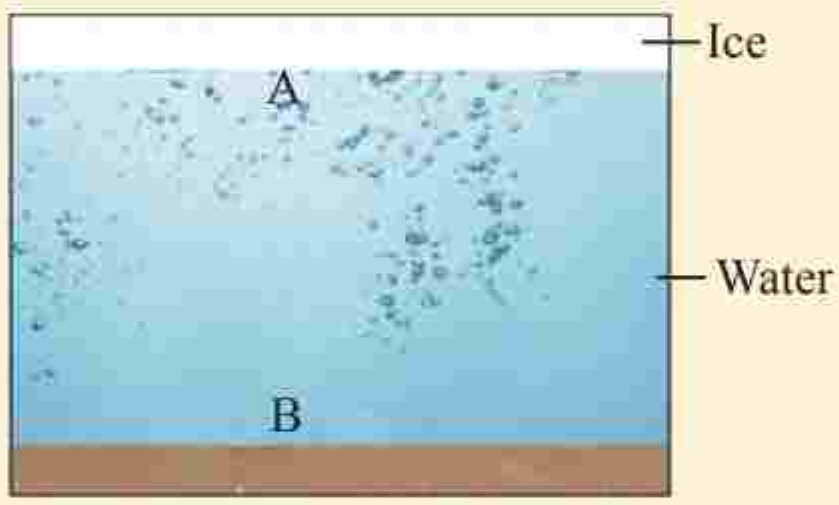


Figure 5.46

(a) State the probable temperatures of the position A and B.

(b) What fact concerning the density of water do these temperatures indicate?

(c) Why will it take a long time for the pond to freeze even if the air temperature falls below 0 °C ?

18. The pressure of 3 m^3 of a gas at 27 °C is 3 atmospheres. What will be the pressure of the gas if it is compressed into half the volume and heated to 102 °C ?

19. The air in a bicycle tyre occupies a volume of 1000 cm^3 when it is at a pressure of 2.5 atmospheres. The air is released to the atmosphere.

(a) Assuming that the temperature of the gas does not change, what volume does it occupy in the atmosphere?

(b) A pump with a volume of 150 cm^3 per stroke is used to inflate the tyre. What is the pressure in the tyre after two strokes?

20. A gas occupies a volume of 2 m^3 when its pressure is 1140 mmHg at a temperature of 27 °C. What volume would it occupy at standard temperature and pressure?

21. A container holds a gas at 0 °C . To what temperature must the container be heated for the pressure of the gas to double? (Assume that the volume of the container does not change).

22. A balloon is filled with air to a volume of 200 ml at a temperature of 20 °C. The balloon is then dipped in water at 80 °C. Assuming no leakage occurs and

ignoring the pressure change due to the water, calculate the new volume of the air.

23. Using the kinetic theory of gases, explain how a rise in the temperature of a gas causes a rise in the pressure of the gas when the volume is held constant.

24. The coefficient of volume expansion of a liquid is $5 \times 10^{-4}\,\text{K}^{-1}$. If the temperature of the liquid is increased by 30 °C, find the percentage change in its density.

25. Helium gas at a temperature of –30 °C is held in a rigid metal container at a pressure of 1.5×10^5 Pa. The container is heated to a temperature of 25 °C. What is the new pressure of the gas?

26. An air bubble at the bottom of a lake 90 m deep has a volume of 1.5 cm^3. What will be its volume just below the surface if the atmospheric pressure is equivalent to10 m of water?

27. A cylinder closed at both ends has an inner diameter of 0.021 m. The cylinder is fitted with a movable piston of a mass 2 kg. The space between the piston and the bottom of the cylinder contains $1.11 \times 10^{-4}\ \text{m}^3$ of air at 25 °C while the space above the piston has been evacuated, as illustrated in Figure 5.47.

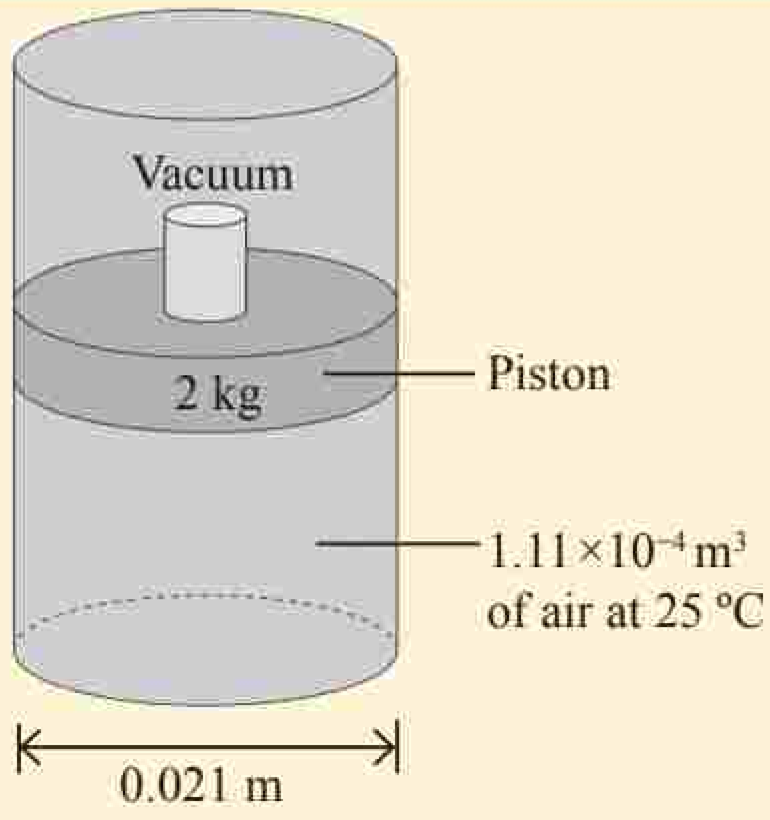


Figure 5.47

(a) Determine the pressure of the air in the cylinder given that the pressure comes from the weight of the cylinder.

(b) The cylinder is placed over a source of heat causing the air to expand and push the piston upward a distance of 3.5 cm as shown in the Figure 5.48.

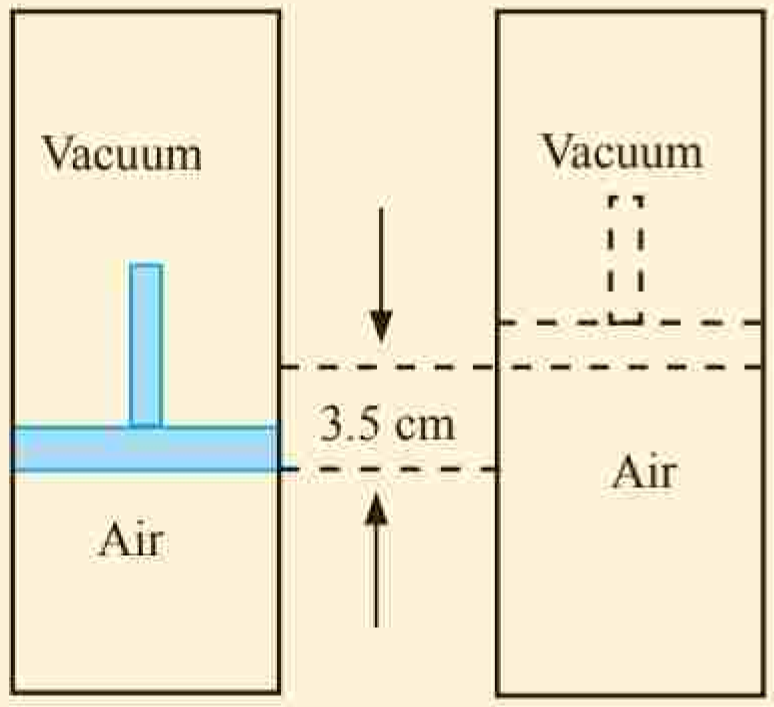


Figure 5.48

Assuming that the pressure of the air remained constant, what was the change in volume of the air? (Volume of a cylinder $= \pi r^2 h$).

Chapter Six

Transfer of thermal energy

Introduction

Heat is a form of energy that cannot be seen but its effects can be felt. Humans get heat from different sources, including the sun, electricity, fire, and gas. On hot days people wear light clothes to allow their bodies to cool. On cold days, they wear heavy clothes or stay indoors to keep themselves warm. In this chapter, you will learn about heat transfer by conduction, convection and radiation. You will identify good and bad conductors of heat, ways of minimizing heat loss and apply the knowledge of heat transfer in daily life. The competencies developed from this chapter will enable you to apply heat transfer concepts in daily life situations, including the use of appropriate cooking tools to minimize energy losses, heat reflectors to maintain temperature and heat absorbers for proper heating of homes.

Methods of transfer of thermal energy

Energy can be transferred by interactions of a system with its surroundings. The form of energy that is transferred by such interactions is called heat. Heat transfer is the transfer of thermal energy due to temperature differences. Whenever a temperature difference exists in a medium or between media, heat transfer must occur. Heat transfer mechanisms can be classified into different types; conduction, convection and radiation, depending on the medium. When a temperature difference (or gradient) exists in a solid object, the transfer mechanism is conduction. Convection refers to heat transfer by the movement of molecules within the fluid under the influence of gravity. In the absence of a medium between two bodies at different temperatures, heat is transferred by radiation.

Heat transfer by conduction

Atoms in a solid material at a given temperature vibrate about their mean equilibrium positions. The higher the temperature the higher the vibration, and hence the higher the thermal energy in form of vibrational kinetic energy.

When a hot object is brought into direct contact with a cold solid object, neighbouring atoms in contact interact. The more energetic atoms of the warm object pass energy to the less energetic atoms of the cold object by collision. As a result, atoms of the cold object begin to vibrate vigorously. These vibrations cause atoms to collide with the nearby atoms, causing electrons in such atoms to vibrate more vigorously. Such interaction is transmitted throughout the cold object, resulting in energy transfer from a warmer end to the relatively cooler end. In the presence of a temperature gradient, energy transfer by conduction occurs in the direction of decreasing temperature. After sufficient time, the entire object will attain a uniform temperature.

Metals are better conductors of heat than insulators because metals have more free electrons which move through the metal easily. Examples of metals include aluminium, copper, silver, iron and steel. In all these metals, free electrons gain the kinetic energy from collisions with hot atoms and pass on the energy when they collide with cold atoms.

Different metals conduct heat with different capacities, for example copper is a better conductor than aluminium and aluminium is a better conductor than iron.

Activity 6.1

Aim: To investigate the transfer of heat through metal solids

Materials: candle, matchbox, metal bar (copper or aluminium), wax, stopwatch

Procedure

1. Light a candle.
2. Hold one end of a metal bar and heat the other end over the lighting candle, as shown in Figure 6.1. Note your observation.

Figure 6.1

3. Remove the candle and let the metal bar cool down to room temperature.
4. Place three waxes A, B and C having the same dimensions at an interval of 5 cm starting from the centre of the metal bar, as shown in Figure 6.2.

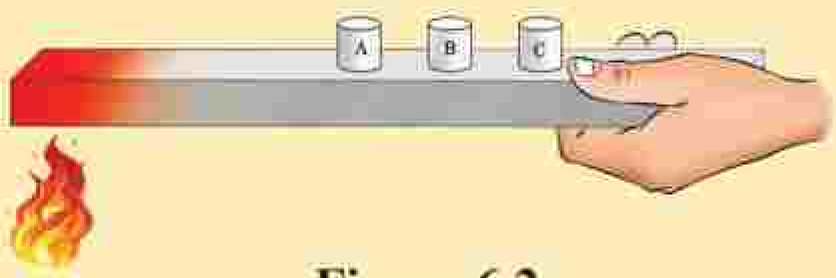


Figure 6.2

5. Heat one end of the bar with wax on it, and immediately start a stopwatch.
6. Record the time taken by wax A, B and C to start melting.

Questions

(a) What do you feel when holding a metal bar in step 2?

(b) What happens to the wax? Explain.

(c) Which part of the rod is hotter than the other? Explain.

Heat from the hot end is transferred to the cold end of the metal bar. Observing the wax, the distance from the heat source affected the melting in the order of A, B and C. This indicates that heat is being transferred from the hot end to the cold end. When you cook food using a stove, heat from the stove is transferred through bottom of the pot to the food in the same manner.

Good and bad conductors of heat

Do all substances conduct heat efficiently? You must have observed that a metallic frying pan have plastic or wooden handle. Can you lift a hot pan by holding it at the base without getting hurt? You probably use insulators without knowing why they are there. The handles are there to prevent your hand from being burned when the pot is hot. The use of plastic or wood prevents heat transfer by conduction from the hot object to your body.

Activity 6.2

Aim: To investigate heat conduction through different objects

Materials: hot water, beaker, pencil, plastic ruler, steel spoon, thermometer

Procedure

1. Pour hot water maintained at 100 °C into a beaker.
2. Hold the three items (pencil, ruler, steel spoon) all together at once, make sure each item touches your hand.
3. Dip them in hot water, as shown in Figure 6.3 and wait for a few minutes.

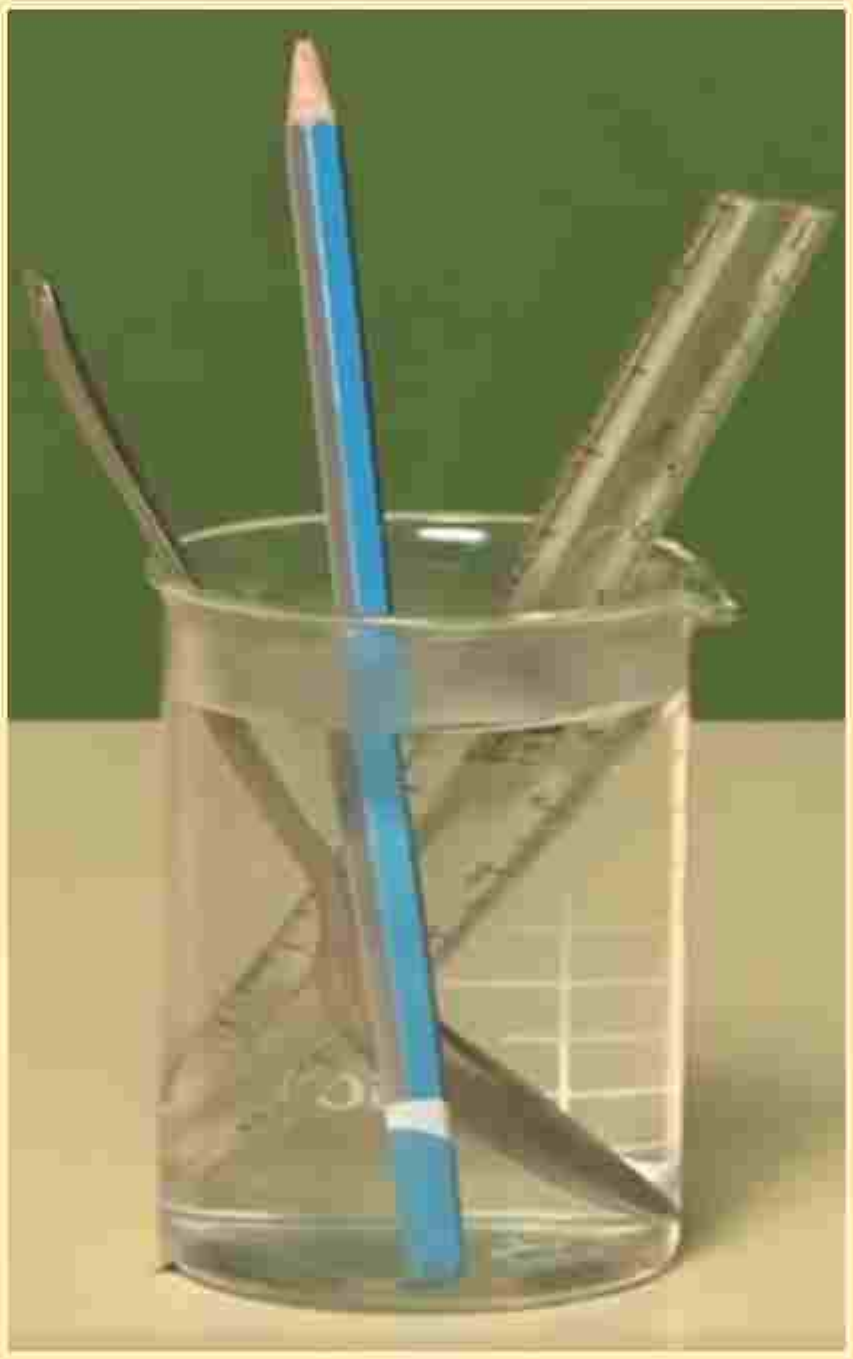


Figure 6.3

4. Record your observation in table as the one shown in Table 6.1.

Table 6.1

Item	Material of the item	Does the other end get hot (Yes/No)
Pencil		
Steel spoon		
Plastic ruler		

Questions

(a) Which material(s) is the best conductor of heat? Explain.

(b) Which material(s) is the poorest conductor of heat? Explain.

In the three investigated materials, the stainless-steel spoon gets hot more quickly and is the only one that transferred heat from the hot water to your hand. This is because steel conduct heat more easily than the wood and plastic. A measure of how good a material allows the flow of heat through conduction is given by its thermal conductivity.

Thermal conductivity of a material is a measure of the ability of the material to conduct heat. The materials that allow heat to pass through them easily such as metals, are good conductors of heat. In a metal, there are free electrons that can move within that metal. When a metal is heated, free electrons carry thermal energy from a high temperature region to a lower temperature region. This makes metals good conductors of heat, and that is why cooking utensils, boilers, and soldering irons, and car engines are made of metals. Materials that do not easily allow heat to flow through them are said to be bad or poor conductors of heat. They are also known as thermal insulators. Examples of insulators are plastic, wood, glass and fabric materials.

Minimization of heat loss by conduction

Heat is lost from buildings through their roofs, windows, walls, floors and through gaps in windows and doors. However, there are several ways in which these losses can be minimized. In a system where heat is to be preserved, losses by conduction have to be minimized. This can be done by covering the walls with poor conductors of heat to increase isolation of interior from exterior of a building, leading to the so-called thermal insulation. Thermal insulation involves the use of poor heat conductors such as air, plastic, and wood. They are used in boilers, hot-water pipes, and in industries like the textile industry, where heat loss has to be minimized. In the house, insulating against heat loss is achieved by using double-glazed windows, carpets, curtains and draught excluders.

Activity 6.3

Aim: To investigate ways of minimizing heat losses by conduction

Materials: a heat source, calorimeter with lid, calorimeter jacket, thermometer, cotton wool, stopwatch, water, beaker

Procedure

1. Put the calorimeter inside the jacket.
2. Add cotton wool inside the jacket to wrap the entire calorimeter, as shown in Figure 6.4.

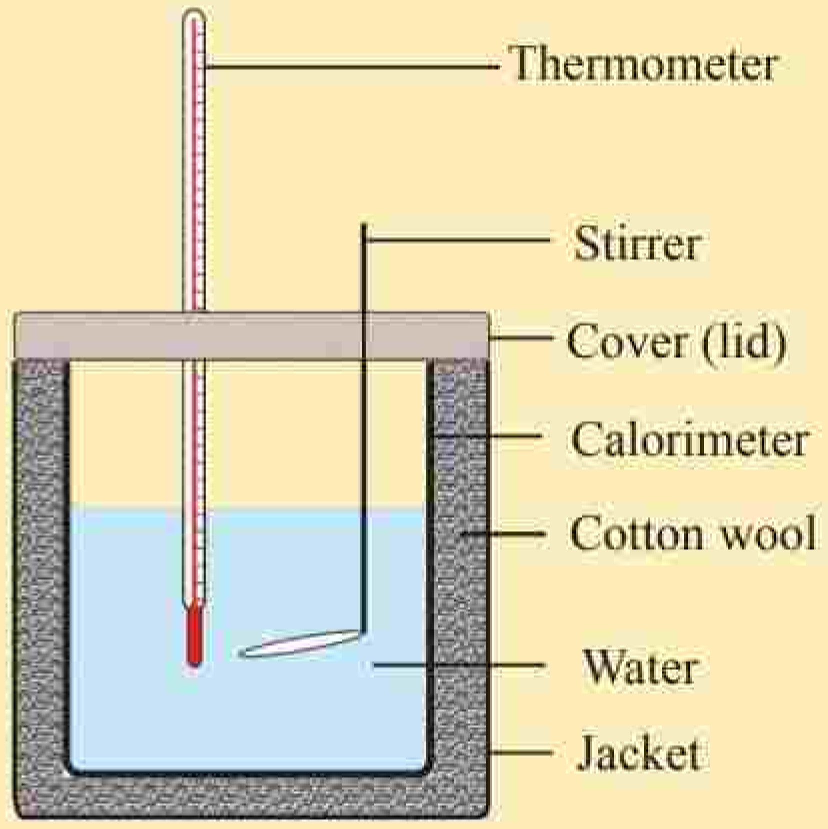


Figure 6.4

3. Boil water to the boiling point (100 °C), and then pour it into the calorimeter, cover it using a lid and insert a thermometer. Make sure that the lid is tightened and the calorimeter is wrapped by cotton wool.

4. While stirring, immediately start a stopwatch and read the temperature every two minutes for ten minutes.

5. Remove the calorimeter from the jacket, then repeat step 3 and 4.

Questions

(a) Which of the two calorimeters lost heat faster?

(b) How long did hot water from each calorimeter take to reach a temperature of 50 °C ?

Cotton wool acts as an insulating material, preventing heat losses from the calorimeter. This kind of material and others that prevent heat loss are used as heat insulators. Using a heat insulating material is also known as lagging. Note that, heat insulators are usually electrical insulators as well.

Activity 6.4

Aim: To determine the best insulating material among plastic, paper and aluminium

Materials: three beakers labelled A, B and C each of 100 ml, three thermometers, aluminium foil, newspaper, plastic, hot water maintained at 100 °C , stopwatch, measuring cylinder

Procedure

1. Wrap beakers A, B and C using newspaper, plastic and aluminium foil, respectively.

2. Pour 80 ml of hot water into each beaker.

3. Place a thermometer in each beaker and record the starting temperature of the water.

4. Measure the temperature of the water after every 5 minutes for half an hour.

5. Record your measurements in a table similar to Table 6.2.

Table 6.2

Time (minutes)	Temperature (°C) of water in beaker A	Temperature (°C) of water in beaker B	Temperature (°C) of water in beaker C
5			
10			
15			
20			
25			
30			

Questions

(a) Which among the three materials is the best conductor of heat? Explain your choice.

(b) Which among the three materials is the best insulator of heat? Explain your choice.

(c) Arrange the three given wrapping materials starting from poor to good insulators.

(d) If you want water to remain cold for a long period, which of the three wrapping materials will you choose? Explain your choice.

The best insulator is the one that prevents heat transfer, while the best conductor allows heat transfer very easily. This experiment reveals the best insulator and best conductor among the three materials, considering the rate at which hot water cools. It is essential to note that both good and bad conductors have significance in our daily life. Clothes prevent heat loss from the body as it is one of the examples of poorly conducting materials. On the other hand, good conductors transfer heat easily, allowing fast cooling of hot objects.

Applications of heat transfer by conduction

You have seen cases that need heat to be transferred quickly, and cases where heat has to be maintained, depending on the need. The following are some of the areas where the knowledge of heat transfer by conduction, is applied:

1. Cooking vessels are made of metal and metal alloys to transfer heat quickly from the cooker to the food placed inside the vessel. This makes the cooking process fast.
2. The computer's central processing unit (CPU) and other electronic devices may be affected by high temperature, causing them to malfunction. Heat sinks are therefore attached to these devices to protect them from overheating. This is because heat sinks quickly transfer heat away from the devices. Heat sinks are made of materials that are good conductors of heat.

3. Aluminium in some cases is used to make motor vehicle engines and other engine accessories because it can easily transfer or exchange heat from a hot engine to the environment.

4. Clinical thermometers use mercury because mercury is a good conductor of heat. Thus mercury can efficiently respond to body temperature.

5. An insulator blocks the transfer of heat. Handles of most kettles, electric iron, frying pans, cooking utensil are made of plastic or wood because they are poor conductors of heat. You can hold a plastic or wood handle of your hot frying pan without burning your hands. Figure 6.5 shows an insulated handle of a steel cooking vessel.

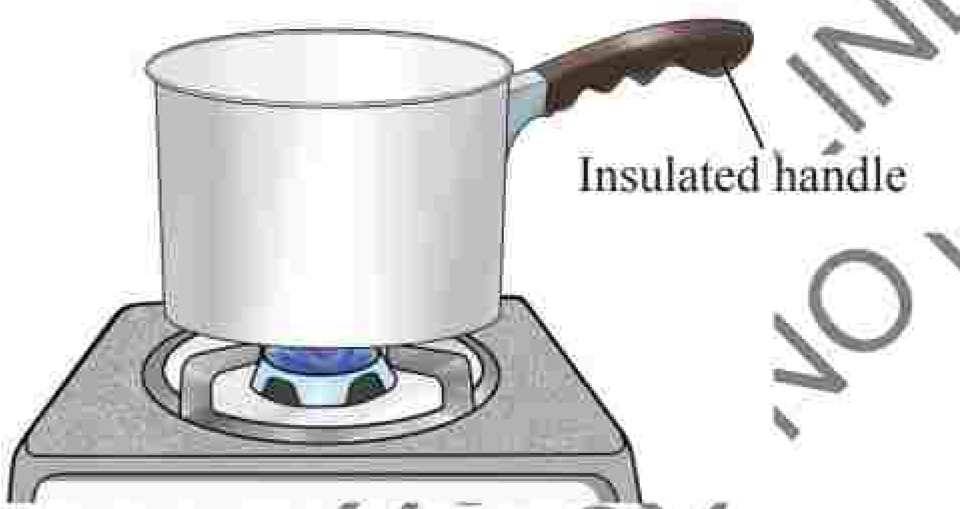


Figure 6.5: *Insulated handle of a cooking vessel*

6. Woollen clothes and blankets are used to reduce heat losses from the body. Different materials including fibreglass, bricks, glass and mud are used for building and insulation in walls and roofs of houses, keeping rooms warm during cold seasons and cool during hot seasons. Sawdust is used to cover up ice blocks. Feathers keep birds warm by minimizing heat that flow out of their bodies. All these materials are poor conductors of heat, thus prevent heat flow in or out of a system.

Exercise 6.1

1. In items (a)-(d), choose the correct answer among the given alternatives.

   (a) Heat energy can be transferred from one point to another through:
   - (i) Conduction, convection and radiation.
   - (ii) Convection, conduction and absorption.
   - (iii) Radiation, conduction and advection.
   - (iv) Conduction, radiation and vibration.

   (b) Thermal energy transferred by physical contact of objects without net displacement of molecules is known as,
   - (i) conduction.
   - (ii) convection.
   - (iii) radiation.
   - (iv) advection.

   (c) Which of the following statements is true?
   - (i) Materials of low thermal conductivities are bad thermal insulators.
   - (ii) Materials of high thermal conductivities are bad thermal insulators.

(iii) Materials of low thermal conductivities are good thermal insulators.

(iv) Both (ii) and (iii).

(d) The rate of flow of heat across the opposite faces of a metre cube of material maintained at the temperature difference of 1 kelvin is called its,
   (i) thermal radiation.
   (ii) flow of energy.
   (iii) thermal conductivity.
   (iv) convection.

2. List down, as many as possible, heat conductors and heat insulators found in your environment.
3. Describe the uses of heat conductors and heat insulators found in your environment. Discuss your results in class.
4. Explain why you feel very cold on bare feet standing on tiled floor during the cold season, while you feel comfortably warm, when standing on a carpeted floor.
5. Why do metals conduct heat better than many other materials?
6. Why do you feel colder when touching a metal lying on the floor than when touching a plastic material?
7. There are two cups, one made from glass and the other from stainless steel. In which cup will you be comfortable to drink hot tea from? Explain.

Heat transfer by convection

Convection is the transfer of heat in fluids (liquids and gases) between regions of different temperatures, and it involves the actual movement of the fluid itself. Thus, heat is carried by the bulk movement of the fluid itself due to differences in density. The higher the fluid velocity, the higher the rate of heat transfer. Therefore, the rate of heat transfer by convection is much higher than that of conduction. However, both conduction and convection require the presence of a material medium. They are different in that, convection involves fluid motion, while in conduction atoms transfer their energy by colliding with their neighbours.

To understand convection, consider heat transfer through water in a pot that is being heated. The heated water expands and becomes less dense and consequently rise upwards. Cold water at the top of the pot sinks downwards to take the place of the rising less dense water under the influence of gravity. The density difference between the water at the bottom and at the top results to the gradual formation of convectional currents. The convection currents form streams of warm moving water, thus carry heat to other parts of the water. Figure 6.6 shows arrows indicating convection currents in water placed on a fire source. Liquids such as water transfer heat by convection.

Convection is classified as either natural (or free) convection or forced convection, depending on how the fluid motion is initiated. In a forced convection, an external agent such as a pump or fan is used to initiate the process. In natural convection, a fluid motion is caused by natural means such as the buoyancy effect which is the rise of warm fluid and the fall of the cold fluid. Forced convection is also classified as external or internal, depending on whether the fluid is forced to flow over a surface or in a channel.

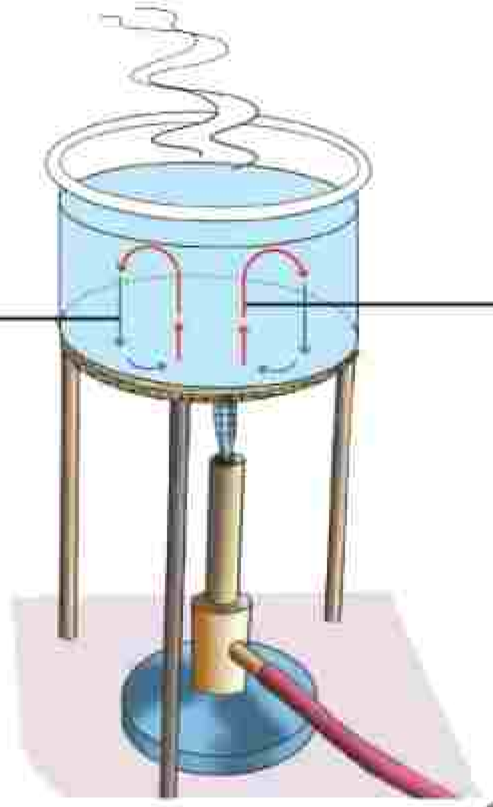


Figure 6.6: *Convectional currents in water*

Activity 6.5

Aim: To demonstrate convectional currents

Materials: water, flat-bottomed flask, crystals of potassium permanganate, source of heat, glass tube or straw

Procedure

1. Fill the round-bottomed flask with water.
2. Set up the apparatus as shown in Figure 6.7.

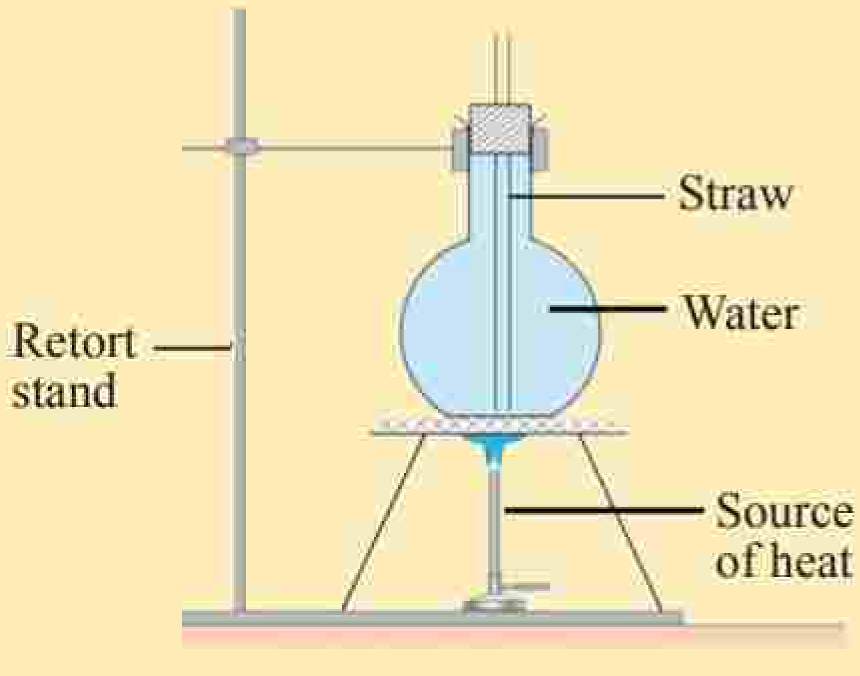


Figure 6.7

3. Using a long glass tube or long straw, drop three tiny crystals of potassium permanganate at the bottom of the flask.
4. Heat the water.
5. Observe what happens to the water with potassium permanganate.

Question

Why does the coloured water rise from the bottom of the flask? Explain.

As the water near the bottom of the flask gets heated, it expands and becomes less dense than the cold water at the top. The hot water moves upwards while the cold water moves downwards. Convection currents are observed through the movement of the purple colour of the dissolved crystals of potassium permanganate.

Convection based on the kinetic theory of matter

Fluids are poor conductors of heat. However, when free to move, fluids can carry heat from one place to another very quickly. According to the kinetic theory of matter, molecules of fluids are in a continuous motion and therefore possess kinetic energy. These molecules have a smaller force of cohesion and are slightly further apart than in solids but close enough to have definite volume. They are free to vibrate, move rapidly over short distances, and slip past each other in all directions. When part of the fluid is heated, the heated molecules at that part move faster, thus colliding with cold molecules. These cold molecules gain energy and in turn start vibrating, thus, transferring energy from one place to another. This results to the convection current of molecules. Usually, convection currents are more pronounced in gases than in liquids.

Minimization of heat loss by convection

One of the typical processes of minimizing heat loss by convection is by using vacuuming. Vacuuming involves the removal of any fluid medium within which convectional currents move. Heat loss by convection can also be reduced by filling air cavities with insulating solid materials. The woollen material between the cavity traps air between its fibres to minimize heat loss by convection. A double-glazed glass window also minimizes heat loss by convection. An air gap in between the glazed glass windows is made too narrow to prevent air circulation, thus minimizing heat loss by convection.

Activity 6.6

Aim: To minimize heat loss by convection

Materials: two 250 ml beakers, a lid, stopwatch, heat source

Procedure

1. Pour an equal volume of water in the two provided beakers.
2. Cover one beaker tightly while leaving the other open, as shown in Figure 6.8.

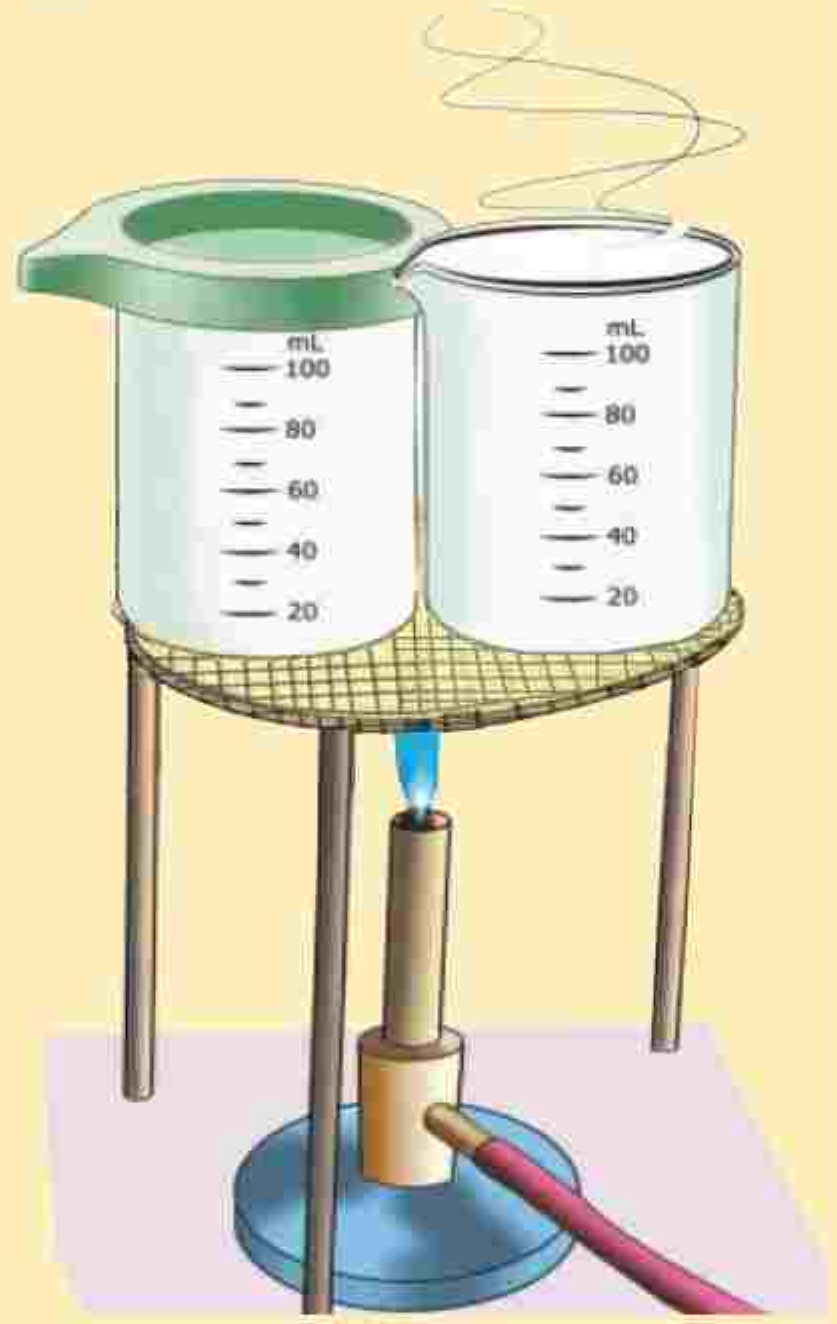


Figure 6.8

3. Put the two beakers on the heat source and use a stopwatch to record the time taken for the water in each beaker to boil.

Question

In which beaker did the water take shorter time to boil? Explain your observations.

The covered beaker takes a shorter time to boil the water compared to the uncovered one. This is because the lid prevents convection currents from escaping the covered beaker, thus preventing energy loss. The steam produced in the process circulates inside the covered beaker, speeding up the boiling process. For the uncovered beaker, convection currents escaped from the beaker, resulting in energy loss and finally taking longer to boil.

Applications of heat transfer by convection

In your life, you might have come across a number of situations where convection of heat is experienced. The following are important examples of situations where heat transfer by convection is applied; land and sea breeze, domestic hot water supply system, air conditioning, cooling of the engine, ventilators and kitchen chimneys.

Land and sea breeze

Land and sea breezes result from unequal heating and cooling of adjacent land and sea surfaces. During the day, the land heats up faster than the sea. This is because the heat capacity of sea water is higher than that of land. Therefore, air over the land surface becomes warmer than that above the sea. This results in a thermally driven circulation with rising air (warm air) over land and sinking air (cool air) over the sea. Thus, a cool air blows from the sea to the land. This phenomenon is called *sea breeze*, (Figure 6.9).

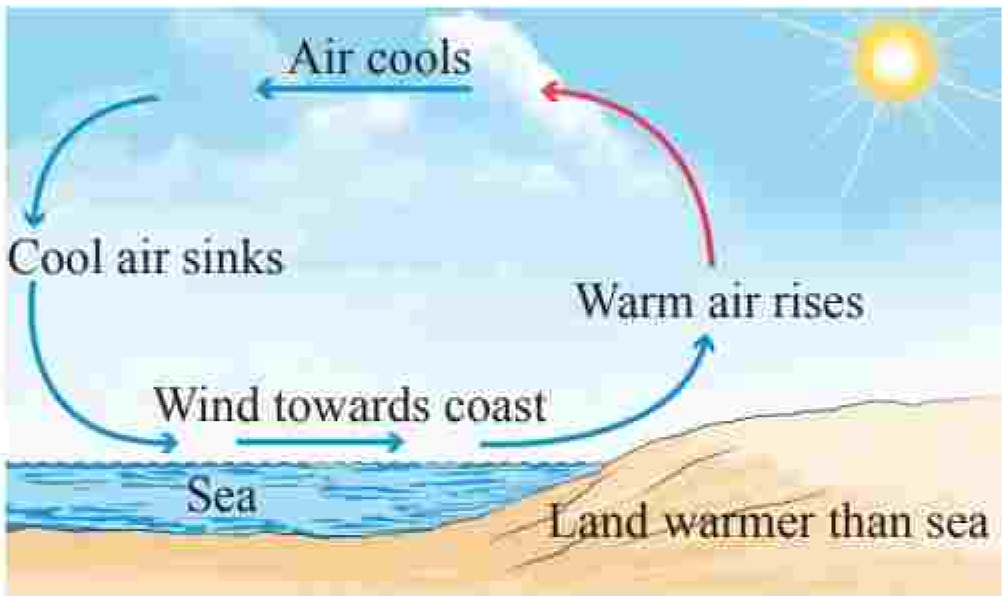


Figure 6.9: *Sea breeze*

At night, the land loses heat faster than the sea water. Thus, air above the sea remains warmer than that of the land. This results in a stream of cool air moving from the land to the sea surface. This phenomenon is called a *land breeze* (Figure 6.10).

Figure 6.10: *Land breeze*

Domestic hot water supply system

Domestic hot water systems rely on convection currents to move warm water from the boilers to where it is used. Typical domestic hot water system consists of a boiler, a hot water reservoir tank and a cold water reservoir, all connected by pipes. A typical system is shown in Figure 6.11.

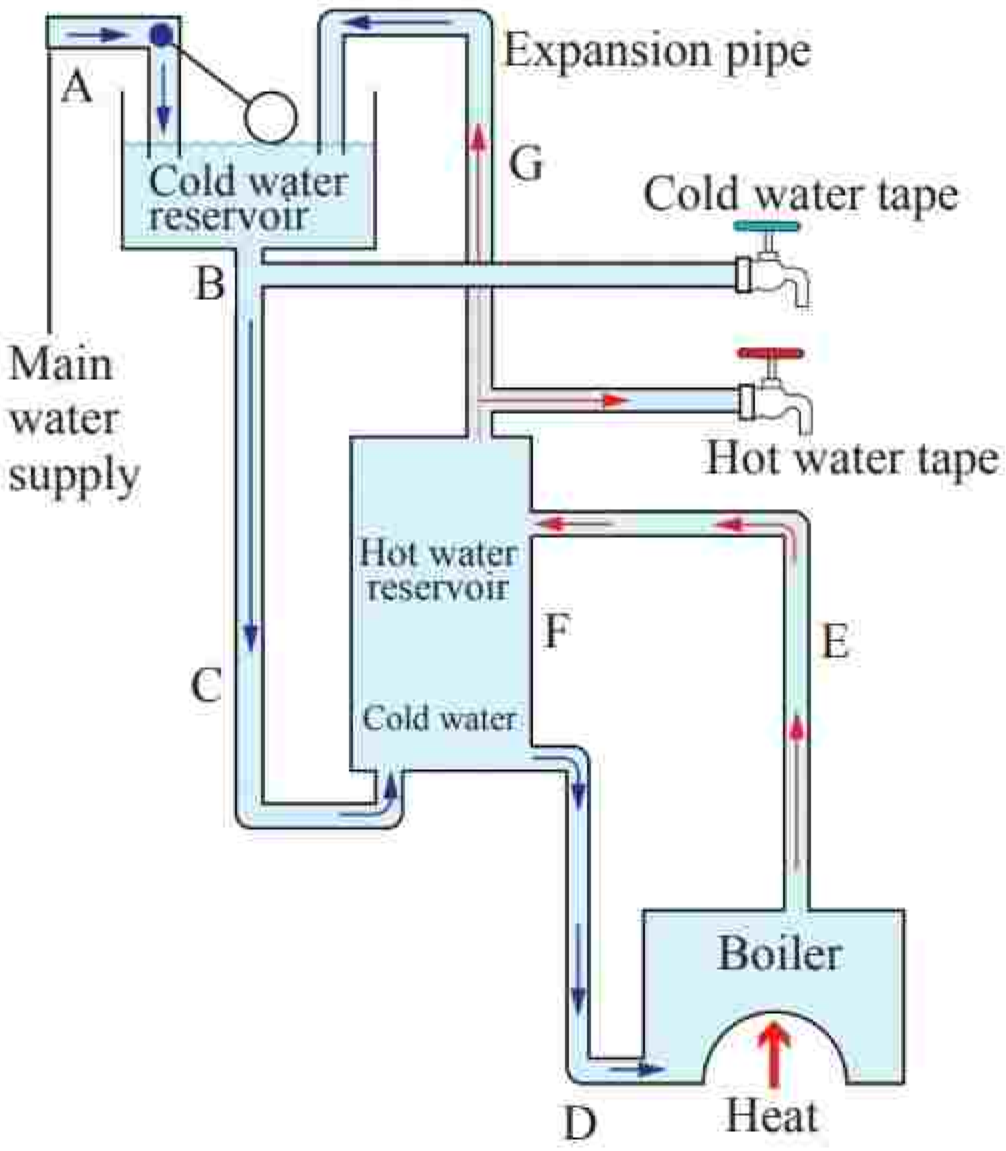


Figure 6.11: *Domestic hot water supply*

Cold water from the main supply enters the cold water reservoir, from the main supply pipe. The water then flows to the boiler through the reservoir, F as shown in Figure 6.11. Inside the boiler, the water is heated, becoming less dense. The water rises and flows into the hot water reservoir, where it is stored until it is needed for use. Water that is drawn out of a hot water tape comes from the hot water reservoir. The steam that is generated during the heating process is vented through an expansion pipe back into the cold water reservoir. The ballcock and the valve maintain a constant water level in the reservoir.

Air conditioners

Air-conditioning systems rely on convectional currents to heat or cool a room. They are strategically placed at a high point in the house. When a room is hot, cool air is blown into the room from the air conditioner. This cool air sinks to the bottom of the room, replacing the less dense warm air. The warm air rises and is pumped out to be cooled from below then re-circulated.

Room heaters, on the other hand, are placed on the floor of a room. When a room is cold, the heater is turned on, heating the surrounding air. The heated air rises up and cold air moves down to take the place of the rising warm air. This forms convectional currents, which continue to circulate until the air in the room is at the desired temperature.

Cooling of a car engine

Motor vehicle engines produce a ernomous amount of heat, which must be removed from the engine; otherwise, the engine will stop working. The car engine cooling system is based on the principle that hot water becomes less dense and rises while cold water replaces warm water. Water that surrounds the engine cylinders becomes less dense as it becomes hot.

Convection currents are therefore formed due to the difference in densities of the water in the engine. These convection currents transfer hot water to the top so as to be pumped to the radiator. In the radiator heat is removed from the hot water by the atmospheric air with the help of a fan. The water is thus cooled and recycled back to the engine as illustrated by Figure 6.12. This is a continuous process as long as the engine is running. Recently engines use special liquids (coolants) that have higher heat capacity for efficient cooling. The use of coolants also prevents the engine from rusting.

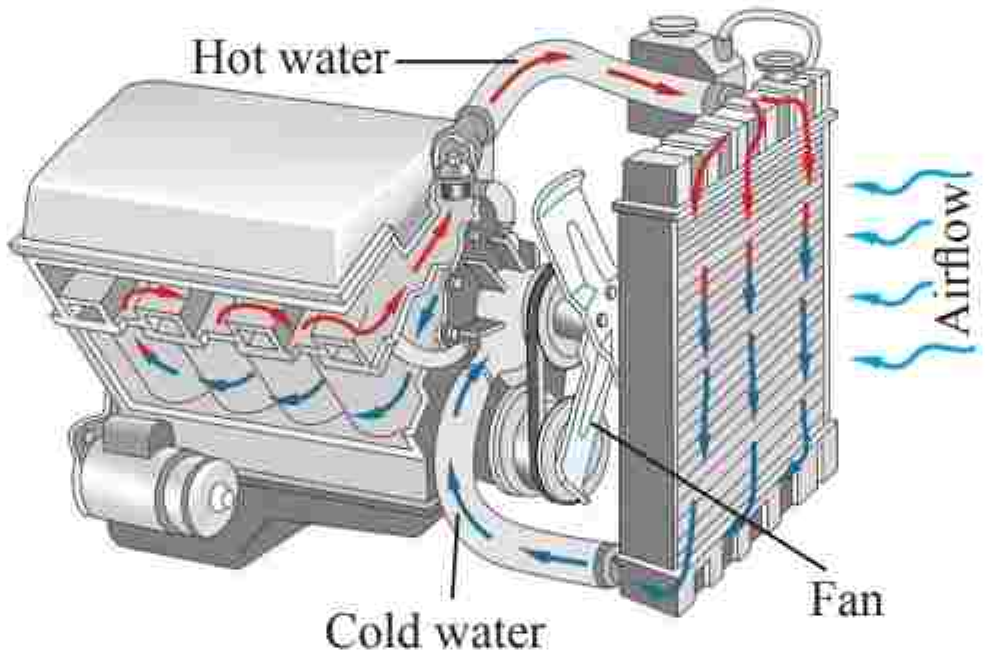


Figure 6.12: *Cooling of the car engine using water*

A ventilator or exhaust fan

Ventilators or exhaust fans are crucial for supplying high quality air throughout a building. A *ventilator fan* helps to bring fresh air from the outside into the enclosed space. It also helps to circulate air throughout a building. The fresh air from outside blows into the room through convection currents set up in the room. When pollutants and other contaminants like dust, smoke, moisture and odours exist in a building, the *exhaust fan* is used to remove them in order to provide clean air.

A chimney in a kitchen room

Kitchen rooms are provided with a chimney through which hot air in the room goes out and fresh air containing oxygen enters through the windows and doors into the kitchen to support the burning of fuel. A chimney serves as an exhaust for smoke that results from fires.

Exercise 6.2

1. Describe the process of convectional currents with reference to a ventilation system in a house.
2. Discuss why wearing several clothes during a cold season keeps us warmer than wearing just one thick piece of cloth.
3. Draw a well labelled diagram of the domestic hot water system and explain its mode of action.
4. Why are houses often built with high ceilings?
5. When cooking using firewood, the smoke produced do not always rise upwards. What is the reason for that?

Heat transfer by radiation

Heat transfer by radiation is another way of transferring thermal energy. In contrast to conduction and convection where material media are required, heat transfer by radiation does not require material media. That is, it occurs in vacuum. Particles of matter are not involved in heat radiation. The mechanism of radiation involves heat transfer between two or more bodies by means of electromagnetic waves. All bodies at a temperature above absolute zero emit some radiant heat. On reaching an object, part of the radiation can be reflected, transmitted or absorbed. The absorbed heat results to the rise in temperature of the object.

Thermal energy from the sun reaches the earth's surface through radiation as the region between the sun and the earth's atmosphere is vacuum. This means that thermal energy from the sun does not reach the earth through conduction or convection. This leaves radiation as the only way through which the sun's heat reaches the earth's surface (Figure 6.13).

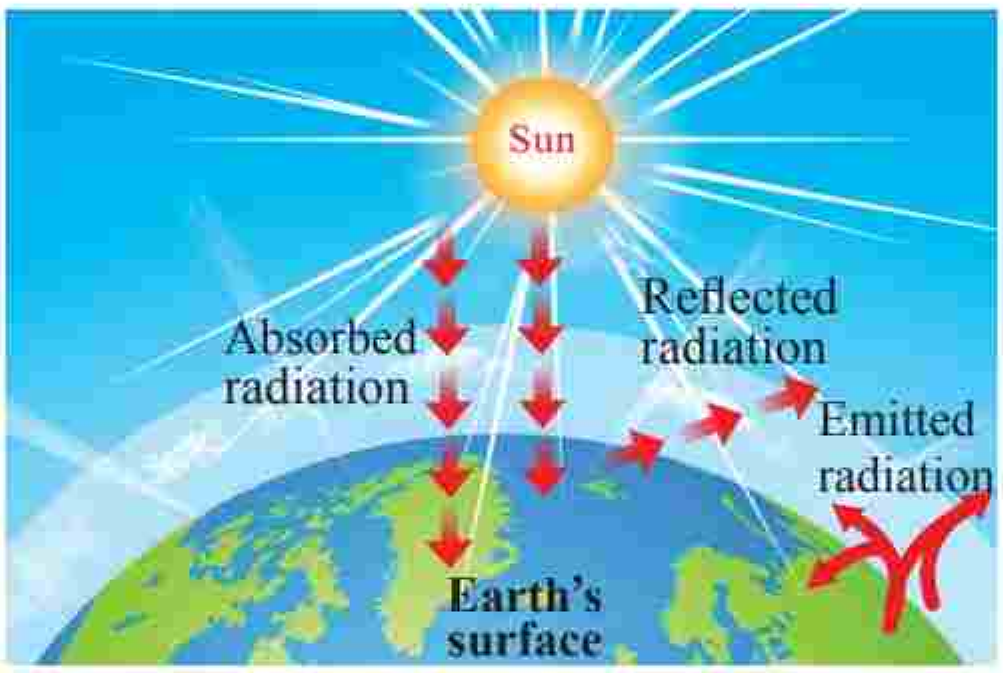


Figure 6.13: *The thermal energy from the sun reaches the earth through radiation*

Activity 6.7

Aim: To investigate how heat reaches us from a distant source

Materials: source of heat (fire), cardboard

Procedure

1. Stand at a place where there is sunlight. How do you feel?
2. Stand in a shade. How do you feel? Do you notice any difference in the two positions?
3. Stand near a fire. How do you feel?
4. Put cardboard between you and the fire. How do you feel? Do you notice any difference in heat? Why?

Heat transfer by radiation does not need a medium. The radiant heat from the sun travels through the vacuum and the atmosphere. The heat from the fire also travels by radiation. Radiant heat energy travels with the speed of light. Radiant heat is used in drying, evaporating water, solar water heaters and solar concentrators like solar cookers.

Absorbers and emitters of radiant heat

The rate at which a body radiates or absorbs radiant heat depends upon the temperature of the emitting surface. Objects that are good emitters are also good absorbers. Blackened surfaces are good absorbers as well as good emitters of radiant heat. A surface that absorbs all radiant energy is referred to as a blackbody. On the other hand, polished or shiny surfaces are good reflectors of radiant energy but poor emitters.

Activity 6.8

Aim: To demonstrate absorption and emission of radiant heat

Materials: two thermometers, two shiny tins, a candle, water, two tripod stands, two wire gauzes, a piece of cardboard

Procedure

1. Cut out two circular covers from the piece of cardboard to fit the circumference of the tins.
2. Make a hole through each of the cardboards through which the thermometers can be inserted.
3. Blacken the outer surface of one of the tins and leave the other one shiny.
4. Heat some water to a temperature of 40 °C.
5. Fill the tins with the heated water up to about three-quarters and cover them with the cardboard covers.

6. Place the two tins on a tripod stand at an equal distance of 5 cm on either side of a heat source, as shown in Figure 6.14. Note the initial temperatures of the water in each tin.

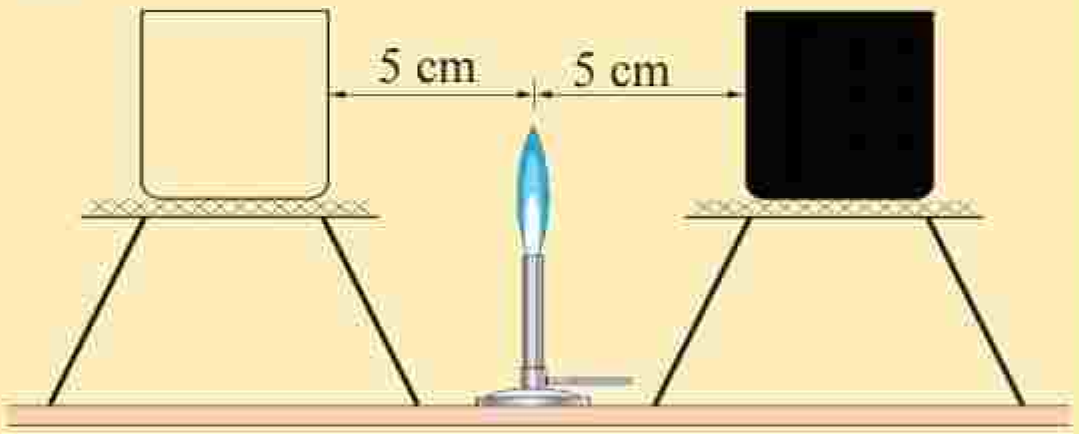


Figure 6.14

7. In the interval of 2 minutes, record the temperature (θ_1) of the water in each tin for ten minutes.

8. Remove the candle and then at the interval of 2 minutes, record the temperature (θ_2) of the water in each tin for ten minutes. Record your results as in Table 6.3.

Table 6.3

Time (min)	Temperature (°C)			
	Shiny surface		Black surface	
	θ_1	θ_2	θ_1	θ_2
0				
2				
4				
6				
8				
10				

Questions

(a) What is your observation when the source of heat was present and when it was removed? Explain.

(b) What conclusion can you draw from this activity?

The temperature of water in the black tin rises to a higher value faster than the water in the shiny tin. When the source of heat was removed, the thermometer readings obtained from the water in the black tin were lower than those obtained from the water in the shiny tin at any time. This means that heat was being lost from the black tin at a higher rate than the shiny tin. We can, therefore, conclude that a black surface absorbs and radiates heat much better than a shiny surface. That is, good emitters of thermal radiation are also good absorbers.

Minimization of heat loss by radiation

There is a number of ways that can be used to minimize heat loss by radiation. Aluminium materials are used to cover items such as hot food, to minimize heat loss by radiation. In fabrication of thermos flasks and cups, the inner and outer container walls are silvered to reduce heat transfer by radiation.

The thermos flask

The thermos flask is designed to minimize heat loss by radiation, convection and conduction. It is designed in such a way that heat loss by the three methods is reduced to a minimum.

A thermos flask is a container designed to keep hot liquids hot and cold liquids cold. Thermos flask is a double-walled flask with the walls joining at the neck. In between the walls is a vacuum, occupying a very narrow space. The inner and outer walls are coated with a thin layer of silver to avoid heat loss by radiation. It has a stopper made of cork or other insulating material and insulated separators keep the inner glass container from touching the outer container. Figure 6.15 shows the main parts of a thermos flask.

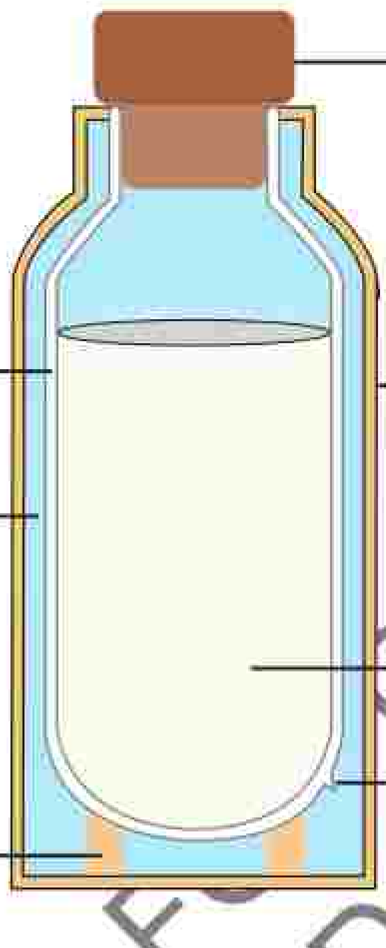


Figure 6.15: *Thermos flask*

Heat loss through conduction is prevented by the vacuum between the glass walls. The vacuum also prevents heat loss by convection. The cork stopper prevents heat from escaping from the flask to the atmosphere by convection. It also prevents heat loss by thermal conduction. Generally, the thermos flask simultaneously demonstrates prevention of heat loss by all the three processes through which heat is transferred. This is contrary to the kettle which demonstrates heat loss by all the processes as illustrated in Figure 6.16.

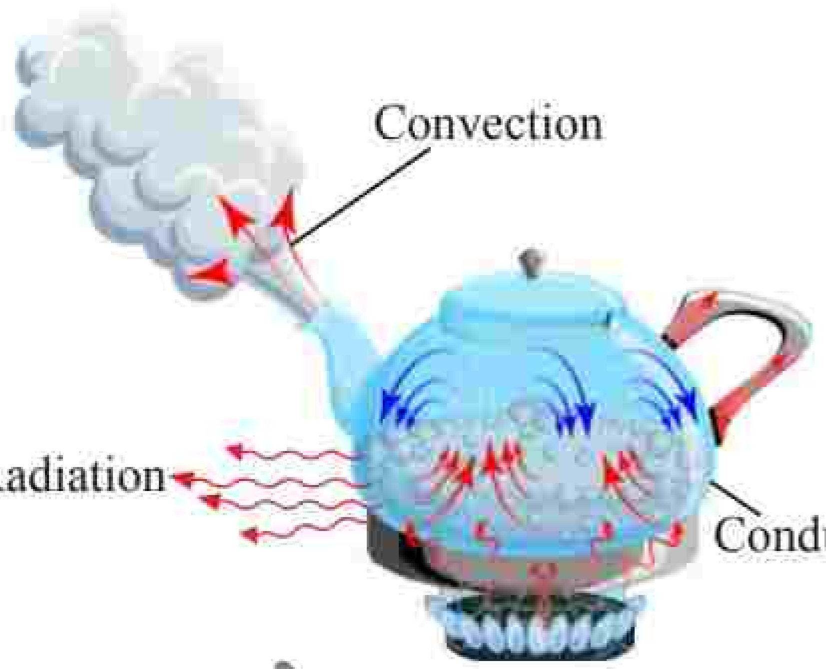


Figure 6.16: *The three mechanisms of thermal energy transfer*

Applications of heat transfer by radiation

Heat transfer by radiation is significant in our daily life. One of the application of radition is the minimization of heat loss as discussed in the preceding section. Other important applications of heat radiation in our daily life include the following:

1. White and light coloured clothes are more suitable in hot season because they absorb very little amount of sun's radiation and thus keep our bodies cool. Dark coloured clothes are more suitable in cold season because they absorb most of the radiant heat from the sun and keep us warm.
2. The base of a box solar cooker is painted black to absorb more radiation and hence cook food quickly.
3. Cooling fins at the back of refrigerators are painted black so as to radiate the maximum amount of heat to facilitate effective cooling.

4. Kettles, cooking pans and electric iron bases are all polished on their surfaces to reduce heat loss through radiation.
5. Car petrol tanks are painted silvery bright to reflect away as much heat as possible.
6. Houses in hot areas have their walls and roofs painted with bright colours to reflect away radiation.
7. In solar concentrators, the incoming heat radiation from the Sun are reflected to a focal point by concave reflectors.

Exercise 6.3

1. Fill in the blanks:
   (a) The hotness of an object is determined by __________.
   (b) Temperature of a substance is expressed in _______.
   (c) No medium is required for transfer of heat by the process of __________.
   (d) A cold steel spoon is dipped in a cup of hot milk. Heat is transferred to its other end by the process of ___________.
   (e) Dark coloured clothes ______ more radiation than light coloured clothes.
2. Distinguish between convection and radiation of heat.
3. Conduct an experiment to show that a dull black surface is a better absorber of radiation than a polished one.
4. Give two ways by which heat loss by a body can be reduced and show how this is effected in one practical example.
5. Why do you often feel warmer during cloudy night than during clear sky night?
6. There are two cars, one is light coloured and the other dark coloured. Which one will you prefer to use on a sunny day with an average temperature of 34°C? Give reasons.

Chapter summary

1. Heat is a form of energy that is transferred from a hot to a cold object.
2. Temperature is a measure of the degree of hotness or coldness of a body.
3. Transfer of heat energy occurs in three ways: conduction, convection and radiation.
4. During conduction, objects must be in contact with each other for heat transfer to take place.
5. Some materials, such as metals, are called good conductors of heat; they conduct heat easily. Others, including plastics and wood, are called insulators or bad conductors of heat; they prevent heat transfer by conduction.

6. Convection is the transfer of thermal energy within liquids or gases with different temperature regions. It involves the bulk movement of the fluids themselves.
7. Convection currents are the streams of warm and cold moving fluids (liquid or gases) due to the difference in densities as a result of temperature differences. The liquid or gas moves upwards as it expands and becomes less dense and the denser liquid on gas moves downwards under the influence of gravity.
8. Radiation is the heat transfer between two or more bodies by means of electromagnetic waves. It transfers energy where objects do not have to be in physical contact. It does not require a medium.
9. The energy from the sun reaches the earth by radiation, not by conduction or by convection.
10. Dark or black surfaces are good absorbers and radiators of radiant energy, whereas light and shiny surfaces are poor absorbers and emitters of radiant energy

Revision exercise 6

Answer all questions.

1. (a) Explain how heat transfer by conduction takes place.
   (b) Why are gases poor conductors of heat?
   (c) Why are cooking vessels made of aluminium and not iron?
   (d) With respect to thermal energy transfer, explain why stadium seats are made of plastic rather than steel.
   (e) Give two ways through which heat loss by convection can be prevented.
2. (a) Explain how heat transfer by convection takes place.
   (b) It is impossible for heat transfer by convection to take place in solids. Explain.
3. Figure 6.17 shows an electric kettle.

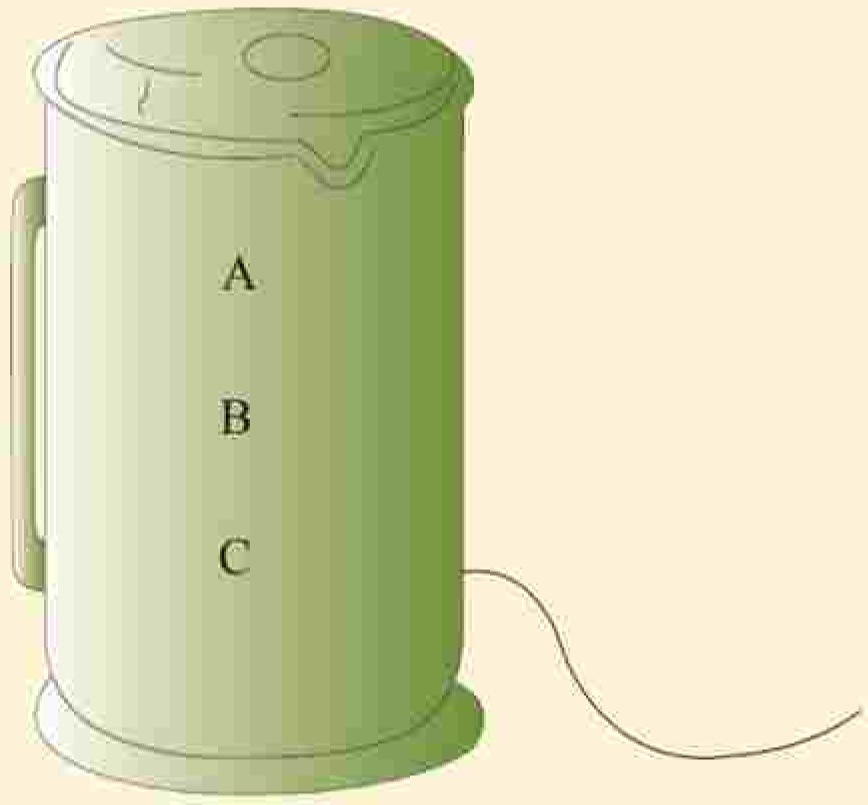


Figure 6.17

   (a) Which is the best position among A, B and C to place the heating element? Explain your answer.
   (b) What happens to the cool air outside the kettle when the kettle is hot?

4. With the aid of diagrams, explain how land breeze and sea breeze occur.
5. (a) Explain how heat transfer by radiation takes place.
   (b) Heat transfer by radiation does not require a medium. Explain.
   (c) A good solar cooking vessel should be black on the outside and not shiny white. Explain.
   (d) Give one way through which heat loss by radiation can be prevented.
6. Aluminium is a good conductor of heat. Why is it sometimes used as an insulator?
7. List three areas where heat transfer through each of the following mechanisms is applied:
   (a) conduction
   (b) convection
   (c) radiation
8. An electric filament lamp with a clear glass bulb is switched on and gives a bright white light. The bulb contains a small quantity of argon. Give an account of the roles played by conduction, convection and radiation in the loss of heat from the lamp filament.
9. Suppose you are assigned to design a thermos flask. Explain how you will minimize heat loss by conduction, convection and radiation.
10. What could be an advantage of using an umbrella when you go out in the sun?
11. There is a set of thin blankets and thick blankets. Will you choose two thin blankets or one thick blanket to wrap yourself during cold night? Explain.
12. Describe the mechanisms of thermal energy transfer from the hot core of Earth to its surface and from Earth's surface to outer space.
13. Which process of heat transfer is involved when using a hairdryer? Explain.
14. A loose-fitting white dress can be used both in the hot sun and during cold evenings. Explain how such dressing is advantageous during both day and night.
15. In hot regions, it is advised that the outer walls of houses should be painted white. Explain.
16. Through which of the following modes is thermal energy transferred at the highest rate?
    (a) Conduction
    (b) Convection
    (c) Radiation
    (d) Conduction and convention

Chapter Seven

Measurement of thermal energy

Introduction

In Chapter Six, you learnt that, cooking utensils such as kettles, frying pans and pots are used because they can quickly heat up. We also learnt that, car engines are cooled using water because they have a high capacity of retaining the heat. All these applications are based on the knowledge of measurement of thermal energy. In this chapter, you will learn the concept of heat capacity, specific heat capacity, melting point, boiling point and evaporation. You will also learn the concept of latent heat of fusion and vapourisation, regelation and mechanism of refrigeration. The competencies developed will enable you to construct different cooling and heating systems, and design thermal radiators.

Heat capacity

Why do some substances require more energy for them to heat up to a certain temperature and some do not? As you learnt in the previous chapters, temperature is a function of the average kinetic energy of molecules in an object. These molecules have kinetic energy due to their translational, rotational and vibrational motion. The kinetic energy makes up the internal energy or thermal energy of the molecules. This thermal energy can be transferred within an object or between two objects. If an object absorbs heat, its internal energy increases and this leads to rise in temperature of the object. The amount of heat required to raise the temperature of a body by one kelvin is called the heat capacity of the body. Bodies made from different materials have different heat capacities. Measurement of changes in thermal energy is an indirect measurement of the molecular kinetic energy.

Factors that determine the heat content of a substance

We have already seen that, heat is a form of energy transferred between bodies due to the difference in temperature between them. It is the quantity of thermal energy that enters or leaves a body and not a property of a material. Heat is energy in motion. It is known from experience that, it takes a longer time to heat a large amount of water in a pot to a certain temperature than to heat a small amount of water to the same temperature. The larger quantity of water requires more energy for it to be heated to the required amount. This means that the amount of heat required depends on the quantity or mass of materials of the object. For example, the heat required to raise the temperature of 1 kg of water by 1 °C or 1 K is not the same as the heat required to raise a temperature of 500 g of water by 1 °C or 1 K. Also, the heat energy required to raise the temperature of 1 kg of water by 60 °C or 60 K is different from the heat energy required to raise the temperature of 1 kg of water by 90 °C or 90 K.

Similarly, the heat required to raise the temperature of 1 kg of water by 1 °C or 1 K is different from heat required to raise the temperature of 1 kg of kerosine by 1 °C or 1 K. Therefore, heat content of an object depends on the mass of the object, temperature change and nature of the material of an object.

Activity 7.1

Aim: To verify the effect of mass and nature of the material on changes in the heat content of a substance

Materials: stopwatch, water, two beakers of 250 ml each, waterbath, source of heat, thermometer, 250 ml measuring cylinder, cooking oil

Procedure

1. Pour 100 cm^3 (100 g) of water in a beaker. Note the initial temperature.
2. Heat the water until the temperature changes by about 10 °C.
3. Record the time taken for this temperature change.
4. Put 200 cm^3 (200 g) of water in a beaker and record its initial temperature.
5. Heat the water until the temperature changes by about 10 °C.
6. Record the time taken for the change.
7. Repeat the experiment using cooking oil.

Questions

(a) Compare the time required to heat different amounts of the same liquid to the same temperature.

(b) Explain the temperature change when equal amounts of different liquids were heated for the same period of time.

(c) What do you conclude from the result in (a) and (b)?

Determination of the heat capacity

The amount of heat required to raise the temperature of an object by one kelvin or one-degree celsius is known as heat capacity. It is a measurable physical quantity that characterizes the amount of heat required to change a body's temperature by 1 kelvin or one degree celsius. It is also called thermal capacity. Heat capacity or thermal capacity is denoted by C.

$$Heat\ capacity = \frac{quantity\ of\ heat\ required\ (\text{J})}{change\ in\ temperature\ (\text{K or °C})}$$

$$C = \frac{Q}{\Delta\theta}\ (\text{J K}^{-1})$$

The SI unit of heat capacity is J K^{-1} or J °C^{-1}. It is also expressed in cal K^{-1} or cal °C^{-1}, where $1\ \text{cal} = 4.184\ \text{J}$ or $1\ \text{J} = 0.239\ \text{cal}$.

The equation of heat capacity is important because it relates the heat gained or lost by an object to its temperature change. An object with a large heat capacity can absorb a large amount of heat energy but its temperature changes only slightly. On the other hand, the object with a small heat capacity absorbs only a small amount of heat and experience a large temperature change.

Example 7.1

In an experiment to determine the heat capacity of steel, 100 kJ of heat was supplied to a block of steel initially at 22 °C. If the final temperature of the block was 219 °C, determine the heat capacity of steel.

Solution

Given $Q = 100\ \text{kJ} = 100\ 000\ \text{J}$

$$\Delta\theta = \theta_2 - \theta_1 = (219 - 22)\ °\text{C} = 197\ °\text{C}$$

$$Heat\ capacity\ (C) = \frac{quantity\ of\ heat\ absorbed\ (Q)}{change\ in\ temperature\ (\Delta\theta)}$$

$$C = \frac{Q}{\Delta\theta} = \frac{100000\ \text{J}}{197\ °\text{C}} = 507.61\ \text{J/°C}$$

Therefore, the heat capacity of steel is $507.61\ \text{J °C}^{-1}$.

Specific heat capacity of a substance

We have noted previously that, the heat capacity of an object depends on the mass of that object. It makes sense to introduce a related quantity so that we can compare the thermal properties of different materials. This is done by using the heat capacity of unit mass of a substance. This suggests that, objects made from different materials gain different amounts of heat when the temperature of unit masses of these objects are to be raised by one degree centigrade. The quantity of heat required to raise the temperature of 1 kg of a substance by 1 °C or 1 K is called the specific heat capacity of the substance. The symbol for the specific heat capacity is c. The word 'specific' is used to show that a 'unit mass' is being considered. That is:

$$c = \frac{heat\ capacity\ (\text{J/K})}{mass\ (\text{kg})}$$

where the heat capacity of a substance is

$$C = \frac{heat\ supplied\ (Q)}{temperature\ change\ (\Delta\theta)}$$

Therefore, the specific heat capacity of a body is given by;

$$c = \frac{heat\ supplied\ (Q)}{mass \times temperature\ change\ (\Delta\theta)}$$

$$c = \frac{Q}{m\Delta\theta}$$

The SI units for specific heat capacity are $\text{J kg}^{-1}\text{ K}^{-1}$ or equivalently, $\text{J kg}^{-1}\,{}^\circ\text{C}^{-1}$.

Rearranging the equation of specific heat capacity, you get,

$$Q = mc\Delta\theta$$

Heat is gained when a substance is heated and lost when a substance is cooled. It can be seen that changes in the heat content of a substance is affected by mass (m), specific heat capacity (c) and the temperature difference ($\Delta\theta$). The greater the specific heat of a substance, the more energy must be added to a given mass of the substance to cause a particular temperature change. Table 7.1 shows the values of specific heat capacities (c) for different materials.

Table 7.1: Specific heat capacities of some materials

Material	Specific heat capacity (J/kgK)
Water	4200
Sea water	3900
Paraffin	2200
Methylated spirit	2500
Ice	2100
Mercury	1395
Aluminium	900
Glass	700
Steel	500
Iron	480
Copper	390
Brass	320
Lead	130

Example 7.2

The temperature of a 6 kg block of copper rises from 15 °C to 30 °C on being heated. Determine the amount of heat supplied to the block. (Specific heat capacity of copper, $c = 390$ J/kg °C).

Solution

$c = 390$ J/kg °C,

$m = 6$ kg, $\Delta\theta = 30\ ^\circ\text{C} - 15\ ^\circ\text{C}$

Given, $Q = mc\Delta\theta$

$= 6\text{ kg} \times 390\text{ J kg}^{-1}\,{}^\circ\text{C}^{-1} \times (30-15)\,{}^\circ\text{C}$

$= 6\text{kg} \times 390\text{ J kg}^{-1}\,{}^\circ\text{C}^{-1} \times 15\,{}^\circ\text{C}$

= 35100 J

Determination of specific heat capacity of substances

If a cold object comes into contact with a hot object, the hot object loses heat and the cold object gains heat until thermal equilibrium is reached. At thermal equilibrium, both objects have the same temperature and there is no heat transfer. This is what happens when we cool hot water, say in a bathtub, by adding cold water. The final temperature of the mixture lies somewhere between the temperature of cold water and that of hot water. The hot water loses heat while cold water gains heat. Some heat is absorbed by the material of bathtub and by the surroundings. If the heat loss is controlled when mixing the water, heat gained by cold water is equal to heat lost by hot water. This is in agreement with the principle of conservation of energy. We can use this principle to determine heat capacity and specific heat capacities of substances.

Suppose a heated sample to some known temperature is placed in a vessel containing water of known mass and temperature such that the temperature of the sample is higher than that of water. If the system of the sample and water does not allow interaction with the environment, the principle of conservation of energy requires that, energy lost, Q_{lost}, by the sample equals to the amount of energy gained, Q_{gained}, by water. That is,

$$Q_{gained} = Q_{lost}$$

It should be noted that, the water container exchanges energy with the sample as it is gaining energy;

$$Q_{gained} = \begin{matrix} heat\ gained \\ by\ water \end{matrix} + \begin{matrix} heat\ gained \\ by\ a\ container \end{matrix}$$

However, if the mass of water is much larger than that of the container, the effect of the container is neglected. The science of measuring amount of heat transferred between bodies is called calorimetry, and a device in which this energy transfer occurs is called a calorimeter. Figure 7.1 shows the features of a calorimeter and the liquid, thermometer.

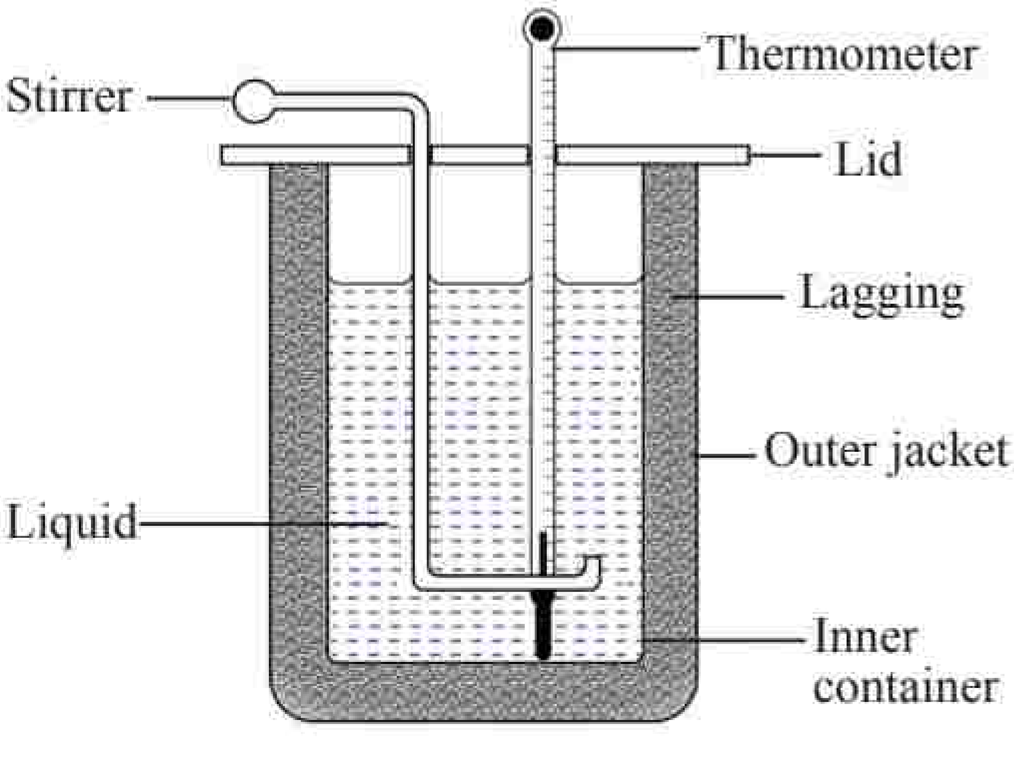


Figure 7.1: *A calorimeter*

The inner container and the stirring rod are both made of the same material, usually aluminium or copper. The heat loss is reduced by the lagging material and the lid. To measure specific heat capacity, a liquid of known mass and temperature is poured in the inner container, and a solid or liquid of known mass and temperature, is added to the liquid. The final temperature is measured when equilibrium is reached.

Activity 7.2

Aim: To determine the specific heat capacity of a solid

Materials: brass block, thermometer, cold water, weighing balance, stirrer, boiling water, thread, source of heat, stopwatch, copper calorimeter

Procedure

1. Measure and record the mass of the copper calorimeter and stirrer.
2. Pour 100 cm^3 (100 g) of water into the calorimeter and then place the calorimeter into its jacket and cover it.
3. Record the temperature of water.
4. Tie a brass block of known mass with a thin thread and place it in boiling water. Make sure the brass block does not touch the sides of the container with the boiling water.
5. After placing the brass block in the boiling water for about one minute,

remove it quickly, wipe off the water with a dry cloth and place it in the calorimeter. Measure the temperature of boiling water.

6. Replace the lid and stir gently until the temperature is steady.
7. Read and record this steady temperature.

Question

Determine the specific heat capacity of brass.

The specific heat capacity of the brass block can be calculated as follows:

Mass of copper calorimeter and stirrer = m_c kg

Mass of cold water = m_w kg

Initial temperature of cold water in the calorimeter = θ_1 °C

Mass of brass = m_b kg

Initial temperature of brass (temperature of boiling water) = θ_2 °C

Final temperature = θ_3 °C

Specific heat capacity of water $c_w = 4200$ J/kgK

Specific heat capacity of copper $c_c = 390$ J/kgK

Specific heat capacity of brass $c_b = ?$

Heat gained by the calorimeter and stirrer, $Q_c = m_c \times 390 \times (\theta_3 - \theta_1)$ J

Heat gained by water, $Q_w = m_w \times 4200 \times (\theta_3 - \theta_1)$ J

Heat lost by brass, $Q_b = m_b \times c_b \times (\theta_2 - \theta_3)$ J

Heat lost by brass = Heat gained by calorimeter and stirrer + Heat gained by water in the calorimeter

$$m_b \times c_b \times (\theta_2 - \theta_3) = m_c \times 390 \times (\theta_3 - \theta_1) + m_w \times 4200 \times (\theta_3 - \theta_1)$$

$$c_b = \frac{m_c \times 390 \times (\theta_3 - \theta_1) + m_w \times 4200 \times (\theta_3 - \theta_1)}{m_b \times (\theta_2 - \theta_3)}$$

$$c_b = \frac{(390m_c + 4200m_w) \times (\theta_3 - \theta_1)}{m_b \times (\theta_2 - \theta_3)}$$

Since the quantities m_c, m_w, m_b, θ_1, θ_2 and θ_3 are all known (from the results of the activity), the specific heat capacity of brass (c_b) can be determined.

Example 7.3

A piece of copper of mass 40 g at 200 °C is immersed into a copper calorimeter of mass 60 g containing 50 g of water at 25 °C. Neglecting heat losses, calculate the equilibrium temperature. (Specific heat capacity of copper, c_c = 390 J/kg°C and specific heat capacity of water, c_w = 4200 J/kg°C).

Solution

Let θ be the equilibrium temperature in °C.

Heat lost by the piece of copper

$$Q_1 = mc\Delta\theta = 0.04 \text{ kg} \times 390 \text{ Jkg}^{-1}\text{°C}^{-1} \times (200-\theta)\text{°C}$$

$$Q_1 = 15.6 \times (200-\theta) \text{ J}$$

Heat gained by the calorimeter

$$Q_2 = 0.06 \text{ kg} \times 390 \text{ Jkg}^{-1}\text{°C}^{-1} \times (\theta-25)\text{°C}$$

$$Q_2 = 23.4 \times (\theta-25) \text{ J}$$

Heat gained by water

$$Q_3 = 0.05 \text{ kg} \times 4200 \text{ Jkg°C}^{-1} \times (\theta - 25) \text{ °C}$$

$$Q_3 = 210 \times (\theta-25) \text{ J}$$

Heat lost by piece of copper = heat gained by the copper calorimeter and water.

$$Q_1 = Q_2 + Q_3$$

$$15.6 \times (200-\theta)\text{J} = 23.4 \times (\theta-25) \text{ J} + 210 \times (\theta-25) \text{ J}$$

By collecting like terms, θ can be calculated

$$\theta = \frac{8955 \text{ °C}}{249} = 35.96 \text{ °C} \approx 36 \text{ °C}$$

Therefore, the equilibrium temperature is 36 °C.

Example 7.4

A brass cylinder of mass, m_b, was heated to 100 °C and then quickly transferred into a thin aluminium can of negligible heat capacity, containing 150 g of paraffin at 11 °C. If the final steady temperature of the paraffin was 20 °C, determine the value of m_b. (Specific heat capacity of brass is 320 J/kg°C and specific heat capacity of paraffin is 2200 J/kg°C).

Solution

Heat gained by the paraffin $Q_p = m_p c_p \Delta\theta$

$$Q_p = \frac{150}{1000}\text{ kg} \times 2200\text{ Jkg}^{-1}\text{°C}^{-1} \times (20-11)\text{°C} = 2970\text{ J}$$

Heat lost by the brass cylinder, $Q_b = m_b c_b \Delta\theta$

$$Q_b = m_b \times 320\text{ J kg}^{-1}\text{°C}^{-1} \times (100-20)\text{°C} = m_b \times 25600\text{ Jkg}^{-1}$$

Heat gained by paraffin = heat lost by brass

$$2970\text{ J} = m_b \times 25600\text{ Jkg}^{-1}$$

$$m_b = \frac{2970\text{ J}}{25600\text{ Jkg}^{-1}} = 0.116\text{ kg}$$

The mass of the brass cylinder was 116 g.

Activity 7.3

Aim: To measure the specific heat capacity of a liquid

Materials: brass mass, thermometer, cooking oil, boiling water, copper calorimeter, copper stirrer

Procedure

Follow the procedure for Activity 7.2 but, instead of cold water, use cooking oil.

Question

Determine the specific heat capacity of cooking oil.

Determination of the specific heat capacity of the cooking oil can be carried out as follows:

Mass of calorimeter and stirrer = m_1 kg

Mass of cooking oil = m_2 kg

Initial temperature of cooking oil and calorimeter = θ_1 °C

Mass of brass = m_3 kg

Initial temperature of brass is the temperature of boiling water = θ_2 °C

Final temperature = θ_3 °C

Specific heat capacity of copper $c_c = 390$ J/kg K

Specific heat capacity of brass $c_b = 320$ J/kg K

Specific heat capacity of cooking oil $c_o = ?$

Heat energy gained by calorimeter and stirrer, $Q_1 = m_1 \times 390 \times (\theta_3 - \theta_1)$ J

Heat energy gained by cooking oil, $Q_2 = m_2 \times c_o \times (\theta_3 - \theta_1)$ J

Heat lost by brass, $Q_3 = m_3 \times 320 \times (\theta_2 - \theta_3)$ J

Heat lost by brass = Heat gained by calorimeter and stirrer + Heat gained by cooking oil

$$m_3 \times 320 \times (\theta_2 - \theta_3) = m_1 \times 390 \times (\theta_3 - \theta_1) + m_2 \times c_o \times (\theta_3 - \theta_1)$$

$$c_o = \frac{m_3 \times 320 \times (\theta_2 - \theta_3) - m_1 \times 390 \times (\theta_3 - \theta_1)}{m_2 \times (\theta_3 - \theta_1)} \text{ J/kgK}$$

Since the quantities $m_1, m_2, m_3, \theta_1, \theta_2$ and θ_3 are all known, the value of c_o can be obtained.

Example 7.5

A block of metal of mass 0.20 kg at a temperature of 100 °C is placed in 0.40 kg of water at 20 °C. If the final steady temperature of the water is 24 °C, determine the specific heat capacity of the metal. (Neglect heat absorbed by the container and heat loss to the surroundings). Specific heat capacity of water is 4200 J/kg°C.

Solution

Heat lost by metal, $Q_m = m_m c_m \Delta\theta = 0.2 \text{ kg} \times c_m \times (100 - 24)\ °\text{C}$, where c_m is the specific heat capacity of the metal.

Heat gained by water, $Q_w = m_w c_w \Delta\theta_w = 0.4 \text{ kg} \times 4200 \text{ J/kg°C} \times (24 - 20)\ °\text{C}$

Heat lost by metal = heat gained by water

$$0.2 \text{ kg} \times c_m \times 76\ °\text{C} = 0.4 \text{ kg} \times 4200 \text{ J/kg°C} \times 4\ °\text{C}$$

$$c_m = \frac{0.4 \text{ kg} \times 4200 \text{ J/kg°C} \times 4\ °\text{C}}{0.2 \text{ kg} \times 76\ °\text{C}} = 442.105 \text{ J/kg°C}$$

Therefore, the specific heat capacity of the metal is 442.11 J/kg°C.

Example 7.6

A block of aluminium of mass 0.5 kg at a temperature of 100 °C is dipped in 1.0 kg of water at 20 °C. Assuming that no thermal energy is lost to the environment, what will the temperature of the water be at thermal equilibrium? (Specific heat capacity of aluminium is 900 J/kg°C and specific heat capacity of water is 4200 J/kg°C.

Solution

Let θ_2 be the final temperature.

Change in the aluminium temperature, $\Delta\theta_1 = (100 - \theta_2)$ °C

Change in the temperature of the water, $\Delta\theta_2 = (\theta_2 - 20)$ °C

Heat lost by aluminium $Q_1 = m_A \times c_A \times \Delta\theta_1 = 0.5 \text{ kg} \times 900 \text{ J/kg °C} \times (100 - \theta_2)$ °C

Heat gained by water $Q_2 = m_w \times c_w \times \Delta\theta_2 = 1.0 \text{ kg} \times 4200 \text{J/kg °C} \times (\theta_2 - 20)$ °C

Heat lost by aluminium, Q_1 = Heat gained by water, Q_2

$$450 \times (100 - \theta_2) = 4200 \times (\theta_2 - 20)$$

By collecting like terms in one side, and then solving for final temperature, you get, $\theta_2 = 27.74$ °C.

Therefore, the final temperature of the mixture is 27.74 °C.

Example 7.7

A brass cylinder of mass 100 g was heated first to a temperature of 100 °C and quickly transferred to an aluminium calorimeter of mass 150 g containing 120 g of paraffin at 10 °C. If the final steady temperature of the paraffin after stirring was 25 °C, calculate the specific heat capacity of paraffin c_p. (Neglect the heat losses to the environment, and use specific heat capacity for brass, $c_b = 320$ J/kg°C and for aluminium, $c_a = 900$ J/kg°C).

Solution

Let c_p be the specific heat capacity of paraffin.

Heat gained by aluminium calorimeter and paraffin

$$Q_1 = m_a \times c_a \times (\theta_f - \theta_i) + m_p \times c_p \times (\theta_f - \theta_i)$$

$$= 0.15 \times 900 \times (25 - 10) + 0.12 \times c_p \times (25 - 10) = 2025 \text{ J} + 1.8c_p$$

Heat lost by brass cylinder

$$Q_2 = m_b \times c_b \times (100 - \theta_f) = 0.1 \text{ kg} \times 320 \text{ J/kg°C} \times (100 - 25) \text{ °C} = 2400 \text{ J}$$

From the principle of energy conservation,

heat gained by aluminium + paraffin = heat lost by the brass

$$2025 \text{ J} + 1.8c_p = 2400 \text{ J}$$

$$c_p = \frac{2400 \text{ J} - 2025 \text{ J}}{1.8 \text{ kg°C}} = 208.33 \text{ J/kg°C}$$

Therefore, the specific heat capacity of paraffin is 208.33 J/kg°C.

Exercise 7.1

1. Why is it so hard to keep your body warm if you are wet?
2. Explain the importance of high specific heat capacity of water.
3. The specific heat capacity of acetic acid is approximately half that of water. Suppose an equal mass of water at 20 °C and acetic acid at 80 °C are mixed in an insulated cup. What will the temperature of the mixture be at thermal equilibrium?
4. An insulated container holds 0.15 kg of water at 85 °C. What mass of water at 22 °C must be added into the water to cool it to a temperature of 60 °C?
5. A block of metal of mass 0.2 kg at a temperature of 100 °C is placed into 0.4 kg of water at 16 °C. If the final temperature of the metal and water is 20 °C, what is the specific heat capacity of the metal? Use specific heat capacity of water c_w = 4200 J/kgºC. Assume no energy losses.
6. A piece of copper of mass 40 g at 200 °C is placed in a copper calorimeter of mass 60 g containing 50 g of water at 10 °C. Assuming there are no heat losses, determine the final steady temperature after stirring. The specific heat capacity of copper is 390 J/kg°C and that of water is 4200 J/kgK.
7. If two objects P and Q are supplied with the same quantity of heat, the temperature change in P is observed to be twice that in Q. Suppose the masses of P and Q are the same, calculate the ratio of the specific heat capacity of P to Q.

Change of state

Have you ever asked yourself how water changes into ice? When changed to ice, is it still water? To answer these questions, we consider what happens when an object is heated or cooled. The temperature of an object rises when it is heated. However, at some point temperature does not change but the phase or state changes. At this point, a solid object changes to a liquid. If heating continues, the liquid will change to a gaseous state. Therefore, an object can exist in three states namely, solid, liquid or gas. The three states of matter are interchangeable through heating or cooling. Figure 7.2 illustrates these changes using water. It is worth noting that, recently the fourth state of matter called plasma has been discovered. However, this state will not be considered here because it is beyond this level.

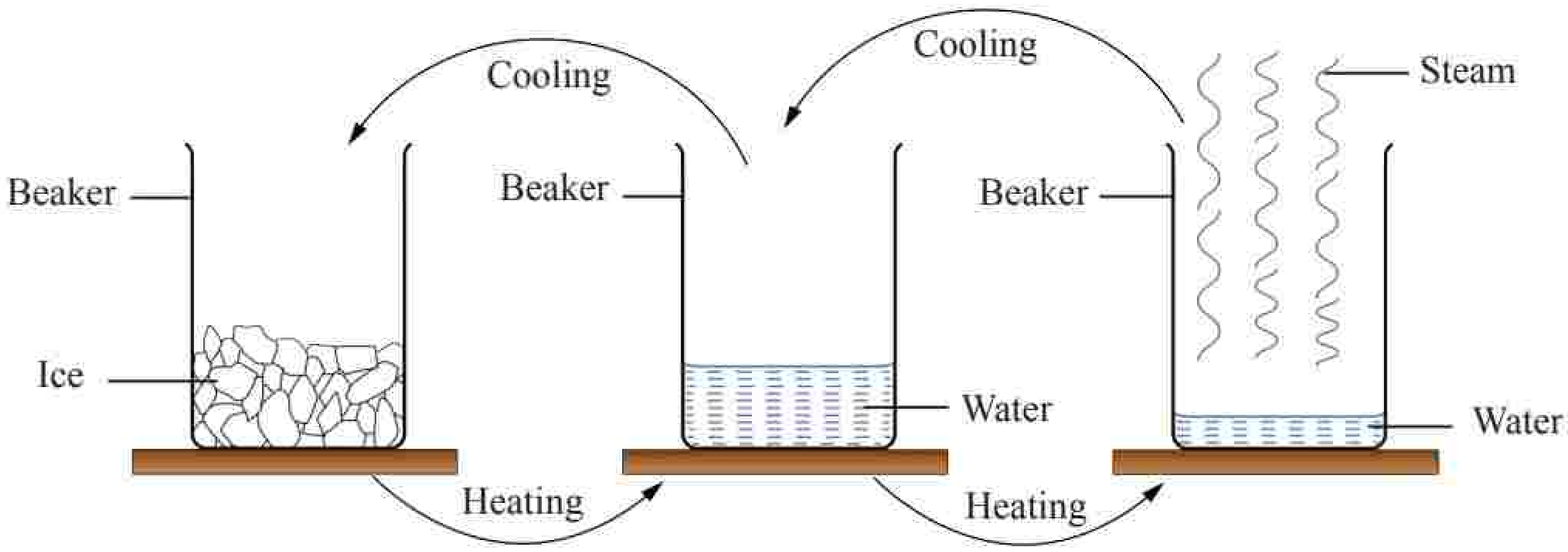


Figure 7.2: *The three states of water*

On heating ice (solid), it changes to water (liquid) at constant temperature which on further heating, the liquid changes to steam (gas) at a constant higher temperature. On cooling steam (gas), changes to water (liquid) and on cooling further, it changes to ice (solid). Generally, at a high temperature, most substances exist in the gaseous state, at intermediate temperature, they exist in liquid state and at the low temperature, they exist in solid state.

Kinetic theory and behaviour of particles of matter

What makes up matter? Why do some matters exist in solid, liquid or gaseous form? The kinetic theory of matter is useful in describing the properties of solids, liquids and gases at the molecular level. The way particles in matter are arranged and how they behave, determines whether the matter is solid, liquid or gas.

In solids, the intermolecular forces between neighbouring molecules are strong enough to keep them locked in position. This makes the solids to maintain their definite shape and size, and they are not easily compressed. Solids can change from solid state to liquid state through melting. The melting needs energy to weaken the force of attraction between molecules of a solid. When the force of attraction is weakened, molecules start moving about as in a liquid. A common example is when ice melts to form liquid water by heating as depicted in Figure 7.3.

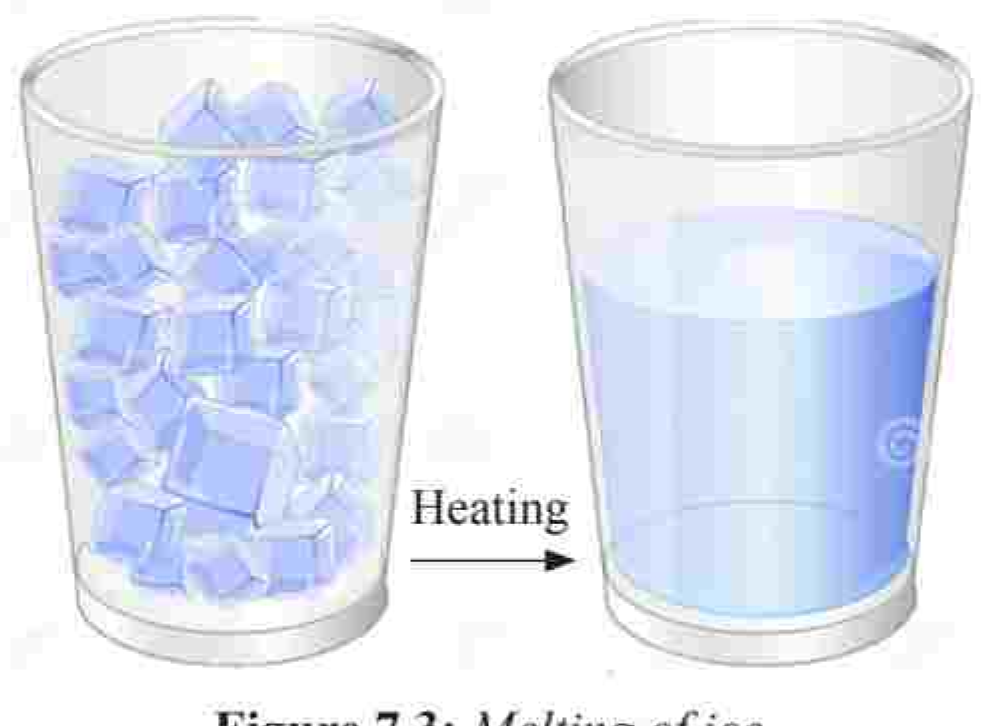


Figure 7.3: *Melting of ice*

To freeze liquid water into ice requires the energy in the liquid be removed. If liquid water is cooled, the particles of liquid lose energy, slow down and eventually stop moving and stick together to form a solid structure.

In liquids, particles possess a larger amount of kinetic energy than particles in a solid. Thus, the particles of a liquid are not in an ordered arrangement, and so a liquid takes the shape of a container in which it is placed. Liquid particles possess enough energy to move throughout a container and past one another. They do not have enough energy to escape the attraction between one another and as a result, they remain loosely connected. Liquids have indefinite shape, a definite volume and are not easily compressed. On heating a liquid, the kinetic energy increases and overcomes the force of attraction between the liquid particles. The particles then become free to move as in gases. The process of changing a liquid state into a gaseous state is called evaporation.

In gases, the particles of a gas possess a very large amount of kinetic energy and move very rapidly and randomly. They move so fast that they have weak attraction between one another. As a result of this kinetic energy, particles of a gas are separated by large distances between them. Just as with liquids, gases take on the shape of the container they are placed in. Gases have an indefinite shape, an indefinite volume and are easily compressed due to the large distance between particles. On cooling, the free molecules in a gas lose energy, slow down and get closer but not close as in a solid. The process of changing the gaseous state into liquid state is called condensation.

The melting and freezing point of a substance

When a solid is heated continuously, it reaches a point where the heat is absorbed but the temperature does not change until the substance has completely melted. This point is called the melting point. At the melting point, the increased thermal energy overcomes the cohesive force holding the molecules.

When a liquid is cooled, energy is released in the form of thermal energy. It reaches a point where the temperature stops changing (remains constant) but the liquid continues to lose energy. At this point, the liquid freezes to solid. This point is the freezing point.

Activity 7.4

Aim: To determine the melting point of naphthalene from its cooling curve

Materials: retort stand and clamp, test tube, beaker, thermometer, solid naphthalene, source of heat, tripod stand, wire gauze, water

Procedure

1. Put some crushed naphthalene in a test tube.
2. Place the test tube in a beaker as shown in Figure 7.4.

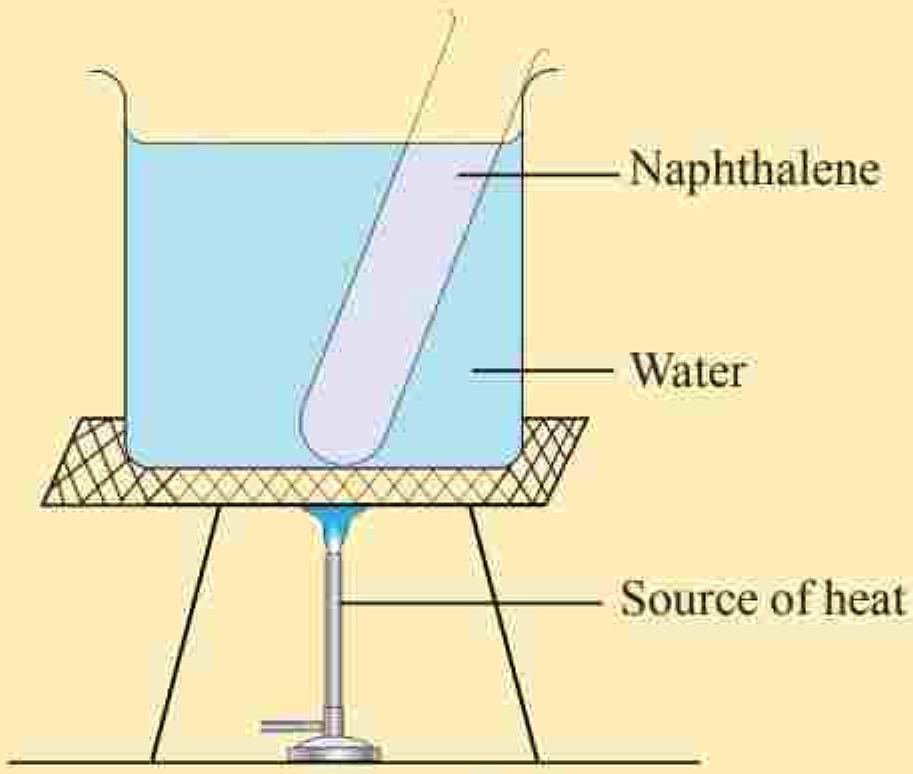


Figure 7.4

3. Heat the water until all the naphthalene has melted and its temperature has reached about 100 °C.
4. Remove the test tube from the beaker and suspend it to a retort stand, with a thermometer inserted in it as shown in Figure 7.5.
5. Gently continue to stir it with the thermometer as it cools and take readings.

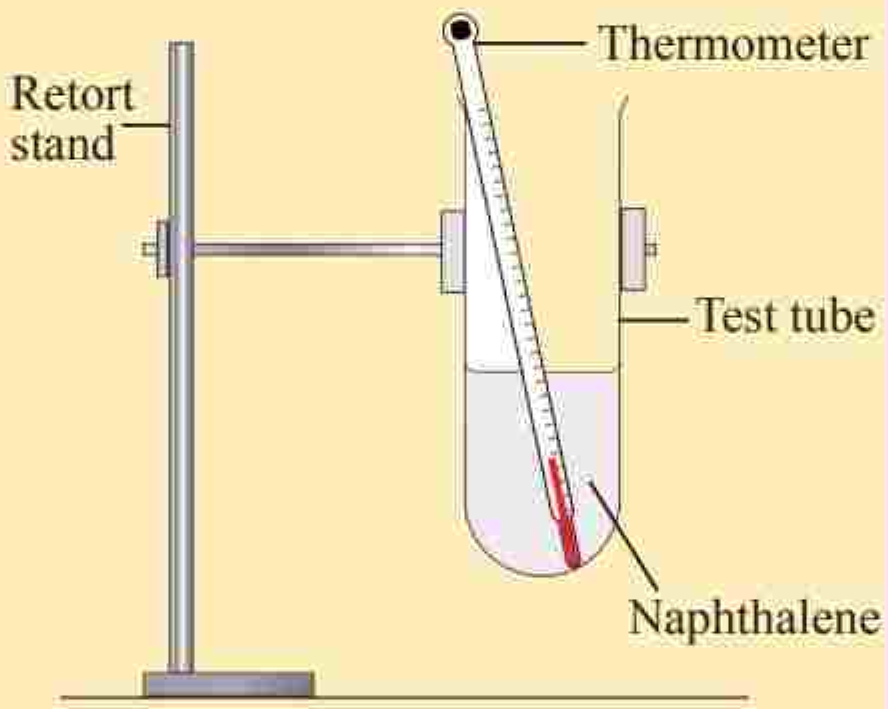


Figure 7.5

6. Record the temperature reading on the thermometer at thirty-seconds intervals until the temperature falls to about 30 °C.

Questions

(a) From the results of the experiment, plot a graph of temperature against time for the cooling of naphthalene.

(b) Explain the shape of the graph in (a) above.

As the naphthalene is cooled, its temperature decreases gradually. After some time, the thermometer reading remains constant for few minutes. At this constant temperature, the naphthalene is solidifying, although heat is being lost by convection and radiation all the time. When all the naphthalene has solidified, its temperature starts to decrease again. Figure 7.6 shows the cooling curve of naphthalene.

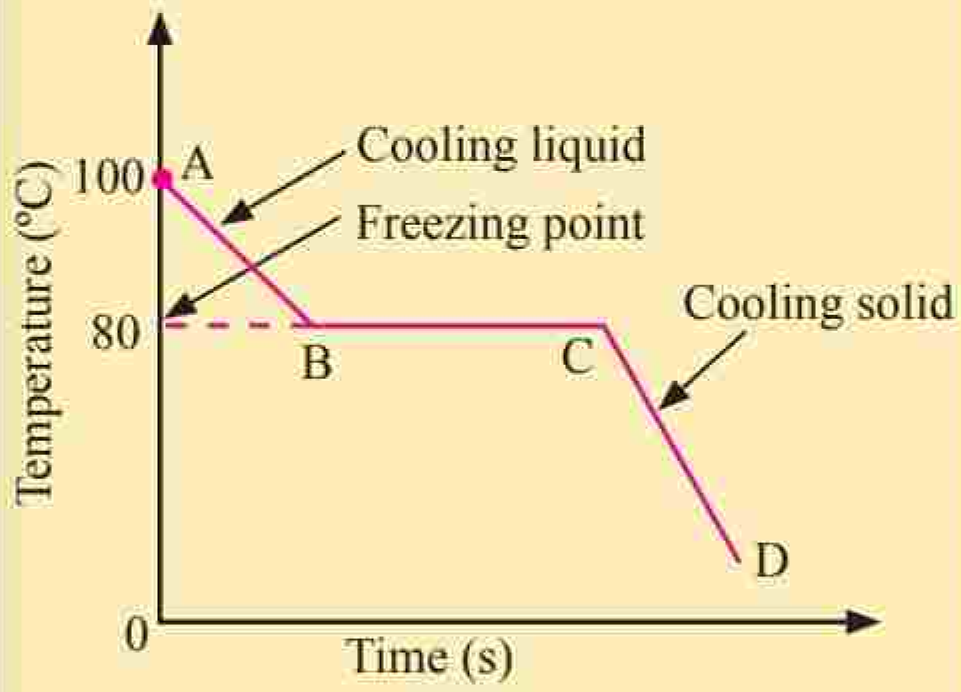


Figure 7.6: *Cooling curve of naphthalene*

From A to B in Figure 7.6, the temperature of naphthalene falls until the naphthalene begins to solidify. The temperature then remains constant from B to C up to the time when solidification is complete at C. The temperature starts to decrease again from C to D. For a pure substance, the flat part B to C occurs at a temperature called the freezing point. This is temperature at which a liquid changes into a solid without a temperature change. The freezing point of a pure substance is the same as its melting point. In the case of pure water, melting and freezing point occur at 0˚C. The melting point of some substances are shown in Table 7.2.

Table 7.2: Melting point of different substances

Substance	Melting point (ºC)
Iron	2080
Oxygen	-218
Mercury	-39
Alcohol	-115
Tungsten	3920
Helium	-273

Effect of impurities on the freezing point of a substance

We have noted that, pure water freezes at 0 °C. This implies that, ice melts at this temperature. Suppose a small amount of impurity is introduced to ice, will the freezing point and boiling point be lowered or raised?

Activity 7.5

Aim: To investigate the effects of impurities on the freezing point of a substance

Materials: thermometer, beaker, retort stand and clamp, funnels, ice, salt

Procedure

1. Put some crushed ice in a funnel.
2. Insert a thermometer in the funnel containing crushed ice as shown in Figure 7.7.

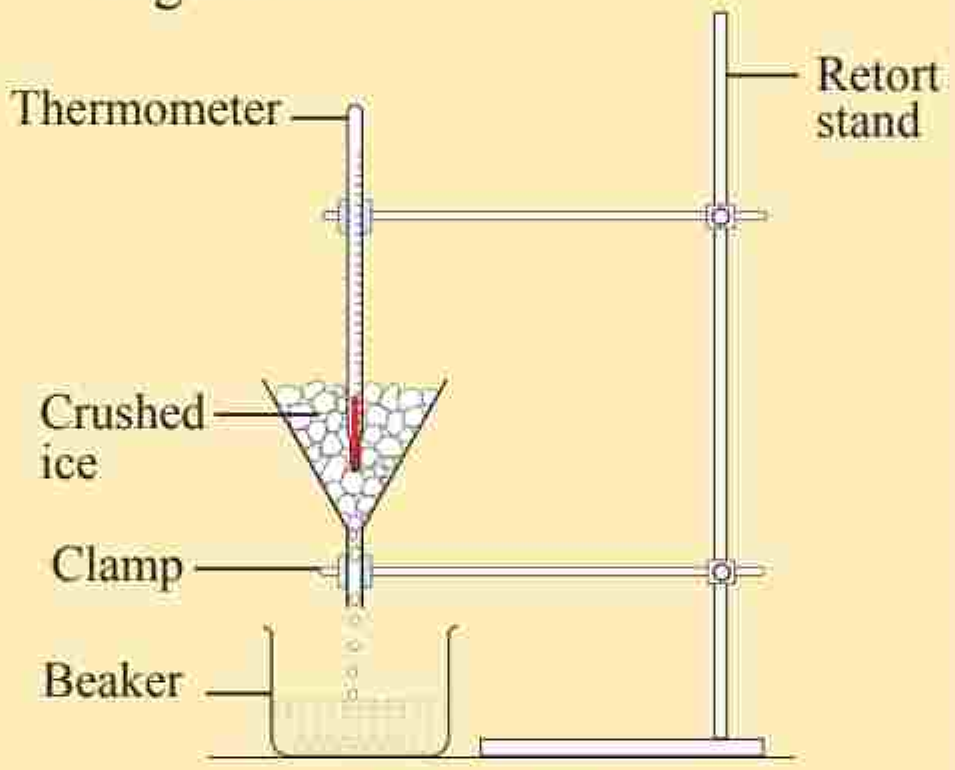


Figure 7.7

3. Stir well until the ice starts to melt.
4. Note and record the temperature of the ice and water in the funnel.
5. Replace the crushed ice with a mixture of crushed ice and salt.
6. Stir until the ice starts to melt.
7. Note and record the temperature of the mixture of ice and salt in the funnel.

Questions

(a) At what temperature did the pure ice start to melt?
(b) At what temperature did the ice mix with salt start to melt?
(c) Compare the melting point of the pure ice with that of the ice-salt mixture.

The melting point of the mixture of ice and salt is lower than that of pure ice. Impurities such as salt lower the melting point of a substance, this is the same as lowering the freezing point. To understand why, consider what happens during the process of freezing a salt solution. In this case, salt is considered as the impurity. The particles of the impurity get between the particles of the liquid and disrupt the organization of molecules in the solid crystals. The introduced impurities weaken the bonds that hold the molecules of the solid. Therefore, less energy is required to separate the molecules in the solid. Thus, the ice containing the impurities melts to a liquid at a lower temperature than the pure ice does. This is referred to as freezing point depression.

Effect of pressure on freezing and melting of a substance

Freezing and melting are two opposing processes that involve state transitions. Variations in pressure can affect both melting and freezing. A solid's volume, on the other hand, is simply the volume of an equal mass of the corresponding liquid. Thus, the effect of pressure on both melting and freezing points is very small unless the pressure change is very large. At the freezing point, however, the rate of freezing equals the rate of melting. By observing the influence of pressure on the melting of a solid, one can study the effect of pressure on the freezing of a substance.

Activity 7.6

Aim: To investigate the effect of pressure on melting of ice

Materials: ice block (thick), two slotted 1 kg masses, a thin copper wire, wooden blocks, two weights of 2 kg each

Procedure

1. Place an ice block of about $10 \text{ cm} \times 15 \text{ cm} \times 5 \text{ cm}$ between two wooden supports as shown in Figure 7.8.

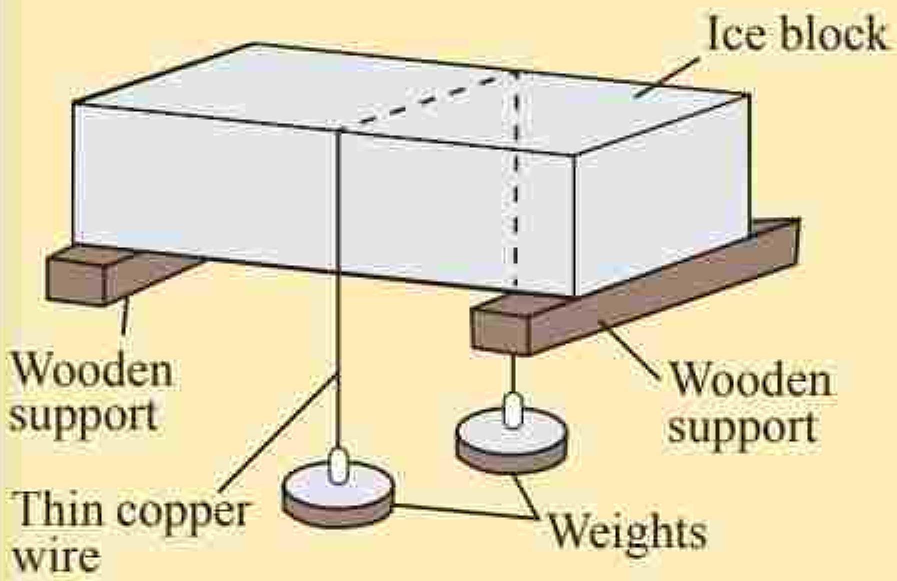


Figure 7.8

2. Attach heavy weights of about 2 kg each at each end of the copper wire.
3. Pass the copper wire with the masses attached to it over the block of ice as shown in Figure 7.8.
4. Observe what happens to the ice block.
5. Repeat step 2-4 using 1 kg.

Question

What have you observed when the 2 kg and 1 kg weights were suspended over the ice block using a copper wire?

The wire cuts through the ice block without breaking it. The weights exert pressure on the wire which exerts pressure on the ice block. This causes the ice under the wire to melt hence allowing the wire to cut through. As the wire moves through the ice, the pressure above is lowered and the molten ice (water) re-freezes. On the other hand, when the 2 kg is used, melting of ice occurs faster than when 1 kg is used. This is because heavy weights exert higher pressure on ice resulting to higher rate of melting and freezing. The process of the liquid re-freezing above the wire is called regelation.

Regelation

When pressure is exerted on ice at constant temperature, the ice melts and re-freezes as the pressure is released. The phenomenon of melting and re-freezing under the influence of pressure is known as regelation. This occurs because pressure lowers the melting point of ice. There are many applications of the knowledge of regelation, one being making snowballs. This is done by compressing snow resulting to slight melting of the ice crystals. This is done by releasing a compressing force on the snow causing the ice to re-freeze and binds the snow forming a snowball.

Boiling and evaporation

As a liquid is heated, its molecules gain kinetic energy. The molecules at the surface of the liquid with more kinetic energy move faster and are able to overcome the intermolecular forces holding them together and hence escape. At boiling point, the vapour pressure of liquid becomes equal to atmospheric pressure. A pure substance has an exact boiling point. At boiling point, heat supplied is used to change water from liquid to vapour state. It does not raise the temperature. Table 7.3 shows the boiling points of some substances.

Table 7.3: Boiling points of some substances

Substance	Boiling Point (ºC)
Helium	–269
Hydrogen	–253
Oxygen	–183
Ethyl alcohol	78.4
Benzene	80.2
Water	100
Mercury	357
Aluminium	2467
Copper	2567
Iron	2750

Activity 7.7

Aim: To investigate the effect of impurities on the boiling point of water

Materials: two round-bottomed flasks, two thermometers, two corks or rubber bungs, pure water, salt solution, retort stand

Procedure

1. Pour some pure water into one of the flasks so that it is half filled.
2. Pour some salt solution into another flask so that it is half filled.
3. Insert the thermometers through the corks.
4. Heat the flask containing the pure water until water boils.
5. Measure and record the temperature at which the water boils.
6. Repeat steps 4-5 using the salt solution.

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Question

Compare the boiling points of the two substances.

The temperature at which pure water boils is lower than that at which the salt solution boils. This is because the impurity (salt) particles alter the intermolecular force between pure water particles such that more energy is required for them to escape from the solution surface.

Activity 7.8

Aim: To investigate the effect of increasing pressure on the boiling point of water

Materials: round-bottomed flask, thermometer, cork or rubber bung, L-shaped glass tube, rubber tube, source of heat, retort stand

Procedure

1. Half-fill the round-bottomed flask with water.
2. Fit the flask with the cork and insert the thermometer and the L-tube.
3. Connect a rubber tube to the glass tube.
4. Place the flask as shown in Figure 7.9. Heat and allow the water to boil.

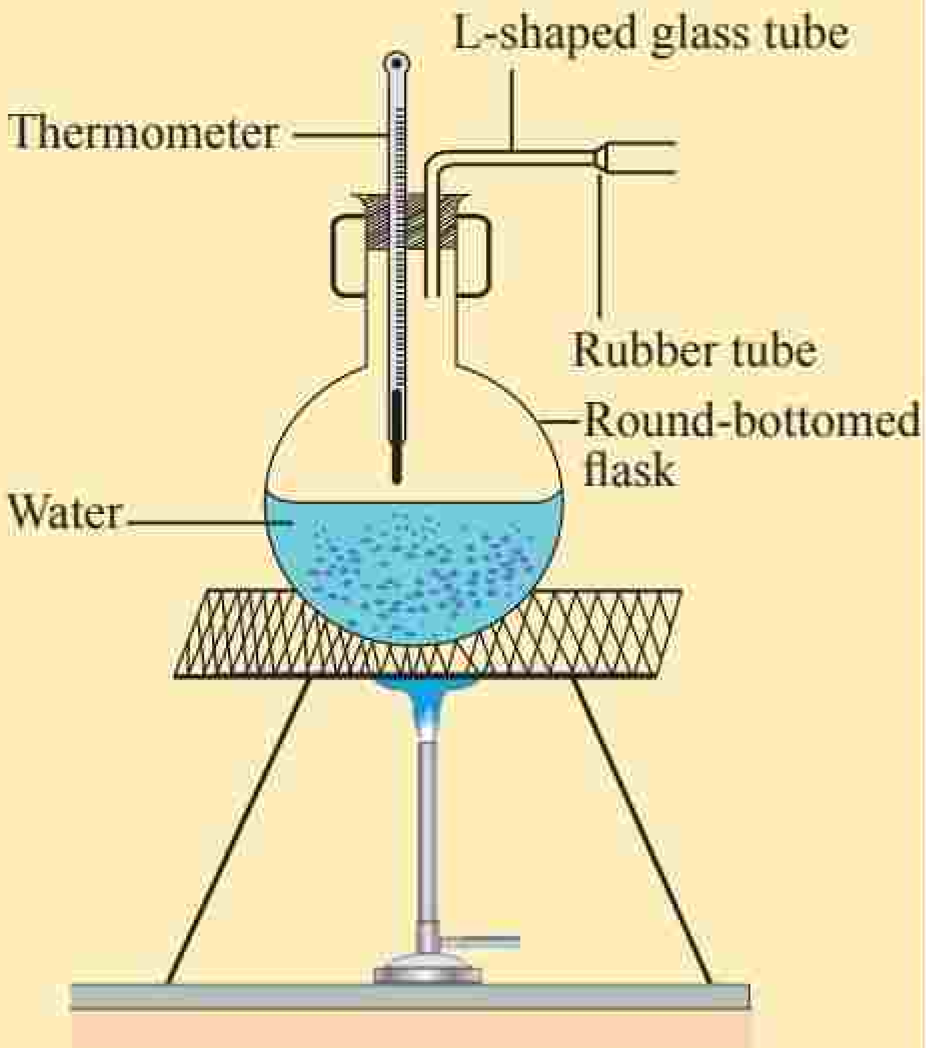


Figure 7.9

5. While the water is boiling, close the open end of the rubber tube by squeezing it. Note the thermometer reading.

Questions

(a) What happens to the temperature reading when the open end of the rubber tube was closed?

(b) Explain your observation in (a) above.

When the tube is squeezed, the temperature reading of a thermometer increases. Squeezing a rubber tube increases the pressure inside the flask. An increase in pressure in turn increases the boiling point of water. This shows that, an increase in pressure raises the boiling point of a liquid. The knowledge of the effect of increasing pressure on boiling point is mainly applied in pressure cookers (Figure 7.10) for cooking rapidly. It is also applied in an autoclave for sterilizing surgical equipment and other pharmaceutical items.

Figure 7.10: *Pressure cooker*

Activity 7.9

Aim: To investigate the effect of reducing pressure on the boiling point of water

Materials: round-bottomed flask, thermometer, cork or rubber bung, L-shaped glass tube, rubber tube, source of heat, retort stand

Procedure

1. Half-fill the flask with water.
2. Insert the thermometer and glass tube through the cork and cover the flask.
3. Heat the water up to 70 °C to 75 °C.
4. Remove the flask from the source of heat, clip the rubber tube and invert the flask as shown in Figure 7.11.

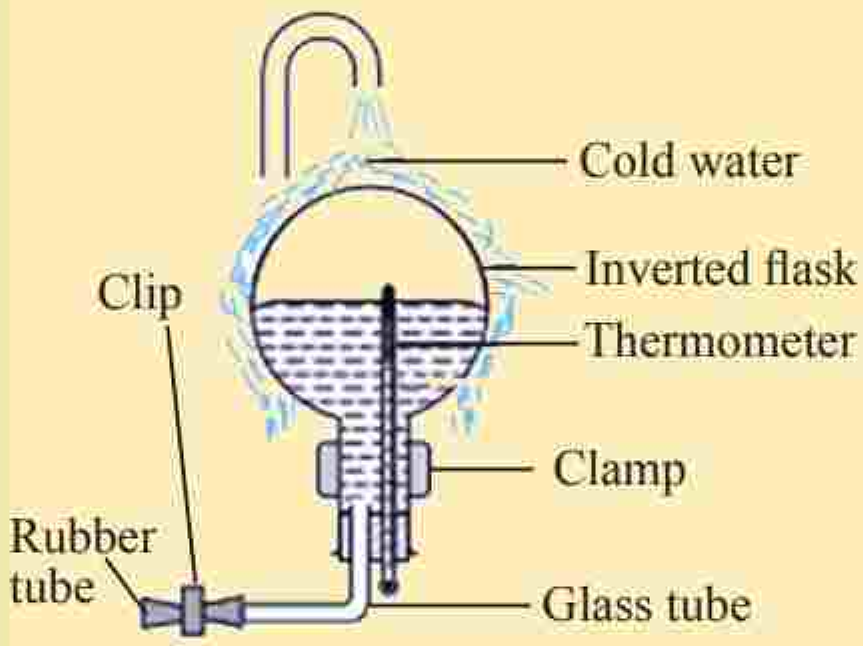


Figure 7.11

5. Run cold water over a flask and observe the reading on a thermometer. Also, note what happens to the water in a flask.

Question

Explain what happened to the thermometer reading on running cold water over the boiling water.

The reading on a thermometer decreases but water in a flask continues to boil. When cold water is run over the flask, a water vapour above the water condenses, thus reducing the pressure inside the flask. Water boils at a temperature below its normal boiling point. If the cooling is continued, the boiling point can drop to about 40 °C. A decrease in pressure, therefore, lowers the boiling point of a liquid. Table 7.4 shows boiling point of pure water at different altitudes, hence different atmospheric pressure.

Table 7.4: Boiling point of water at different altitudes

Altitudes (m)	Pressure (mmHg)	Boiling Point (°C)
300	732	99.0
600	706	98.0
900	681	97.0
1200	656	96.0
1500	632	95.0
1 800	609	93.9
2 100	586	92.9
2 400	565	91.9
2 700	543	90.8
3 000	523	89.8

It is important to note that, at sea level water boils at 100 °C and for every 152. 4 m increase in elevation, boiling point of water is lowered by approximately 0.5 °C. The boiling point of a liquid depends on the vapour pressure of liquid. At a given temperature, different liquids have different vapour pressure because of the strength of their intermolecular forces. You learnt that, for molecules to escape into vapour phase, they must move fast with more kinetic energy to overcome the intermolecular forces holding them at the surface. This implies that, liquid with weaker intermolecular forces will allow more molecules to escape into vapour phase. It can be said that, this liquid has higher vapour pressure. On the other hand, liquids with strong intermolecular forces possess lower vapour pressure. This is because few molecules will posses enough kinetic energy to overcome intermolecular forces and escape into vapour phase at a given temperature.

Evaporation is the process through which a liquid changes to vapour (gas) at a temperature even below its boiling point. When a liquid is heated, its molecules gain kinetic energy. The molecules at the surface which have gained more kinetic energy move faster and are able to escape from the liquid's surface. This takes place at all temperatures even below a boiling point. Water evaporates from a pool more rapidly when it is windy, sunny and less humid. Evaporation is different from boiling. Table 7.5 shows the differences between boiling and evaporation.

Table 7.5: Differences between boiling and evaporation

Basic comparison	Boiling	Evaporation
Meaning	Is vapourisation process where liquid changes into gas system when heated	Is a process for which liquid changes its form into gas even without heating
Temperature	Occurs at a definite temperature which is the boiling point	Occurs at a temperature even below the boiling point
Occurrence	Occurs throughout the liquid with the formation of bubbles	Occurs at the surface of the liquid only and no bubbles are formed
Temperature effect to the liquid	Has no cooling effect (temperature remain constant)	Has a cooling effect (Temperature decreases)
Occurring rate	Takes place rapidly	Takes place slowly

Latent heat of fusion and vapourisation

We have seen that, objects change their states when heated or cooled. Phase change may be from solid to liquid or liquid to vapour and vice versa. As an object changes its phase, some heat is released or absorbed. The heat required to convert a given mass of a substance from one state to another is termed as the latent heat, which means concealed or hidden heat. For example, in order to convert 1 kg of water at its boiling point to steam at the same temperature, it requires 2260 kJ.

According to the kinetic theory of matter, when a solid object is heated, its temperature increases. As the temperature increases, it reaches a point where an object melts. At the melting point, the temperature of the object does not change even though it absorbs heat. Heat is absorbed by a solid during melting and an equal amount of thermal energy is liberated by the liquid during freezing. The heat absorbed or liberated during this change of state is called the latent heat of fusion.

During melting, the absorbed thermal energy increases the kinetic energy of the particles of the solid, enabling them to overcome the forces that hold the molecules in their fixed positions. This gives them freedom of motion required in liquid form. At boiling and melting points, the heat supplied does not increase the kinetic energy of the particles, instead, it breaks the intermolecular force of attraction so that the particles are set free and an object changes its state. If the molten liquid of mass, *m,* is cooled to a solid state, it gives up its latent heat of fusion. The energy required to change the phase of a pure substance of mass, *m*, is given by,

$$Q = mL$$

where L is the specific latent heat.

When the thermal energy is added or removed from objects with different properties, the objects respond differently as they change phase. This also applies to the amount of energy transferred in different objects as it is the function of the amount of substance of the object. For example, it takes a large amount of energy to melt a frozen lake than to melt an ice cube. Suppose a system containing a substance in two phases in equilibrium, such as water and ice, has an initial mass, m_i, of water, and the final mass of water is m_f at the same temperature. If the energy, Q, enters the system, then the amount of ice that melts is equal to the change in the amount of new water. Therefore, the latent heat for this phase change is given by,

$$Latent\ heat(L) = \frac{Heat\,(Q)}{Change\,in\,mass\,(m_f - m_i)}$$

$$= \frac{Heat\,(Q)}{Change\,in\,mass\,(\Delta m)}$$

$$L = \frac{Q}{\Delta m}$$

The value L represents the specific latent heat of a substance. It is also represented by the symbol L_f. If the entire amount of ice undergoes a phase change, change in mass(Δm), $\Delta m = m_f - 0$. The quantity of heat required to change completely a unit mass (1 kg) of a solid to liquid at its melting point is called the specific latent heat of fusion.

The SI unit for the specific latent heat is joules per kilogram (J/kg). The specific latent heat of fusion of ice is 3.34×10^5 J/kg. The quantity of heat energy required to change a unit mass of a solid into liquid at its melting point depends on the material of the solid. Latent heat of fusion of different substances are shown in Table 7.6.

Table 7.6: Melting/freezing points and specific latent heats of fusion of some substances at S.T.P

Substance	Melting or freezing point (°C)	Specific latent heat of fusion ($\times 10^5$ J/kg)
Aluminium	659	3.96
Copper	1086	2.06
Iron	1535	2.47
Water	0	3.34
Mercury	–39	0.11
Ethyl alcohol	–117	1.05

Similarly, for a liquid to change to a gas, energy must be absorbed by the liquid during vapourisation. This energy is equal in amount to heat liberated by the vapour during condensation. The heat absorbed or liberated here is called the latent heat of vapourisation.

Latent heat of vapourisation is the heat required to cause vapourisation or condensation. The heat supplied at this point does not go towards increasing the temperature of the liquid. Instead, this energy is used to overcome the molecular forces of attraction between the particles of the liquid and bring them to a vapour state. Figure 7.12 shows a relationship between the temperature and the heat supplied to an object.

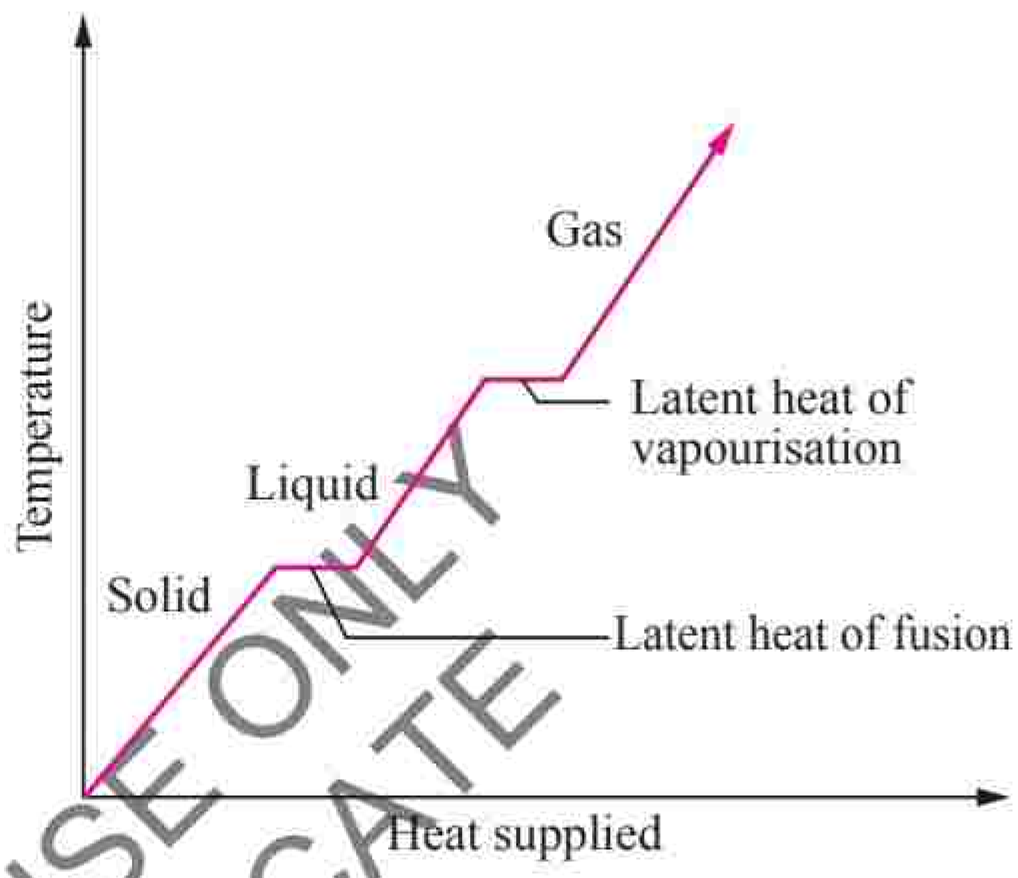


Figure 7.12: *Latent heat*

Activity 7.10

Aim: To demonstrate the latent heat of vapourisation of water

Materials: beaker, stopwatch, water, source of heat, thermometer

Procedure

1. Pour water into a beaker.
2. Heat water in a beaker and record its temperature after every minute until it starts to boil.
3. Continue heating and note the temperature two minutes after the water has started to boil.
4. Record your results in a table similar to Table 7.7.

Table 7.7

Time (min)	0	2	4	6	8	10
Temperature (ºC)						

Questions

(a) Using the results obtained, plot a graph of temperature against time.

(b) Explain the shape of the graph.

As the water is heated, its temperature rises. However, once the boiling point is reached, the temperature remains constant even though heat energy is still being supplied to water. A sketch of the graph of temperature against time is shown in Figure 7.13.

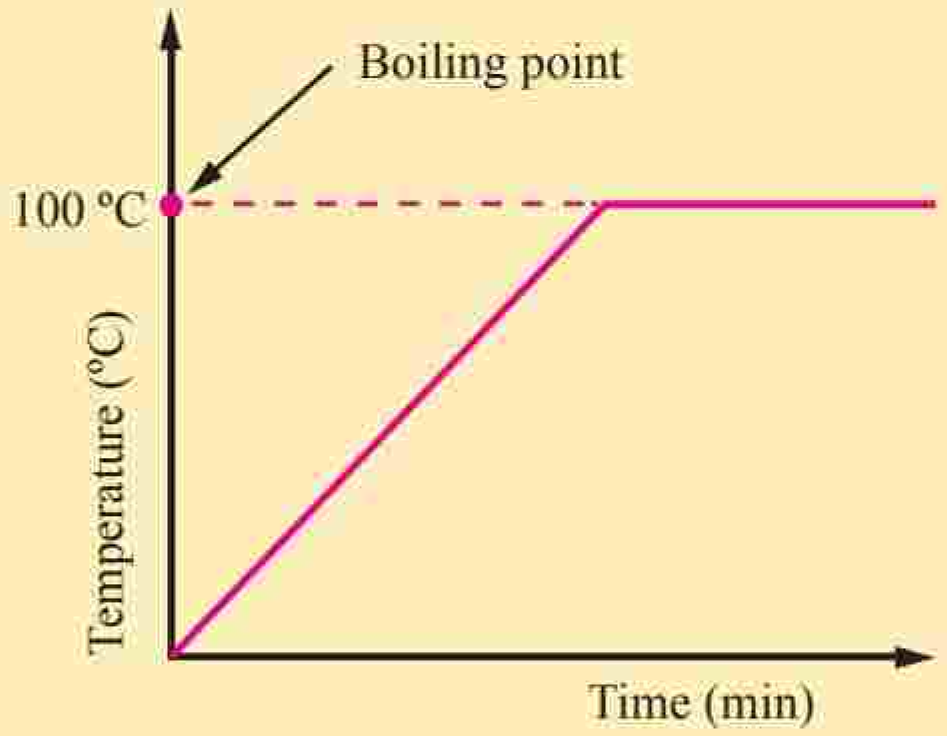


Figure 7.13: *Temperature–time graph for heating water*

The portion of the graph with zero gradient represents the boiling point of the material. However, the boiling point depends on the place where the experiment was carried out. This is because the boiling point of the water is affected by atmospheric pressure. When the atmospheric pressure is high, the water molecules require additional heat so as to acquire the velocity necessary to escape from the surface. At low atmospheric pressure, the molecule escapes at low speed, thus, lowering the boiling point. That is why, it is difficult to boil eggs at the top of Mount Kilimanjaro. When water vapourises to form steam, it uses up latent heat of vapourisation.

Example 7.8

How much heat is required to raise the temperature of 1.5 kg of ice at −10 °C to steam at 120 °C? (The specific heat capacities of ice, water and steam are: 2144 J/kg°C, 4186 J/kg°C and 2010 J/kg°C respectively. Latent heat of fusion of ice and vapourisation of steam are 3.35×10^5 J/kg and 2.27×10^6 J/kg, respectively.

Solution

The process takes place in stages as shown in Figure 7.14.

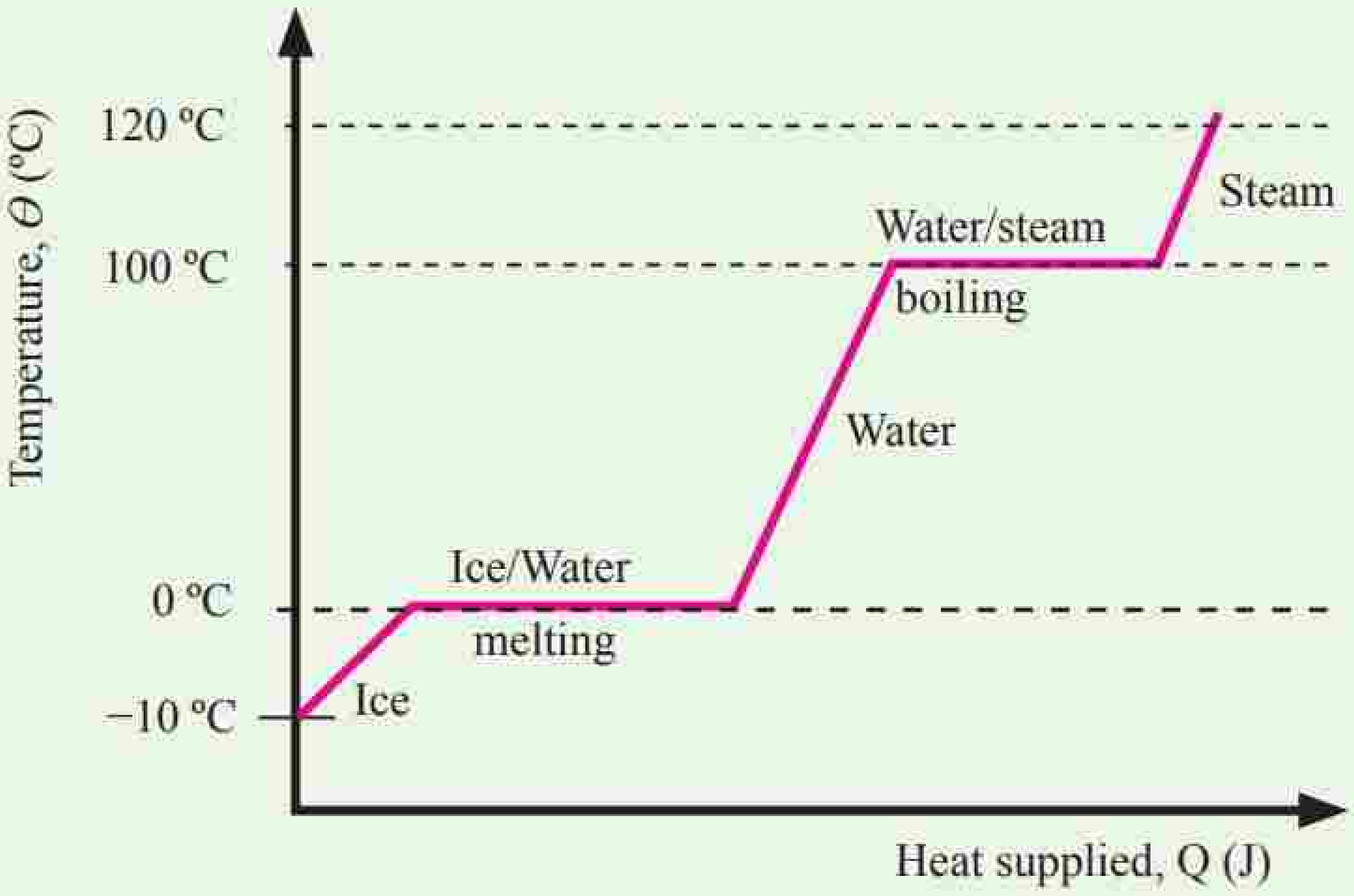


Figure 7.14

Stage 1: Energy required to raise the temperature of ice from –10 °C to 0 °C, Q_1

Mass of ice, $m_{ice} = 1.5$ kg

Specific heat capacity of ice, $c_{ice} = 2144$ J/kg°C

$\text{Change in tempearture, } \Delta\theta_1 = \text{Final temperature} - \text{Initial temperature}$

$\Delta\theta_1 = 0\ °\text{C} - (-10\ °\text{C}) = 10\ °\text{C}$

$Q_1 = m_{ice} \times c_{ice} \times \Delta\theta_1$

$Q_1 = 1.5 \text{ kg} \times 2144 \text{ J/kg°C} \times 10\ °\text{C} = 3.216 \times 10^4 \text{ J}$

Stage 2: Energy required to melt the ice, Q_2 at constant temperature, i.e., $0\ °\text{C}$.

$Q_2 = m_{ice} L_f$

$Q_2 = 1.5 \text{ kg} \times 3.35 \times 10^5 \text{ J/kg} = 5.025 \times 10^5 \text{ J}$

Stage 3: Energy required to raise the temperature of the water from 0 °C to 100 °C, Q_3

$Q_3 = m_w \times c_w \times \Delta\theta_2$

$Q_3 = 1.5 \text{ kg} \times 4186 \text{ J/kg°C} \times 100\ °\text{C} = 6.279 \times 10^5 \text{ J}$

Stage 4: Energy required to convert the water to steam, Q_4

$Q_4 = m_w L_v$

$Q_4 = 1.5 \text{ kg} \times 2.27 \times 10^6 \text{ J/kg} = 3.405 \times 10^6 \text{ J}$

Stage 5: Energy required to raise the temperature of the steam from 100 °C to 120 °C (Q_5)

$$Q_5 = m_s \times c_s \times \Delta\theta_3$$

$$\Delta\theta_3 = (120 - 100)\,°\text{C} = 20\ °\text{C}$$

$$Q_5 = 1.5\ \text{kg} \times 2010\ \text{J/kg°C} \times 20\ °\text{C} = 6.03 \times 10^4\ \text{J}$$

The total heat required for the entire process (Q_T) is given by,

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_T = (3.216 \times 10^4 + 5.025 \times 10^5 + 6.279 \times 10^5 + 3.405 \times 10^6 + 6.03 \times 10^4)\ \text{J} = 4.63 \times 10^6\ \text{J}$$

The total heat required for the process is 4.63×10^6 J.

Note: The most amount of heat was required in step 4 to convert the water to steam. Steam at 100 °C has much more thermal energy than water at 100 °C. This is why steam is used in engines to convert thermal energy to mechanical energy.

Example 7.9

A copper calorimeter together with a stirrer of total heat capacity 60.5 J/°C, contains 300 g of water at 15 °C. Dry steam at 100 °C is passed in while the water is stirred until the whole system reaches a temperature of 30 °C. Calculate the mass of water condensed. Use specific heat capacity of water as 4200 J/kg°C and specific latent heat of vapourisation of water as 2260000 J/kg.

Solution

Let C_c be the heat capacity of copper and Q_1 be heat gained by copper and water.

Heat gained by copper calorimeter and stirrer $= C_c \times (\theta_f - \theta_i)$

Heat gained by water $= m_w \times c_w \times (\theta_f - \theta_i)$

Thus,

$$Q_1 = C_c \times (\theta_f - \theta_i) + m_w \times c_w \times (\theta_f - \theta_i)$$

$$= 60.5\ \text{J/°C} \times (30\ °\text{C} - 15\ °\text{C}) + 0.3\ \text{kg} \times 4200\ \text{J/kg°C} \times (30\ °\text{C} - 15\ °\text{C})$$

$$= 19807.5\ \text{J}$$

Again

Let m_s be the mass of condensed steam and Q_2 be heat lost by steam.

Q_2 = Heat lost when steam is at 100 °C + Heat lost when steam is condensed to 30 °C

But

Heat lost when steam is at 100 °C $= m_s L_v$

Heat lost when steam is condensed from 100 °C to 30 °C $= m_s \times c_w \times (\theta_i - \theta_f)$

Thus,

$$Q_2 = m_s L_v + m_s \times c_w \times (\theta_i - \theta_f)$$

$$Q_2 = m_s \times \left[2260000 \text{ J/kg} + 4200 \text{ J/kg°C} \times \left(100 \text{ °C} - 30 \text{ °C}\right)\right]$$

$$= 2554000 m_s \text{ J/kg}$$

From the principle of energy conservation,

Heat gained by copper + stirrer + water = Heat lost by steam + condensed water

$$19807.5 \text{ J} = 2554000 m_s \text{ J/kg}$$

$$m_s = \frac{19807.5 \text{ J}}{2554000 \text{ J/kg}} = 7.76 \times 10^{-3} \text{ kg or } 7.76 \text{ g}$$

Therefore, the mass of water condensed is 7.76×10^{-3} kg or 7.76 g.

Example 7.10

Naphthalene of mass 0.5 kg contained in a copper calorimeter of mass 0.4 kg is melted in a waterbath and raised to a temperature of 100 °C. Calculate the total heat given out when the calorimeter and its contents are allowed to cool to room temperature of 30 °C. Neglect the heat losses by evaporation during the heating process. Use the following information: Melting point of Naphthalene is 80 °C, the specific heat capacity of Naphthalene for both solid and liquid is 2100 J/kg°C and specific latent heat of fusion 170000 J/kg°C. For copper, specific heat capacity is 390 J/kg°C.

Solution

Heat given out when Naphthalene cools from 100 °C to 80 °C,

$$Q_1 = m_n c_n (100 \text{ °C} - 80 \text{ °C})$$

$$= 0.5 \text{ kg} \times 2100 \text{ J/kg°C} \times (100 \text{ °C} - 80 \text{ °C})$$

$$= 21000 \text{ J}$$

Heat given out when copper calorimeter cools from 100 °C to 80 °C,

$$Q_2 = m_c c_c (100 \text{ °C} - 80 \text{ °C})$$

$$= 0.4 \text{ kg} \times 390 \text{ J/kg°C} \times (100 \text{ °C} - 80 \text{ °C})$$

$$= 3120 \text{ J}$$

Heat given out when Naphthalene freezes at 80 °C, $Q_3 = m_n L_f$

$= 0.5 \text{ kg} \times 170000 \text{ J/kg}$

$= 85000 \text{ J}$

Heat given out when Naphthalene cools from 80 °C to 30 °C,

$Q_4 = m_n c_n \Delta\theta$

$= 0.5 \text{ kg} \times 2100 \text{ J/kg°C} \times (80 \text{ °C} - 30 \text{ °C})$

$= 52500 \text{ J}$

Heat given out when copper calorimeter cools from 80 °C to 30 °C,

$Q_5 = m_c c_c \Delta\theta$

$= 0.4 \text{ kg} \times 390 \text{ J/kg°C} \times (80 \text{ °C} - 30 \text{ °C})$

$= 7800 \text{ J}$

Thus, total heat given out,

$Q_T = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$= 21000 \text{ J} + 3120 \text{ J} + 85000 \text{ J} + 52500 \text{ J} + 7800 \text{ J}$

$= 169420 \text{ J}$

Therefore, the total heat given out is 169420 J.

Example 7.11

A piece of lead of mass 500 g initially at room temperature of 28 °C is to be melted. If the melting point of lead is 328 °C.

(a) Find the heat needed to melt the lead completely. Given, the specific heat capacity of lead is 130 J/kg°C and its specific latent heat of fusion is 270000 J/kg.

(b) If the heat is supplied to the lead at the rate of 50 J/s,

(i) find the time taken to bring the lead to its melting point.

(ii) what extra time is required for it to melt?

Solution

Given

$L_f = 270000 \text{ J/kg}$, $c_{Pb} = 130 \text{ J/kg°C}$, $\Delta\theta = 328 \text{ °C} - 28 \text{ °C} = 300 \text{ °C}$

(a) Let the total heat supplied to the lead, be Q,

Q = heat required to raise the temperature of lead from 28 °C to 328 °C + heat required to melt the lead at constant temperature.

$$Q = m_{Pb}c_{Pb}(\theta_f - \theta_i) + m_{Pb}L_f$$

$$= 0.5 \text{ kg} \times 130 \text{ J/kg°C} \times 300 \text{ °C} + 0.5 \times 270000 \text{ J/kg}$$

$$= 19500 \text{ J} + 135000 \text{ J}$$

$$= 154500 \text{ J}$$

Therefore, the total heat supplied is 154500 J.

(b) The rate of heat supplied, $P = 50$ J/s

(i) Let t_1 be the time taken to reach the melting point. From the definition of power.

$$t_1 = \frac{Q_1}{P} = \frac{154500 \text{ J}}{50 \text{ J/s}} = 3090 \text{ s or } 51.5 \text{ minutes}$$

(ii) Let t_2 be the time taken to melt the lead. From the definition of power.

$$t_2 = \frac{Q_2}{p} = \frac{135000 \text{ J}}{50 \text{ J/s}} = 2700 \text{ s or } 45 \text{ minutes}$$

Activity 7.11

Aim: To determine the specific latent heat of vapourisation of water

Materials: water, thermometer, copper calorimeter, stirrer, steam trap, beam balance, L-shaped glass tube

Procedure

1. Put cold water of known mass into a calorimeter.
2. Record the temperature of the water and then place the calorimeter to its jacket.
3. Set the apparatus as shown in Figure 7.15.

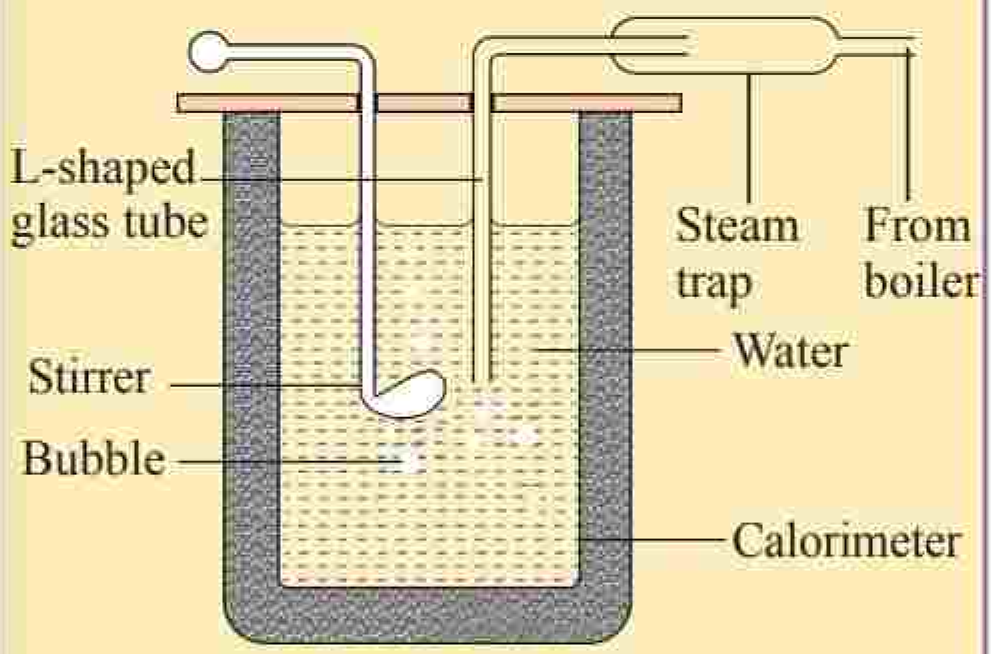


Figure 7.15

1. Pass steam from a boiler into the cold water using the glass tube fitted with a steam trap.

2. Allow the steam to run for about five minutes and then remove the steam pipe.
3. Stir the water gently and record the new temperature.
4. Remove the calorimeter from the jacket and measure its mass (including the contents).
5. Obtain the mass of steam absorbed.

Questions

(a) Determine the mass of steam absorbed.

(b) Determine the specific latent heat of vapourisation of water.

From the data collected, the specific latent heat of vapourisation can be determined as follows:

Mass of calorimeter and stirrer = m_1 kg

Mass of cold water = m_2 kg

Mass of steam absorbed = m_3 kg

Temperature of cold water $= \theta_1$ °C

Final temperature of the mixture $= \theta_2$ °C

Specific heat capacity of copper, $c_c =$ 390 J/kgK

Specific heat capacity of water, $c_w =$ 4200 J/kgK

Specific latent heat of vapourisation of steam = L_v

Temperature of steam = 100 °C

Heat gained by calorimeter and stirrer $= m_1$ kg × 390 J/kgK × $(\theta_2 - \theta_1)$ K

Heat gained by cold water, $Q_2 = m_2$ kg × 4200 J/kgK × $(\theta_2 - \theta_1)$ K

Heat lost by a steam change to water, $Q_3 = m_3 \times L_v$

Heat lost by steam (converted into water) cooling to θ_2

$Q_4 = m_3$ kg × 4200 J/kgK × $(100 - \theta_2)$ K

Heat gained = Heat lost

$$m_1 \times 390 \times (\theta_2 - \theta_1) + m_2 \times 4200 \times (\theta_2 - \theta_1) = m_3 L_v + m_3 \times 4200 \times (100 - \theta_2)$$

Since m_1, m_2, m_3, θ_1 and θ_2 are known, the value of Lv can be calculated from,

$$L_v = \frac{(390m_1 + 4200m_2) \times (\theta_2 - \theta_1) - 4200m_3 \times (100 - \theta_2)}{m_3}$$

The cooling effect of evaporation and mechanism of refrigeration

When a liquid evaporates, it uses up latent heat of vapourisation. If no heat is supplied from outside, cooling occurs. The human body uses evaporation to control its temperature. When it is very hot, sweat glands release water which then evaporates from the skin taking away latent heat of vapourisation. This causes cooling of the body.

If a volatile liquid like alcohol or ether is poured on the skin, the body feels much colder than when water is poured on the skin at the same temperature. This is because the alcohol evaporates quickly, taking latent heat of vapourisation from the skin, thus cooling the skin. The refrigerator works on this principle. Cooling in a refrigerator takes place when a volatile liquid like Freon evaporates inside a copper coil surrounding the freezing unit or compartment. Figure 7.16 shows the main features of a domestic refrigerator.

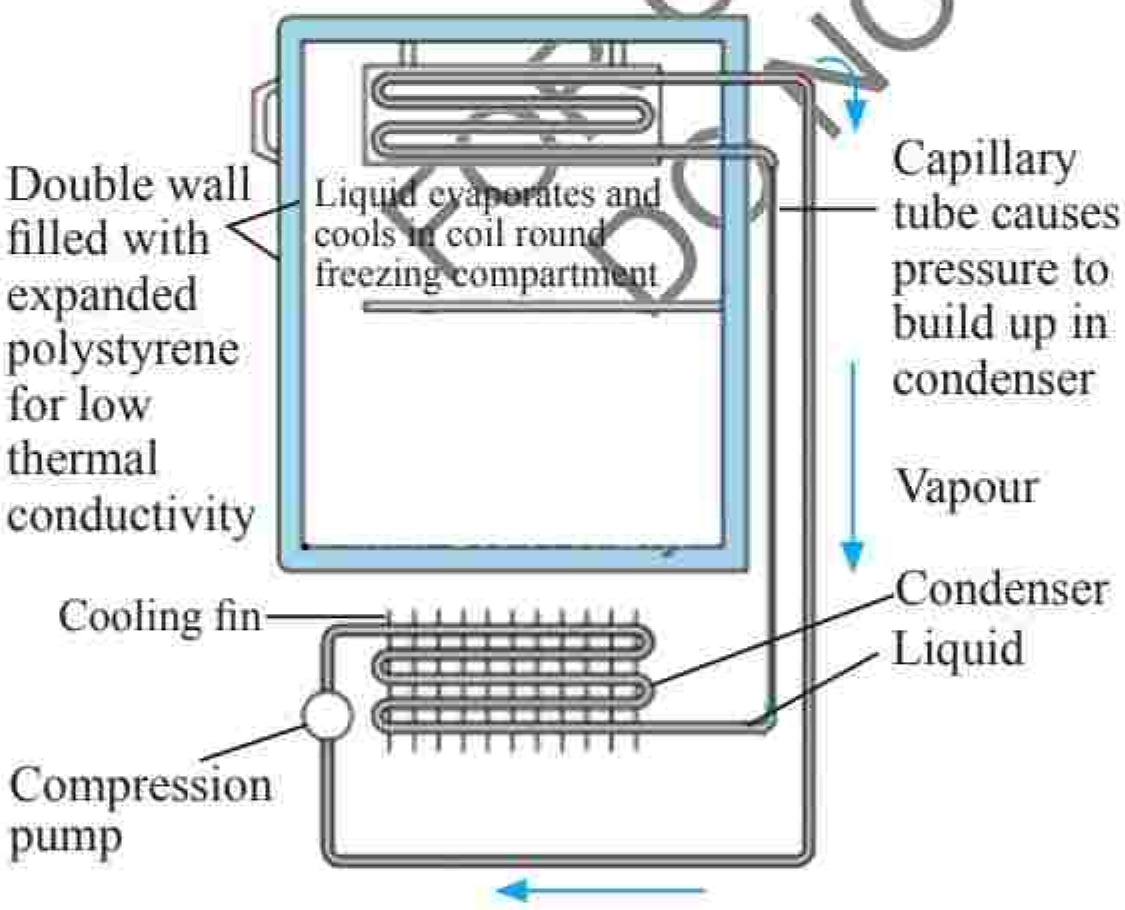


Figure 7.16: *The domestic refrigerator*

As quickly as vapour is formed, it is pumped off to the outside coil (condenser) where it is compressed. In the condenser, the compressed vapour gives up its latent heat of vapourisation to the cooling fins and so condenses. The capillary tube also causes pressure to build up in the condenser so that the vapour changes to a liquid.

In the freezing compartment, the tube carrying the fluid widens while, at the same time, the fluid is sucked by the pump. This reduces the pressure in the compartment and the fluid suddenly evaporates. When the fluid evaporates, it draws the latent heat of vapourisation required from the surrounding (the compartment) causing the temperature to go down. The vapour is then pumped back to the condenser, which is outside the refrigerator, where it is compressed back to a liquid and gives up the latent heat of vapourisation to the cooling fins. This process is repeated with time the temperature in the refrigerator becomes quite low.

The circulation process may be controlled by an installed thermostat in the freezing unit(s). The thermostat stops the circulation when the desired temperature is attained and restarts when the temperature exceeds the desired one. The refrigerator is sealed with double walls filled with polystyrene so that it does not gain or lose heat other than through the fluid.

Exercise 7.2

1. Students are performing two experiments. One is on latent heat of vapourisation and the other is on latent heat of fusion. In which experiment will they be required to use a large amount of energy against the environment?
2. How much heat is required to change 2 kg of ice at 0 °C into water at 20 °C? Specific latent heat of fusion of water = 334 000 J/kg and specific heat capacity of water = 4200 J/kg°C.
3. How much thermal energy must be removed from 5 kg of water at 0 °C to turn the entire mass into ice? Specific latent heat of fusion of ice is 334 000 J/kg.
4. A pan full of crashed ice at 0 °C is heated by a 167 W heater. Assuming there is no heat loss, what mass of the ice will melt if the heater is switched on for 100 s? Specific latent heat of fusion of ice is 334 000 J/kg.
5. A pan contains 2.0 kg of water at 0 °C. A jet of steam at 100 °C is passed through the water. What will be the temperature of the water when 0.1 kg of steam has condensed in it? Assume no heat is lost or absorbed by the pan. Specific latent heat of vapourisation of water is 2260 kJ/kg. Specific heat capacity of steam is 2144 J/kg°C and that of water is 4200 J/kg°C.
6. How much thermal energy is needed to turn 3.0 kg of water at 50 °C into steam at 100 °C? The specific heat capacity of water is 4200 J/kg °C. Specific latent heat of vapourisation of water is 2260 kJ/kg.

Chapter summary

1. The heat capacity of a substance is the amount of heat required to raise the temperature of the substance by 1 °C or 1 K.
2. Specific heat capacity is the quantity of heat required to raise the temperature of 1 kg of the substance by 1 °C or 1 K.
3. Matter exists in three states; solid, liquid and gas. The physical state of a substance depends on the heat content of the substance.
4. As the temperature of a substance is increased, the average kinetic energy of the constituent particles of the substance also increases, hence, the particles move faster and further apart.
5. When heat flows in or out of a substance, the magnitude of the change in temperature is directly proportional to the quantity of heat, Q, and inversely proportional to the mass, m, and specific heat capacity, c, of the substance.
6. When two or more substances exchange heat in an insulated environment, the heat lost by the substance(s) at a higher temperature is equal to the heat gained by the substance(s) at a lower temperature.

7. At the melting point, the inter-molecular forces can no longer hold the particles in fixed positions and the solid melts.
8. The specific latent heat of fusion, L_f, of a substance is the heat absorbed or released when 1 kg of the substance changes from liquid to solid state and vice versa, at its melting or freezing point.
9. The specific latent heat of vapourisation, L_v, is the heat absorbed or released when 1 kg of a substance changes from the liquid to gas state and vice versa, at its boiling point.
10. The freezing point and the boiling point of substances are affected by a change in pressure and the presence of impurities.

Revision exercise 7

Choose the letter which corresponds to a correct answer in the items of question 1. Then answer the remaining questions.

1. Figure 7.17 shows the temperature of a 2 kg sample of an unknown material as it is heated. The sample was initially a solid at –200 °C.

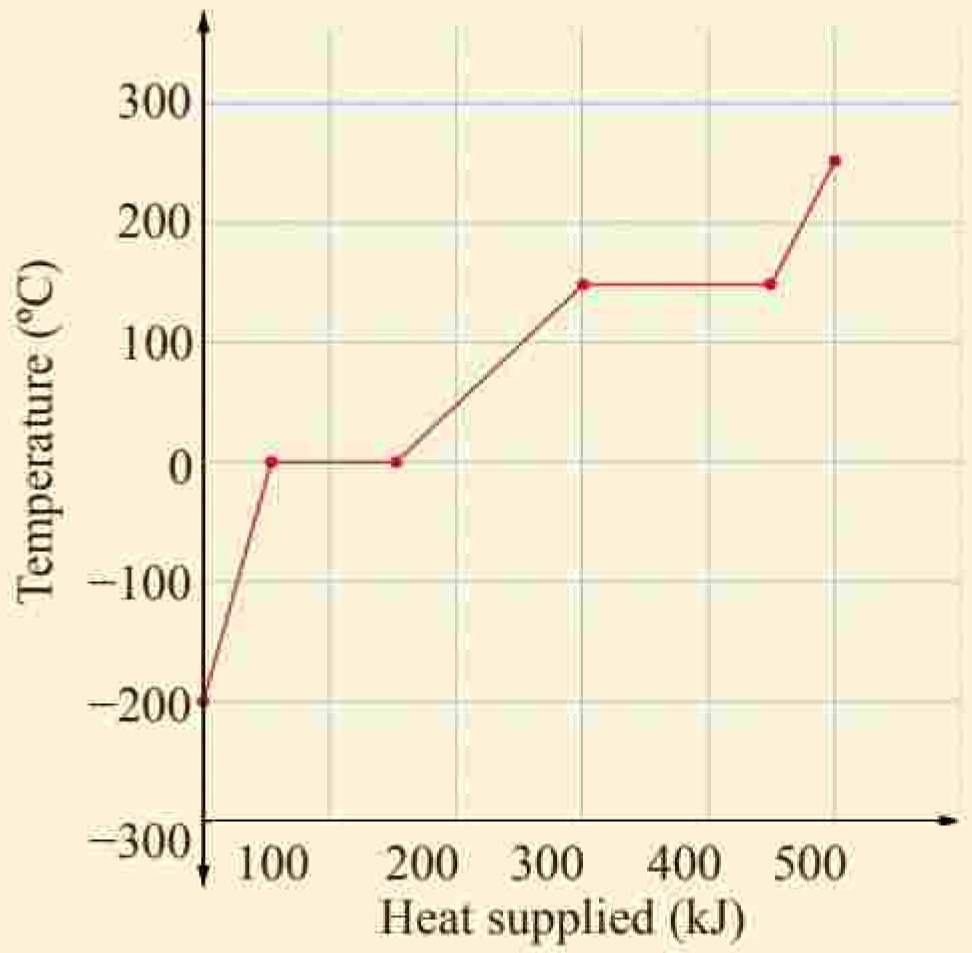


Figure 7.17

Use the graph to answer questions (i) to (iv).

(i) The melting point of the substance is ____.

A –200 °C
B –100 °C
C –50 °C
D 0 °C

(ii) The boiling point of the substance is ____.

A 100 °C
B 150 °C
C 200 °C
D 250 °C

(iii) The specific heat capacity of the solid phase in kJ/kg°C is ____.

A 0.167
B 0.250
C 0.375
D 0.503

(iv) The latent heat of vapourisation of a substance in kJ/kg is ____.

A 50
B 20
C 75
D 90

2. Define the following:
   (a) Specific heat capacity.
   (b) Specific latent heat of vaporization.
3. Use the kinetic theory of matter to explain the concept of boiling and evaporation.

4. Describe two methods by which a quantity of liquid may be made to evaporate more quickly.

5. Some water is filled in a bag made of porous materials, such as canvas. The bag is hung where it is exposed to a draught of air. Explain why the temperature of the water in the bag becomes lower than that of the air.

6. Explain the factors which affect the boiling point of water.

7. A 0.2 kg block of ice at 0 °C is dropped carefully into a Styrofoam calorimeter cup with an unknown mass of water at 20 °C. When thermal equilibrium is reached, the final temperature is measured to be 5 °C. What was the mass of the water initially in the cup? Specific latent heat of fusion of ice = 334 000 J/kg, specific heat capacity of water = 4200 J/kg K.

8. A container holds 1.5 kg of ice initially at $-40\ °C$. Heat is supplied to the container at the rate of 12.6 kJ per minute for 120 minutes.
   (a) Plot a graph of temperature versus time for 120 minutes during which heat is supplied.
   (b) What will be the temperature of contents of a container at the end of 120 minutes?
   (c) What will a mass of steam in a container be at the end of 120 minutes?

9. A 0.15 kg aluminium cup holds 0.2 kg of water at 18 °C. A 0.12 kg iron block at 85 °C is carefully dropped into the water and the entire system surrounded by an insulating jacket. What will be the final temperature of the system when thermal equilibrium is reached? Specific heat capacities are: aluminium is 900 J/kg °C, iron is 480 J/kg °C and water is 4200 J/kg°C.

10. How much thermal energy is required to raise the temperature of 3 kg of aluminium from 15 °C to 25 °C? Specific heat capacity of aluminium is 900 J/kg°C.

11. When a certain quantity of heat was supplied to a substance, its temperature rose from 5 °C to 20 °C. What will the final temperature of the substance be if twice the amount of heat is then removed from the sample?

12. Why is water used as a coolant in car engines?

13. Two substances A and B have the same mass and are at the same temperature. Substance A has a higher specific heat capacity than substance B. Which substance will have a higher final temperature if the same amount of heat is supplied to each substance?

14. An insulated cup holds 0.1 kg of water at 0 °C. The amount of 0.1 kg of boiling water at a temperature of 100 °C is poured into the cup. What will be the final temperature of the mixture at thermal equilibrium?

15. How much heat is needed to change 340 g of ice at 0 °C to water at 0 °C? Specific latent heat of fusion is 334000 J/kg.

16. A metal sphere of unknown composition has a mass of 0.4 kg. The sphere is heated in a furnace to a temperature of 150 °C and then dropped into an insulated cup holding 0.35 kg of water at 20 °C. Upon reaching thermal equilibrium, the temperature of the system is measured to be 32.4 °C.
   (a) Calculate the specific heat capacity of the metal.
   (b) Use the values of specific heat capacity in Table 7.1 to identify the metal.

17. Explain why a piece of ice at 0 °C is more effective in cooling a drink at room temperature than an equal mass of water at 0 °C ?

18. A refrigerator can convert 600 g of water at 50 °C to ice at 0 ºC in 3.5 hours. Find the average rate of heat absorbed from water in watt. Specific heat capacity of water $c_w = 4200$ J/kg°C, specific latent heat of fusion $L_f = 334000$ J/kg and specific heat capacity of ice $c_{ice} = 2100$ J/kg°C.

Chapter Eight

Vapour and humidity

Introduction

You have studied in Chapter Seven that, a liquid can change to gas even when the temperature is below the liquid's boiling point. In our daily life we experience different situations where such change occurs. These include, disappearing of water droplets on plant leaves when the sun shines, drying wet hair using hair drier, evaporation of sweat from the skin and drying up of water bodies during hot seasons. The change from liquid to gas increases moisture content in the atmosphere. The presence of moisture in the atmosphere indicates the potential capacity of air to precipitate. The moisture content in the atmosphere also helps a pilot to decide when and where to fly an aeroplane. In this chapter you will learn the concept of vapour and humidity. The competencies developed from this chapter will enable you to measure the relative humidity and dew point. You will also be able to store pharmaceutical products and food items safely.

Concept of vapour

Liquids are made up of particles (atoms or molecules) that are in a state of random motion. These particles have different kinetic energies as they move rapidly in all directions, and collide with each other. The average speed and kinetic energy of the particles depend on the temperature of the liquid. If a molecule in a liquid acquires enough thermal energy, it can overcome the liquid cohesive force and escapes to the atmosphere. Water in gaseous phase in the atmosphere is known as vapour. Water vapour in the atmosphere increases the amount of water in the atmosphere. The process of changing liquid to vapour is called evaporation. The amount of water vapour present in the air affects the rate of evaporation.

Process of evaporation of a liquid

In our everyday life we observe the disappearance of small pools of water just after raining stops. In this case, the water changes to gaseous or vapour state. Evaporation takes place at any temperature depending on the amount of water vapour already present in the atmosphere. Evaporation stops only when the air is saturated with water vapour. The driving force for evaporation, is therefore, the difference in vapour concentration between the air and the evaporating surface. For a liquid to evaporate, sufficient energy is required to overcome liquid-phase intermolecular forces. The particles that are left have low kinetic energy on average. The kinetic energy of the remaining liquid particles decreases as more of the liquid evaporates. Consequently, the liquid cools as it evaporates. For liquids in which the molecules are held together by reasonable strong force (bond), more energy is required for a molecule to break up. Such a liquid will take a long time to evaporate. Examples of such liquids include water, diesel and lubricating oil. Liquids with weak forces of attraction between their molecules evaporate very quickly. Examples of liquids with weak forces are, methylated spirit and petrol.

Factors affecting evaporation of a liquid

The main factors that affect the rate of evaporation of a liquid are nature of liquid, temperature, surface area, wind and amount of water vapour in air.

Nature of liquid

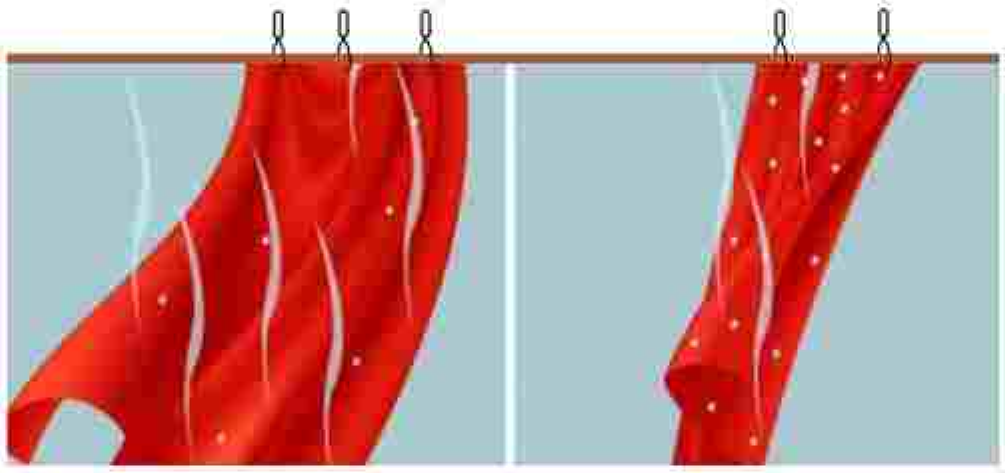
The rate of evaporation depends upon the nature of the liquid. Some liquids evaporate more quickly compared to others. Liquids that have low boiling point, evaporate in a short period of time at ordinary temperature. These liquids are called volatile liquids. Examples of these include ether and methylated spirit. To change from liquid to vapour, liquids require latent heat. If a volatile liquid is poured on your body, you feel cold because the liquid takes heat from your body and evaporates. Water also causes the body to become cold but not noticeably as methylated spirit does. This is because the spirit evaporates more quickly than water at the temperature of the body. It is worth noting that, some solids such as iodine, naphthalene and solid carbon dioxide evaporate to gas through sublimation.

Temperature

The ability of an air to hold water vapour depends on its temperature. As the air temperature increases, the capacity of air to hold water vapour increases too. This is because at high temperatures more molecules are moving faster, hence it is more likely for a molecule to have enough energy to break away from a liquid to become a gas. Moreover, wet clothes dry faster if the air is at an elevated temperature.

Surface area

If a wet cloth is dried in air by hanging it on a clothes line as shown in Figure 8.1(a) and the same is folded as shown in Figure 8.1(b), it will take shorter time for unfolded to dry than the folded one. When surface area is increased, then the number of molecules that are exposed to air increases. This means that, the larger the surface area that is exposed to the air, the more molecules will escape into the air. Another example is when a liquid is placed in a wide-mouthed container, the large number of molecules are exposed to air. As a result, a large number of molecules escape away to the gaseous phase. The rate of evaporation, therefore, increases with increase in the surface area of an evaporating surface.



(a) Cloth with large surface (b) Cloth with small surface

Figure 8.1: *Drying clothes with different evaporating surfaces*

Wind

The rate of flow of air determines the rate of evaporation into the surrounding air. In a windy environment, molecules near the evaporating surface are carried away as soon as they leave the surface of an evaporating liquid. This increases the difference in concentration of vapour molecules between air and the surface of the liquid, resulting in increased rate of evaporation.

Figure 8.2: *Clothes drying on a windy day*

Amount of water vapour in air

As already stated, air can hold water vapour to a certain limit. This limit depends on the air temperature. If air is humid, it has high content of water vapour. The more humid the air is, the lower the rate of evaporation. This can be demonstrated by drying clothes in dry and humid air. Wet clothes dry faster in dry air than in humid air. This is because humid air already has a high amount of water vapour. This effect is a decrease in the vapour concentration gradient between the air and the evaporating surface, hence reduced evaporation rate.

Activity 8.1

Aim: Demonstration of evaporation from different surface areas

Materials: a 10 litre empty plastic bucket, 20 litre empty plastic bucket, metre rule, water, stopwatch

Procedure

1. Fill the two plastic buckets with same amount of water.
2. Label the level of water in each bucket.
3. Place the buckets containing water in a sunny place.
4. Record the level of water in buckets after every 3 hours.
5. Measure the diameter of each bucket and calculate its surface area.

Questions

(a) Which bucket has higher surface area?

(b) Is there any relationship between the final level of water and surface area of the bucket?

Saturated and unsaturated air

We have seen that, water molecules escape into air during evaporation. There is a maximum limit of water vapour that air can hold at a given temperature and pressure. When this limit is attained, the number of molecules escaping from the liquid is equal to the number of molecules returning to the liquid. At that particular temperature and pressure, the air is said to be saturated. On the other hand, the air is unsaturated when it does not contain the maximum amount of water it can hold. Unsaturated air can hold more vapour at the same temperature.

Saturation vapour pressure

If a hot liquid is poured into an open container, the liquid will tend to evaporate. If the container is closed as shown in Figure 8.3, and the temperature is kept constant, the level of water in the container will drop for a certain period of time and then become constant.

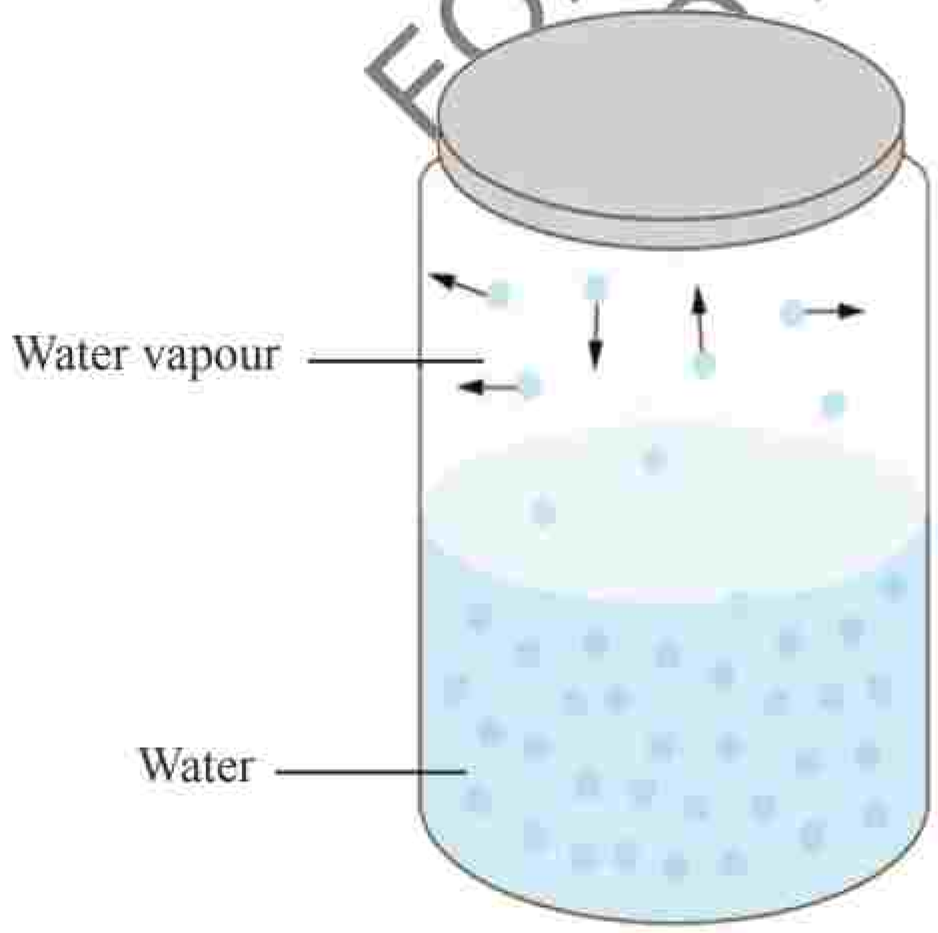


Figure 8.3: *Water in a sealed beaker with an air space*

Initially, the water in the container will evaporate rapidly. As soon as there is some water vapour in the space above the water, some of the vapour will also begin to return into liquid state, a process known as condensation. At first, the rate of evaporation will be higher than the rate of condensation. In this case, the amount of water vapour in the air space above the container increases while the amount of water in the container decreases. As the number of vapour molecules increases, the rate at which they condense also increases. An equilibrium is reached when the number of molecules which have kinetic energy high enough to escape from liquid phase to form vapour equals to the number of molecules returning to the liquid from the vapour. At this point, the number of gaseous molecules above the surface of water is constant and the rate of evaporation equals the rate of condensation. In this situation the vapour is said to be saturated with water vapour.

When the water vapour molecules collide with the surface of the container, they exert pressure. The pressure exerted by the vapour when the rate of evaporation equals to the rate of condensation is called the *saturation vapour pressure (S.V.P)* at that temperature. It is measured in pascal (Pa). The saturation vapour pressure is a measure of the rate of evaporation of liquid at a given temperature.

A substance with a high saturation vapour pressure at room temperature is said to be *volatile*. Table 8.1 shows the saturation vapour pressure for various liquids for temperatures range from 20 °C to 25 °C.

Table 8.1: Saturated vapour pressure for various liquids in the range 20 °C – 25 °C

Liquid	S.V.P (kPa)
Acetone	30.0
Benzene	14.0
Ethyl alcohol	9.0
Acetic acid	3.3
Water	2.4
Milk	2.4
Kerosene	0.5

Partial vapour pressure

Partial vapour pressure is the pressure exerted by vapour molecules above the surface of a liquid when the surface of the air contains vapour less than the maximum amount that it can hold. Under this condition, the air is said to be unsaturated. When air is unsaturated, a large number of molecules leave the liquid compared to those returning into the liquid per unit time. The vapour pressure above the liquid keeps on rising with time.

Beside the vapour molecules, other gas molecules present above the liquid exert their own pressure. Atmospheric gases exert pressure which is called atmospheric pressure. For example, when water is held in an open container, the total pressure on the surface of the water is the sum of water vapour pressure and atmospheric pressure.

Activity 8.2

Aim: To measure saturation vapour pressure

Materials: mercury barometer, ether, a ruler, thermometer, special bent pipette

Procedure

1. Using a barometer, measure the atmospheric pressure as shown in Figure 8.4.

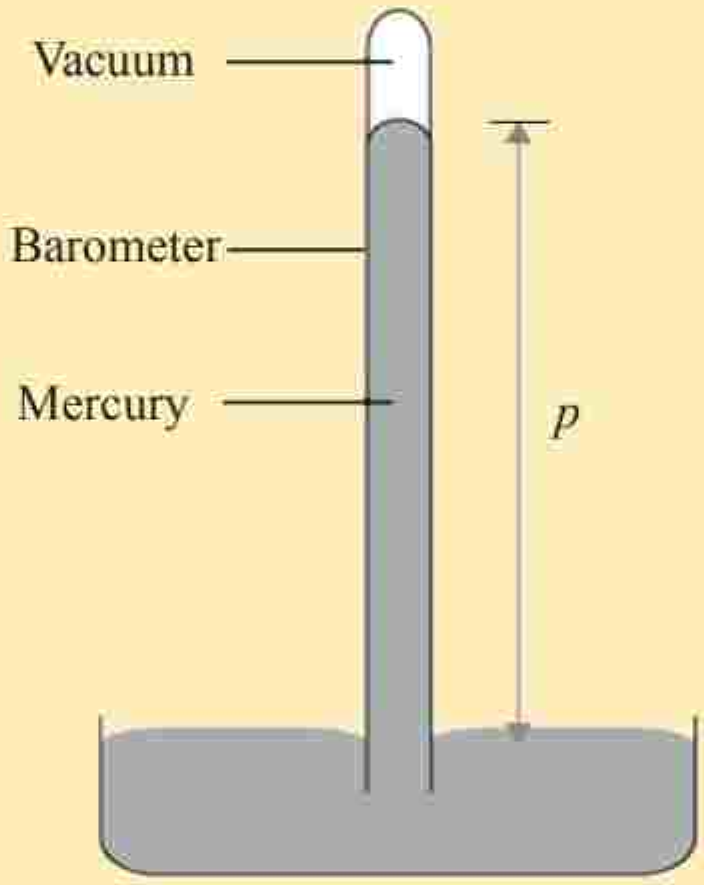


Figure 8.4

2. Record the length of mercury column p and the air temperature.
3. Using a special bent pipette, add few drops of ether in the tube through the lower end until the vapour is saturated.
4. After the addition of ether, observe the change of mercury level.
5. Measure the height h (Figure 8.5).

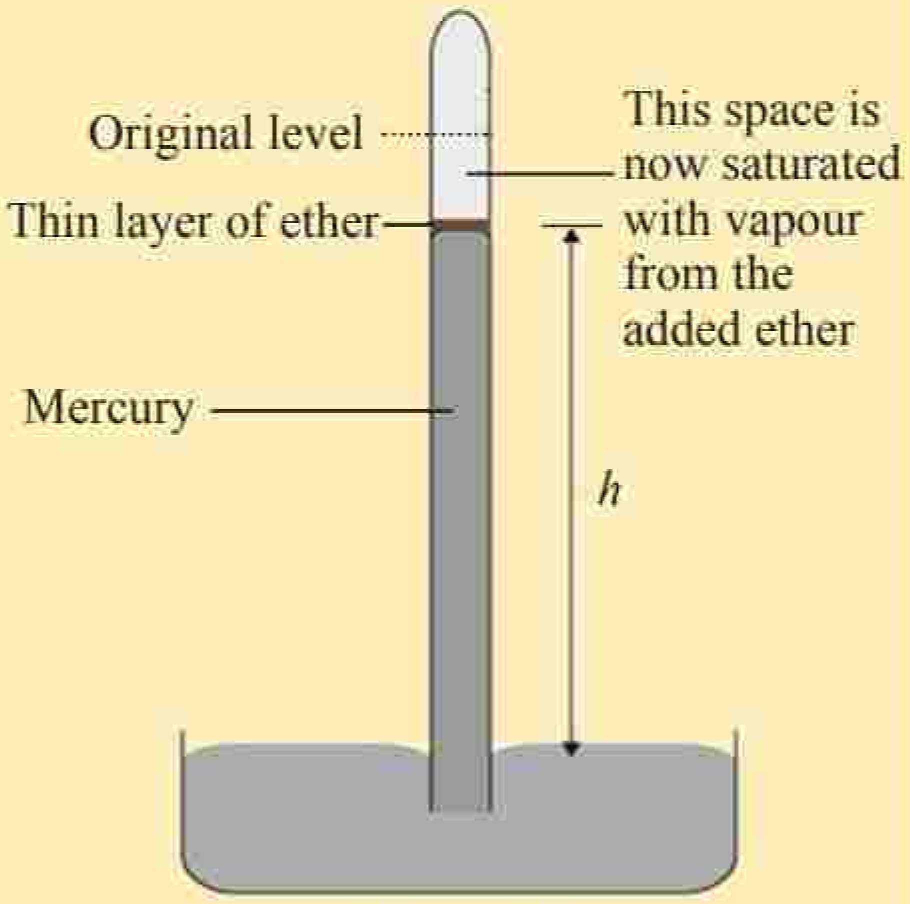


Figure 8.5

Questions

(a) Compare the height, p, and h.

(b) What type of pressure is associated with the length, p?

(c) What did you observe when ether was introduced?

(d) What is the saturation vapour pressure at the measured temperature?

Mercury column length, p, is longer than the mercury column length, h. Since the space above the mercury in the tube for length, p is vacuum, the length, p is sustained by atmospheric pressure only. The saturation vapour pressure created after addition of ether, pushes the mercury down the tube to level h. The difference between mercury length p and h represent the saturation vapour pressure.

Effect of temperature on saturated vapour pressure (S.V.P) of a liquid

If the temperature of a liquid increases, the speed of its molecules increases and so the number of molecules escaping from the liquid surfaces increases. The increase in evaporation increases the vapour, so the vapour pressure becomes high. It is worth to note that, at low temperature, fewer molecules have sufficient energy to escape from the surface of the liquid. Figure 8.6 shows a graph of saturation vapour pressure (S.V.P) of water at various temperatures.

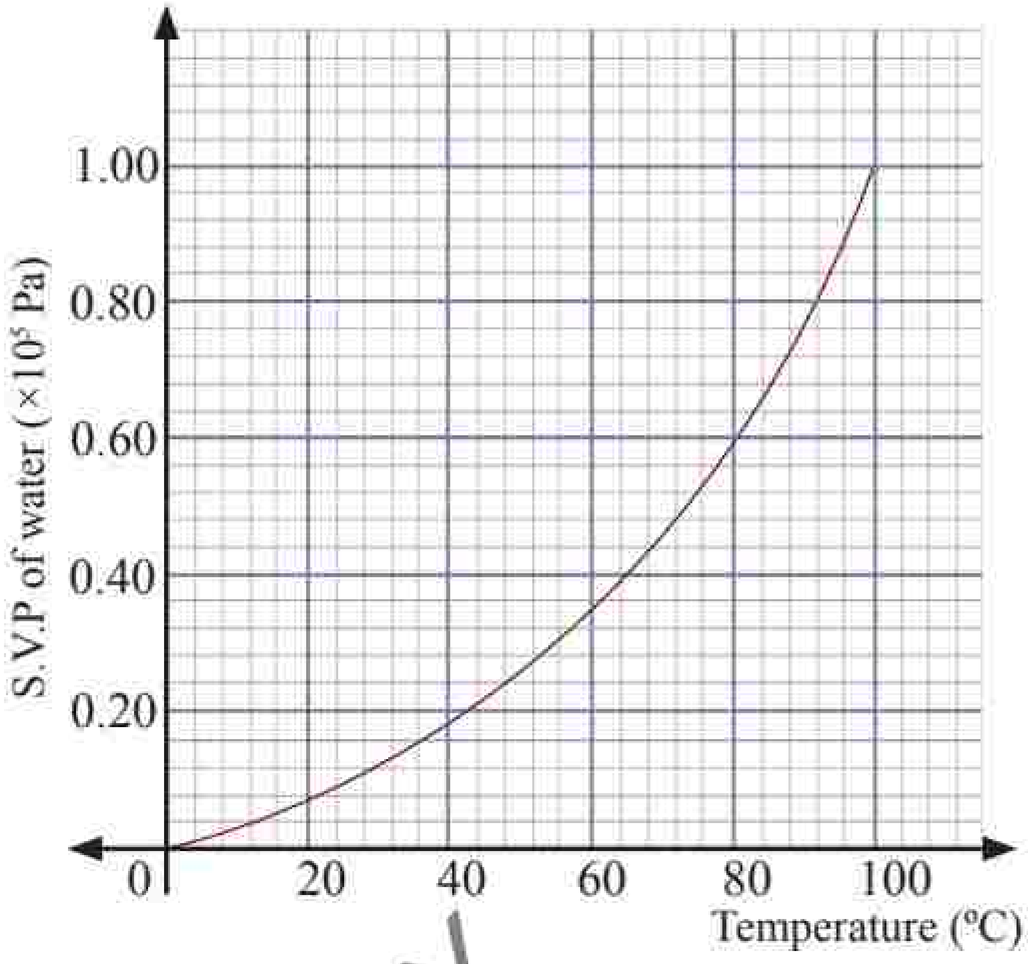


Figure 8.6: *Saturated vapour pressure of water against temperature*

Activity 8.3

Aim: To demonstrate the relationship between saturation vapour pressure and temperature

Materials: source of heat, round bottomed flask, a cork, mercury manometer, thermometer, wire gauze, tripod stand, water

Procedure

1. Pour some water in a round bottomed flask so that it is half filled.
2. Insert a cork fitted with a mercury manometer and a thermometer as shown in Figure 8.7.

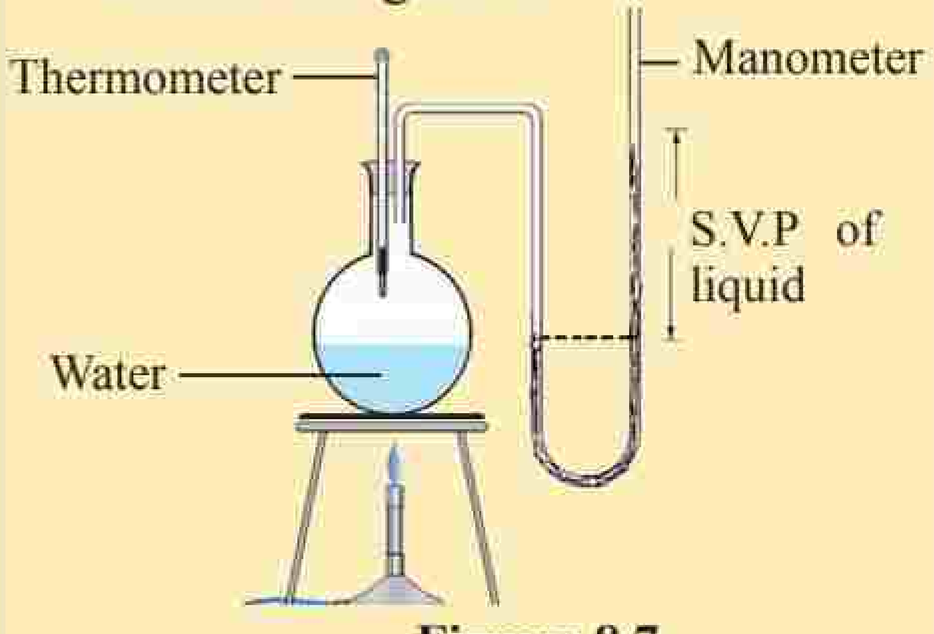


Figure: 8.7

3. Record the reading of the mercury manometer and thermometer. Note that, the height between the mercury levels in a manometer gives the saturation water vapour.
4. Heat the water and record the temperature and S.V.P readings at interval of 10 ºC.
5. Record your results in the table similar to Table 8.2.

Table 8.2

Temperature (ºC)									
S.V.P (mm of mercury)									

Question

What is the relation between the S.V.P and temperature?

At a given temperature, the saturation vapour pressure of water is the same regardless of any air in the space above it. As the temperature increases, so is the S.V.P. Note that, saturation vapour pressure is independent of the volume of the vapour, provided that there is some free liquid present to ensure saturation conditions. If the temperature is increased until the boiling point of water is reached, the S.V.P equals the external atmospheric pressure.

Exercise 8.1

1. Which of the following statements are "TRUE"?
   (a) Water evaporates into air from the rivers, lakes, and oceans but not from soil.
   (b) The process of water changing into its vapour is called evaporation.
   (c) Evaporation of water cannot take place in the absence of sunlight.
   (d) Tiny droplets of water vapour in the upper layer of air are formed due to condensation of water vapour.
2. For each of the following, state whether it is due to evaporation or condensation:
   (a) Water drops appear on the outer surface of a glass containing cold water.
   (b) Dew appearing on a cold surface early in the morning.
   (c) Wet floor dries up after mopping it.
   (d) Steam rising from a hot girdle when water is sprinkled on it.
3. Is evaporation a cooling or a warming process? Explain.
4. Explain the statement 'the saturation vapour pressure of water at 15 °C is 13.0 mmHg'.
5. Briefly describe two ways by which a quantity of liquid may be made to evaporate more quickly.
6. (a) State two ways in which a saturated air differs from unsaturated air.

(b) Describe an experiment to determine how saturation vapour pressure of water depends on temperature. Draw a graph showing the relation between them.

7. Form III physics students are having debate on the density of dry air and that of saturated air. Group A argues that, density of dry air is greater than that of saturated air, while group B argues otherwise. Conclude the discussion.

Concept of humidity

When water evaporates, its vapour gets into the air. This vapour contributes to the moisture content of the air. Air which contains water vapour is called humid air. It is also called moist air. A measure of the amount of water vapour present in a given air is called humidity.

In daily life, you experience humid air when working in open air on a sunny day following a rain shower. Your body will be covered with sweat and you will feel uncomfortable. We say that the air is very humid. High humidity in the atmosphere is a result of excessive evaporation from oceans and land surfaces.

Task 8.1

Stand a bottle containing water at about 0 ºC in air. Quickly dry the outside of the bottle using blotting paper. Carefully observe what happens on the outer surface of the bottle after some minutes has elapsed. Discuss your observations.

Assuming the ambient air is not dry, water droplets will form on the surface of the bottle. This water results from the condensation of the water vapour in the air. The collected water confirms that the air contained some water vapour.

Dew point

Temperature has influence on the quantity of water which the air can hold. If the temperature of the air drops to the point that air becomes saturated with water vapour, the vapour condenses and water droplets are deposited on cooler surfaces such as leaves and blades of grass. Such droplets are called dew. The temperature at which air must be cooled to become saturated with water vapour is called dew point. When air has reached the dew point at a particular pressure, the water vapour in the air is in equilibrium with liquid water, meaning that, water vapour is condensing at the same rate at which liquid water is evaporating. Below the dew point, water vapour will begin to condense on solid surfaces (such as blades of grass) or around solid particles in the atmosphere (such as dust or salt) forming clouds or fog. Above dew point, the liquid exists in the gaseous state and condensation will not occur. The dew can be vividly observed in the morning on grass, or leaves of trees as shown in Figure 8.8.

Figure 8.8: *Dew formed on grass blades*

Factors that influence the formation of dew

The formation of dew is mainly influenced by temperature, wind, cloud cover and water vapour.

Temperature

The warmer the air, the more humid is the air. For this case, at low temperature, air will take shorter time to be saturated. Since the air saturate quickly at low temperature than at high temperature, dew forms quickly at low temperature than at high temperature. The temperature of the atmospheric air must fall below the dew point for dew to be formed.

Wind

In windy day, the movement of air blows away water vapour. Consequently, it becomes difficult for the air to get saturated, hence, no dew is formed. Wind also increases the rate of evaporation, thus preventing the formation of water droplets (dew).

Water vapour

The air must be saturated with water vapour for dew to be formed. A high concentration of water vapour increases the dew point while a small amount of water vapour lowers dew point.

Cloud cover

When a surface loses heat to the atmosphere, it cools, and can form dew on that surface. Cloud cover prevents the heat from escaping and thus, hinders dew formation. The absence of clouds will allow heat to escape to outer space hence promoting dew formation.

Activity 8.4

Aim: To measure dew point

Materials: ice cubes, water, thermometer, can (beaker)

Procedure

1. Fill a beaker with water.
2. Measure the temperature of the water.
3. Add ice cubes gradually while stirring with a thermometer.
4. Continue adding the cubes until condensation just forms on the surface of the beaker.
5. Read the temperature on the thermometer.

Questions

(a) What was the temperature of the water before adding ice cubes in the beaker?

(b) What was the temperature when water just condensed on the beaker?

(c) Determine the dew point.

When ice is added, the temperature of water decreases. Temperature of the outer surface of the beaker is practically equal to that of the water. The content in the beaker is stirred in order to keep the beaker surface at the same temperature as the water. When the dew point is reached, condensation forms on the outer surface. The temperature at which dew forms on the beaker is the dew point of the surrounding air.

Relative humidity

The humidity of air can be quantified and expressed as either absolute humidity, specific humidity or relative humidity. Absolute humidity refers to the actual amount (mass) of water vapour in a unit volume of moist air. Specific humidity is the ratio of mass of vapour in a unit volume to the total mass of moist air in the volume.

The actual moisture content of a sample of air as a percentage of that contained in the same volume of saturated air at the same temperature is called the relative humidity. It is the measure of degree of wetness of the air. In practice, absolute and specific humidity are seldom used in everyday life; instead, the relative humidity is more commonly used.

Relative humidity (RH) is a measure of the amount of water vapour in a unit volume of air at a given temperature compared to the maximum possible amount of water vapour the air can hold at the same temperature, usually expressed as a percentage. Thus, relative humidity (RH) is the ratio of the amount of water vapour present in air (*m*) to the amount of water vapour required to saturate the same amount of air (M) at the same air temperature multiplied by 100. Hence:

$$RH = \frac{m}{M} \times 100\%$$

Air that is totally saturated with water vapour at a given temperature has a relative humidity of 100%. A relative humidity of 50% means that the water vapour in the air (at a given temperature) exerts a partial pressure half the saturated vapour pressure at that temperature.

Since the mass of water vapour in a given volume corresponds almost proportionally to its pressure, the relative humidity can also be expressed as the ratio of the partial vapour pressure to the saturated vapour pressure at the same air temperature.

$$RH = \frac{partial\ vapour\ \ pressure}{S.V.P\ at\ air\ temperature} \times 100\%$$

Relative humidity is also calculated with respect to saturated vapour density, that is:

$$RH = \frac{actual\ vapour\ density}{saturated\ vapour\ density} \times 100\%$$

If you recall that, absolute humidity is the density of water vapour in saturated air, then,

$$RH = \frac{actual\ vapour\ density}{absolute\ humidity} \times 100\%$$

Example 8.1

The relative humidity of a place was measured at 25 °C and found to be 53.6%. If the absolute humidity is 23.05 g/m^3, determine the actual water vapour density at this temperature.

Solution

$$RH = \frac{actual\ vapour\ density}{absolute\ humidity} \times 100\%$$

Actual water vapour density

$= RH \times$ Absolute humidity

Actual water vapour density

$$= \frac{53.6}{100} \times 23.05\ \text{g/m}^3$$

$$= 12.35\ \text{g/m}^3$$

Therefore, actual water vapour density = 12.35 g/m^3.

Measurement of RH using wet and dry bulb hygrometer

An instrument for measuring humidity is called a hygrometer. The relative humidity can be measured using the wet and dry bulb hygrometer. The wet and dry bulb hygrometer consists of two thermometers mounted side by side. A piece of muslin or cotton cloth is wrapped around the bulb of one of the thermometers and immersed in a container filled with distilled water (Figure 8.9). This keeps the wrapped bulb of thermometer moist while the bulb of the other thermometer remains dry.

Figure 8.9: *Wet and dry bulb hygrometer*

The wet bulb is cooled by the evaporation of water from the cotton cloth. The cooling continues to the extent which depends on the humidity of the surrounding air (dryness of the surrounding air). The drier the atmosphere, the higher the rate of evaporation of the water. In this way, the wet-bulb thermometer generally reads a lower temperature than the dry-bulb thermometer. The dry-bulb thermometer measures the temperature of the surrounding air. A hygrometer of the type described above is also called a psychrometer.

If the relative humidity is 100%, no water can evaporate because the air is saturated and the two thermometers would read the same temperature. On the other hand, if the relative humidity is 0%, evaporation rate would be so high that the wet-bulb thermometer would read a much lower temperature value than the dry-bulb thermometer. So, the difference between the two temperatures determines a measure of the relative humidity. A small difference between the reading of the two thermometers means high relative humidity and vice versa.

Conversion of the temperature difference to the relative humidity involves tedious calculations. However, the conversion has been simplified and the results can be presented in the form of charts called psychrometric charts. The charts give information on relative humidity, dew point, the energy contained in air, moisture contained in air and the space the air takes up. A table accompanying the hygrometer gives the relative humidity in terms of the reading of the wet-bulb and dry-bulb thermometers. Such a table is called a *psychrometric table.* Table 8.3 shows an example of a psychrometric table for relative humidity.

Table 8.3: Relative humidity psychrometric table

DBT*	Relative humidity (%)																					
	Depression of the wet bulb thermometer (dry-bulb temperature minus wet-bulb temperature)																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
0	81	63	45	28	11																	
2	83	67	51	36	20	6																
4	85	70	56	42	27	14																
6	86	72	59	46	35	22	10	0														
8	87	74	62	51	39	28	17	6														
10	88	76	65	54	43	33	24	13	4													
12	88	78	67	57	48	38	28	19	10	2												
14	89	79	69	60	50	41	33	25	16	8	1											
16	90	80	71	62	54	45	37	29	21	14	7	1										
18	91	81	72	64	56	48	40	33	26	19	12	6	0									
20	91	82	74	66	58	51	44	36	30	23	17	11	5	0								
22	92	83	75	68	60	53	46	40	33	27	21	15	10	4	0							
24	92	84	76	69	62	55	49	42	36	30	25	20	14	9	4	0						
26	92	85	77	70	64	57	51	45	39	34	28	23	18	13	9	5						
28	93	86	78	71	65	59	53	47	42	36	31	26	21	17	12	8	2					
30	93	86	79	72	66	61	55	49	44	39	34	29	25	20	16	12	8	4				
32	93	86	80	73	68	62	56	51	46	41	36	32	27	22	19	14	11	8	4			
34	93	86	81	74	69	63	58	52	48	43	38	34	30	26	22	18	14	10	8	5		
36	94	87	81	75	69	64	59	54	50	44	40	36	32	28	24	21	17	13	10	7	4	
38	94	87	82	76	70	66	60	55	51	46	42	38	34	30	26	23	20	16	13	10	7	5
40	94	89	82	76	71	67	61	57	52	48	44	40	36	33	29	25	22	19	16	13	10	7

DBT* is Dry-bulb temperature (°C)

Example 8.2

The dry-bulb temperature reading of a psychrometer is 22 °C and the wet-bulb temperature reading is 18 °C. With the use of psychrometric table, find the relative humidity.

Solution

The difference between the two temperature readings is 4 °C. From the psychrometric Table 8.3, the relative humidity reading that corresponds to a dry-bulb temperature reading of 22 °C and a difference of 4 °C is 68%. Therefore, the relative humidity is 68%.

Task 8.2

Visit the nearest meteorological weather station if available, and study how humidity is measured at the station. If the weather station is not available, search through the internet. Present your findings.

Measurement of relative humidity using Regnault hygrometer

The Regnault hygrometer is an instrument used to measure dew point and relative humidity. It consists of an enclosed thin silver tube containing ether and a thermometer to measure the temperature of the ether. There is also a tube through which air can be pumped into the ether (Figure 8.10).

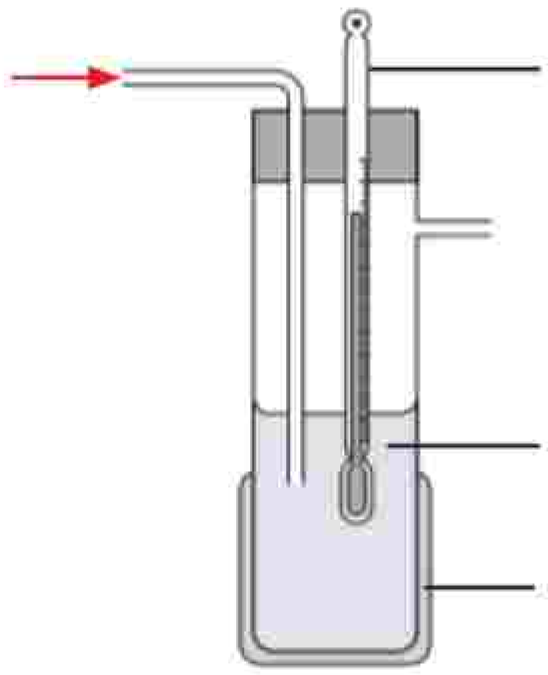


Figure 8.10: *Regnault hygrometer*

Air is pumped into the ether; the ether evaporates and its temperature and that of the silver tube surface falls. Cooling continues until the air adjacent to the outside surface of the tube becomes saturated with water vapour. Some water vapour condenses on the outside surface of the tube at this temperature (dew point).

Suppose the dew point is T_1 and the actual temperature is T_2. If the values of S.V.P for water at T_1 and T_2 are X and Y millimetres of mercury, respectively then,

$$RH = \frac{S.V.P \text{ at } T_1}{S.V.P \text{ at } T_2} \times 100\%$$

$$= \frac{X}{Y} \times 100\%$$

Example 8.3

You are given that the temperature of a dry bulb (air) is 16.5 ºC and the dew point is 7.4 ºC. If the saturated vapour pressures at these temperatures are 14 mmHg and 6.7 mmHg, respectively, calculate the relative humidity of the air.

Solution

$$RH = \frac{S.V.P\ at\ dew\ point\ T_1}{S.V.P\ at\ temperature\ of\ the\ air\ T_2} \times 100\%$$

$$= \frac{6.7\ \text{mmHg}}{14\ \text{mmHg}} \times 100\%$$

$$= 48\%$$

Therefore, the relative humidity is 48%.

Applications of the knowledge of humidity

The knowledge of humidity has various applications in everyday life. These include; weather forecasting, piloting, textile processing, storage of pharmaceuticals, transportation and storage of food items.

Weather forecasting

The knowledge of humidity is used by meteorologists to forecast the weather. Knowing the relative humidity and dew point in the atmosphere helps the meteorologist to determine the formation of clouds and the probability of weather events such as precipitation as illustrated in Figure 8.11, and storms. Relative humidity determines how moist you feel when you are outside.

Figure 8.11: *Weather event*

Piloting

Pilots use humidity levels to determine safe altitudes to fly. Low humidity levels favour safe flying than high humidity levels. As the humidity goes up, the air density goes down. This means that, the wings of an aeroplane will have fewer air molecules for it to be lifted as they are pushed through the air mass. So, humidity decreases the performance of aircraft, not only because of its effect on the wings, but also it affects the engine performance.

Textile processing

Relative humidity is used to determine the appropriate site to store cotton fibres. Cotton fibres require very high humidity. In low humidity, the fibres become very dry and pose difficulties in spinning. Also, the flying fluff of cotton materials could be inhaled by workers. In preventing this, water vapour is sprayed in the air. The vapour is absorbed by the fluff and the fluff drops down to the floor. As the humidity increases the more fluff absorbs the vapour.

Pharmaceutical storage

Knowledge of humidity is important in hospitals, since it determines safe keeping of pharmaceutical products. Moisture causes desterilization, irreversible destabilization and oxidative degradation because proteins found in many drugs are sensitive to humidity. In extreme cases, humidity may result in toxic degradation that is not easily detectable.

Storage and transportation of food items

The knowledge of relative humidity is used in the storage and transportation of food items, especially perishable foods (Figure 8.12). Dry air can cause dryness of the product while very damp air can cause the growth of moulds and bacteria. Food items are usually transported and stored under an optimum relative humidity.

Figure 8.12: *Perishable fruits*

Exercise 8.2

1. Define the following terms:
   (a) Evaporation
   (b) Condensation
2. Why can warm air hold more moisture than cold air?
3. Explain how a gas exerts pressure on the walls of its container.
4. A mass of air at 20 °C has a relative humidity of 36%. If the air is cooled to 16 °C, what will its relative humidity be? Saturation vapour pressure at 20 °C and 16 °C are 17.5 mmHg and 13.6 mmHg , respectively.
5. The temperature and relative humidity of air on a certain day are 20 °C and 80%, respectively. Find the fraction of mass of water vapour that will condense if the temperature falls to 5 °C. Saturation vapour pressure at 20 °C and 5 °C are 17.5 mmHg and 6.5 mmHg, respectively.
6. A closed vessel contains moist air at 20 °C, the relative humidity being 30%. What should be the relative humidity if the vessel were cooled to 10 °C? Saturation vapour pressure of water at 20 °C is 17.5 mmHg and at 10 °C is 9.2 mmHg.
7. The air in a room has 15 g of water vapour per cubic metre. However, for saturation, one cubic metre of volume requires 20 g of water vapour. What is the relative humidity?
8. On a particular day, the relative humidity is 100% and the room temperature is 30 °C. What is the dew point? The saturated vapour pressure of water at 30 °C is 42400 Pa.

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Chapter summary

1. Evaporation is the change of state from liquid to gas that occurs at the surface of a liquid at constant temperature.
2. When liquid is heated, the molecules gain kinetic energy, hence random speed of the molecules increases.
3. The rate of evaporation is affected by nature of a liquid, temperature, surface area, concentration of vapour in the surrounding air and rate of flow of air.
4. Vapour pressure is the pressure exerted by the vapour of a substance that forms above a liquid of the same substance.
5. Saturation vapour pressure is the pressure exerted by vapour when vapour reaches a state of equilibrium with its evaporating liquid. Saturation vapour depends on temperature.
6. Volatile liquids have high saturated vapour pressures indicating higher rates of evaporation at the same temperature.
7. Humidity refers to the water vapour content of atmospheric air.
8. Condensation is the process by which water vapour changes into liquid at constant temperature.
9. The dew point of air is the temperature at which the air becomes saturated with water vapour.
10. The water vapour density of saturated air is called the absolute humidity. The absolute humidity increases with increasing temperature.
11. Absolute humidity is the mass of water vapour present in a unit volume of air and is expressed in grams per cubic metre (g/m^3).
12. Relative humidity is the ratio of the mass of water vapour in a unit volume of air to that required to saturate it at the same temperature.
13. Relative humidity can be measured with a wet and dry bulb hygrometer and Regnault hygrometer.

Revision exercise 8

Choose the most correctitem in question 1 to 4, then answer question 5-13.

1. Suppose equal quantities of water are placed in four uncovered containers with different shapes and left on a table at room temperature. From which of the following containers in Figure 8.13, will the water evaporate most rapidly?

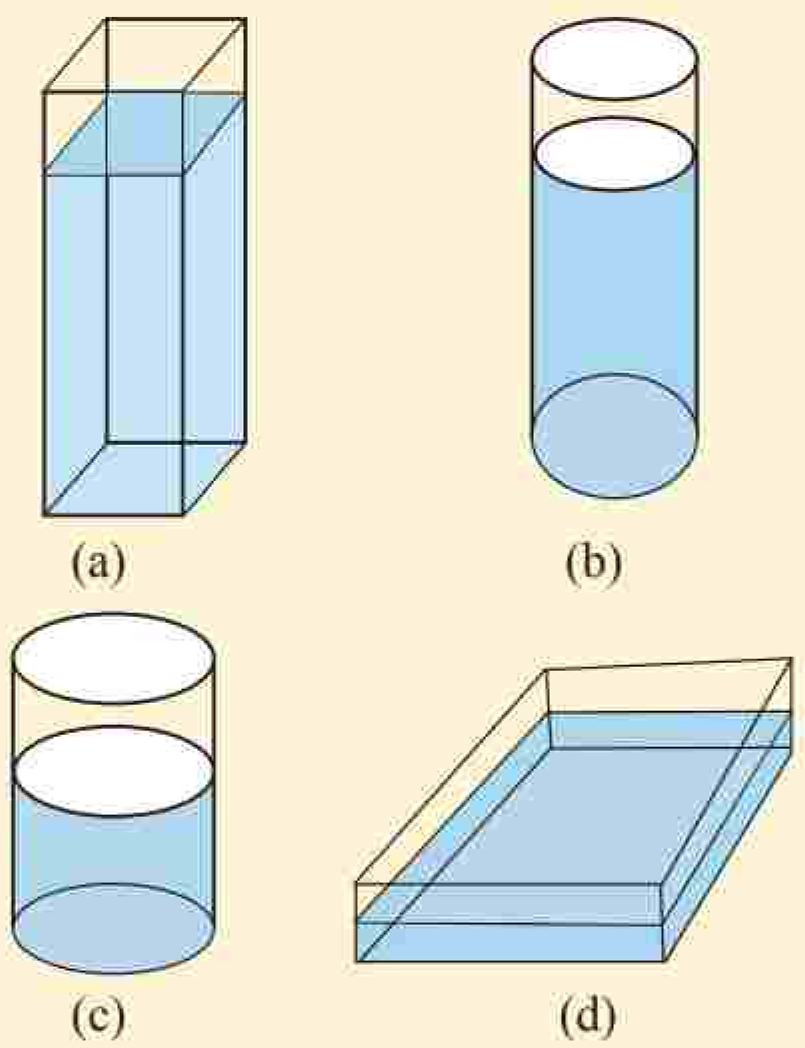


Figure 8.13

2. When air has reached its dew point, it:
   (a) is saturated.
   (b) has low humidity.
   (c) has to be cold.
   (d) can hold more water.
3. Which term among the following best explains how close the air is to saturation?
   (a) Absolute humidity
   (b) Relative humidity
   (c) Vapour pressure
   (d) All of the above
4. What would happen to the relative humidity of air if no water vapour is added or removed from cooling air?
   (a) remain constant.
   (b) decrease.
   (c) increase.
   (d) keep fluctuating.
5. Explain the difference between
   (a) vapour and a gas.
   (b) saturated and unsaturated vapour.
   (c) evaporation and boiling.
6. State the factors that influence the dew point.
7. How does temperature affect absolute humidity?
8. Why do small droplets of pure water evaporate in air even when the relative humidity is 100%?
9. At a certain temperature and pressure, air can hold a maximum of 120 g of water vapour. If at this temperature and pressure the air is holding only 40 g of water vapour, what is the relative humidity of the air?
10. A room with dimensions of $7\ \text{m} \times 10\ \text{m} \times 2\ \text{m}$, holds air that is saturated with water vapour. The saturation vapour pressure of the water vapour is 7.37 kPa. If all of the water vapour in the room was condensed, what volume would the water occupy? Give your answer in m^3.
11. (a) Use Figure 8.14 to estimate the temperature at which the saturation vapour pressure of water is half that of atmospheric pressure.
    (b) At what temperature is the saturation vapour pressure of water equal to atmospheric pressure?

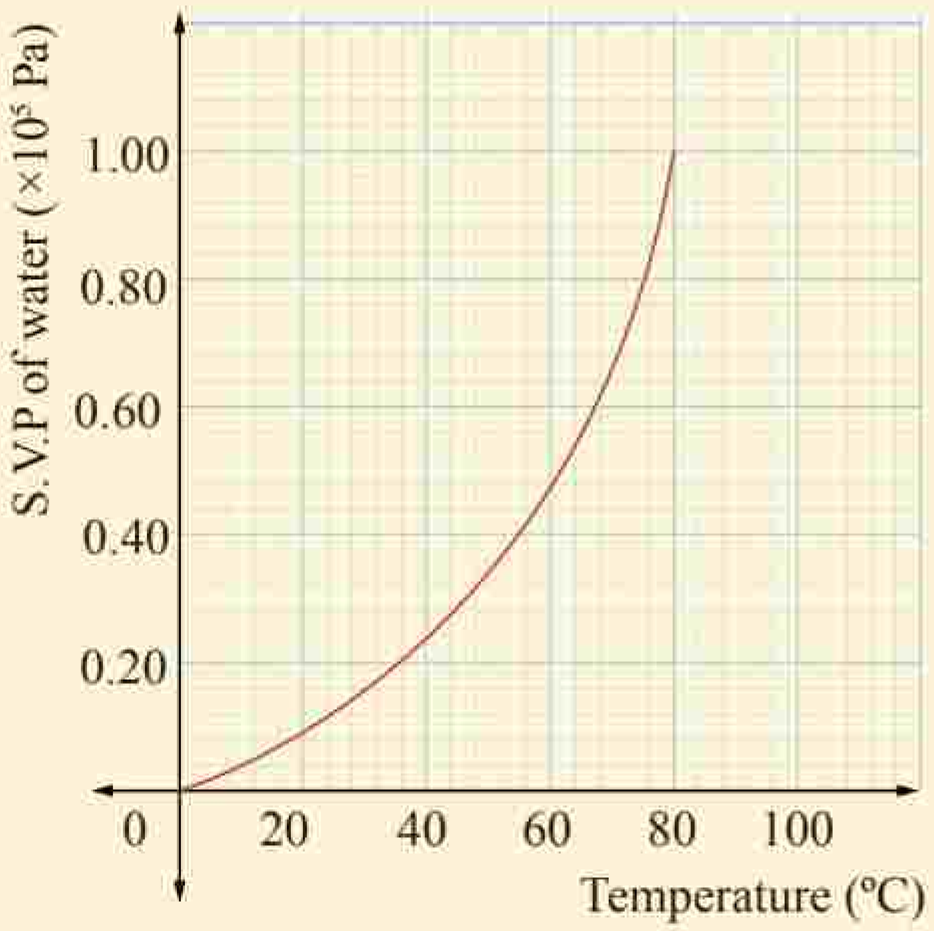


Figure 8.14

12. Figure 8.15 shows a wet and dry bulb psychrometer that is hanging in a room. What is the relative humidity in the room? Use Table 8.3.

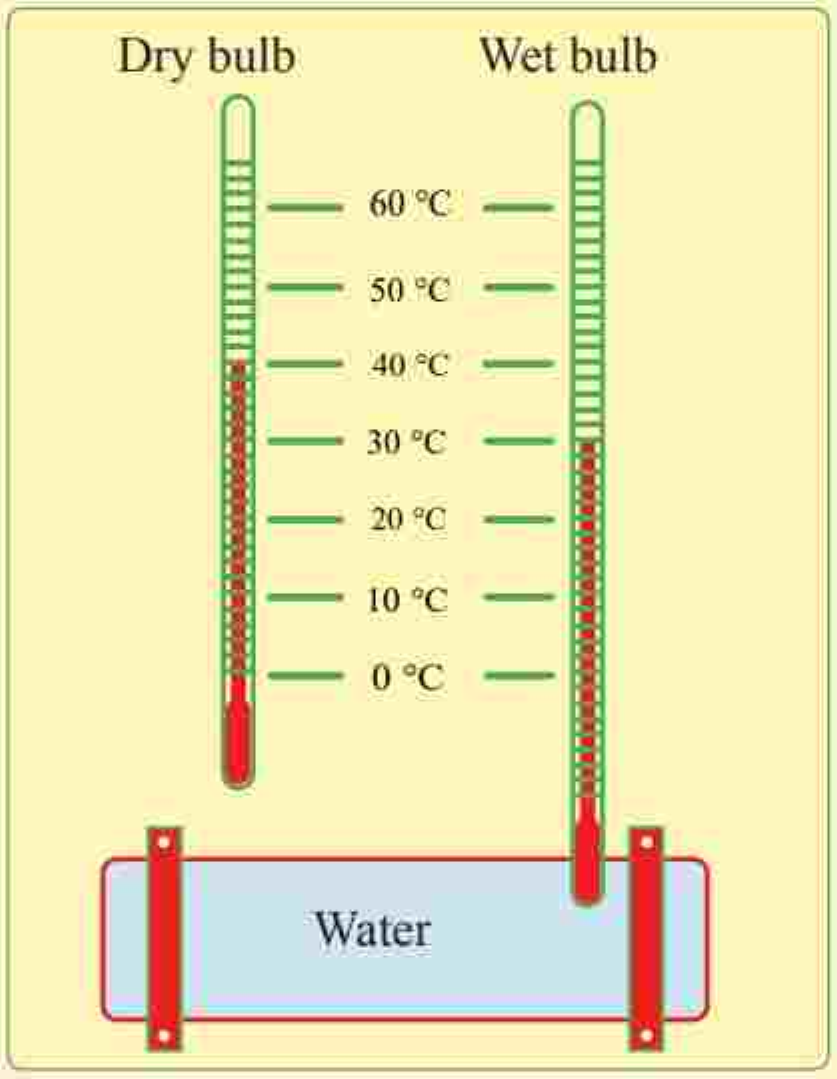


Figure 8.15

13. Table 8.4 shows the dew point in four towns at 07: 00 a.m.

Table 8.4

City	Dew point (°C)
Arusha	1
Morogoro	12
Zanzibar	5
Dar es Salaam	4

(a) In which town is the relative humidity the highest?

(b) In which town is the relative humidity the lowest?

Chapter Nine

Current electricity

Introduction

Current electricity is widely used in industries, offices, communication and in domestic activities. Domestic appliances such as electric fans, electric cookers, lighting devices, fridges, televisions and radio make use of current electricity. Therefore, current electricity plays a very important role in our daily life. For example, you cannot study comfortably at night if the electricity goes off. Understanding the concept of current electricity is therefore very important for human development. In this chapter you will learn the concept of electromotive force and the potential difference, resistance to the flow of electric current and heating effect of an electric current. You will also learn about electrical installation in buildings and the mode of action of different types of cells. The competencies developed will enable you to connect and analyse simple electric circuits and perform domestic electrical installation. You will also be able to effectively use and interpret power ratings of different electrical appliances found in your home and understand their power consumption.

Electromotive force and potential difference

Electromotive force (e.m.f) and potential difference (p.d) are common terms used in current electricity. Although e.m.f and p.d are two different quantities, they are both measured in volts (V). While e.m.f is the energy generated when any other form of energy is transformed into electrical energy, p.d is a measure of how much energy is transferred between two points in a circuit. Both e.m.f and p.d are important concepts for understanding current electricity.

Electromotive force (e.m.f)

Electromotive force (e.m.f) is the maximum potential difference (p.d) between the two terminals of a battery or a cell when no current is drawn from the battery or cell. The electromotive force is a force of energy source required to push electrons to flow through a conductor leading to electric current. Note that, the word “force” in this case does not mean force of interaction between bodies. One may draw an analogy of e.m.f to water pressure. When the pressure is high, more water flows through a pipe. Similarly, with higher e.m.f, more electrons flow through a conductor. In other words, e.m.f is the electrical pressure developed by a source of electrical energy. A few examples of sources of electromotive force include electrochemical cells such as dry cells and car batteries, thermoelectric devices, solar cells and electric generators. Labels of values with SI unit written on a battery or cell refer to its e.m.f (E). Figure 9.1(a) shows a dry cell whose e.m.f is 1.5 V and (b) a car battery whose e.m.f is 12 V.

(a) (b)

Figure 9.1: *A dry cell and car battery*

Potential difference (p.d)

When an electric device such as a bulb is connected to a cell, electric current flows through the device and some of the electrical energy is converted into light and heat. The amount of energy converted per unit charge is equal to the potential difference (p.d) across the device. Potential difference (p.d) is also called *voltage*. Electric potential difference (p.d) is the work done in moving a unit charge in a circuit from one point to another.

$$p.d(V) = \frac{work\ done\,(W)}{charge\,(Q)}$$

Like the e.m.f, the SI unit for p.d is the volt (V). One volt is equivalent to one joule per coulomb. Figure 9.2 shows a circuit diagram with the dashed box indicating a region where potential difference between two points of a resistor can be measured. Therefore, the difference between e.m.f and p.d is that, e.m.f is the potential difference across the cell terminals when no current flows or no load is connected while p.d is measured when current flows through the circuit or when a load is connected. The load here refers to the resistance, R of the electrical appliance connected which converts electric current into another form of energy. Figure 9.2 shows a circuit having an e.m.f source and a resistor as a load.

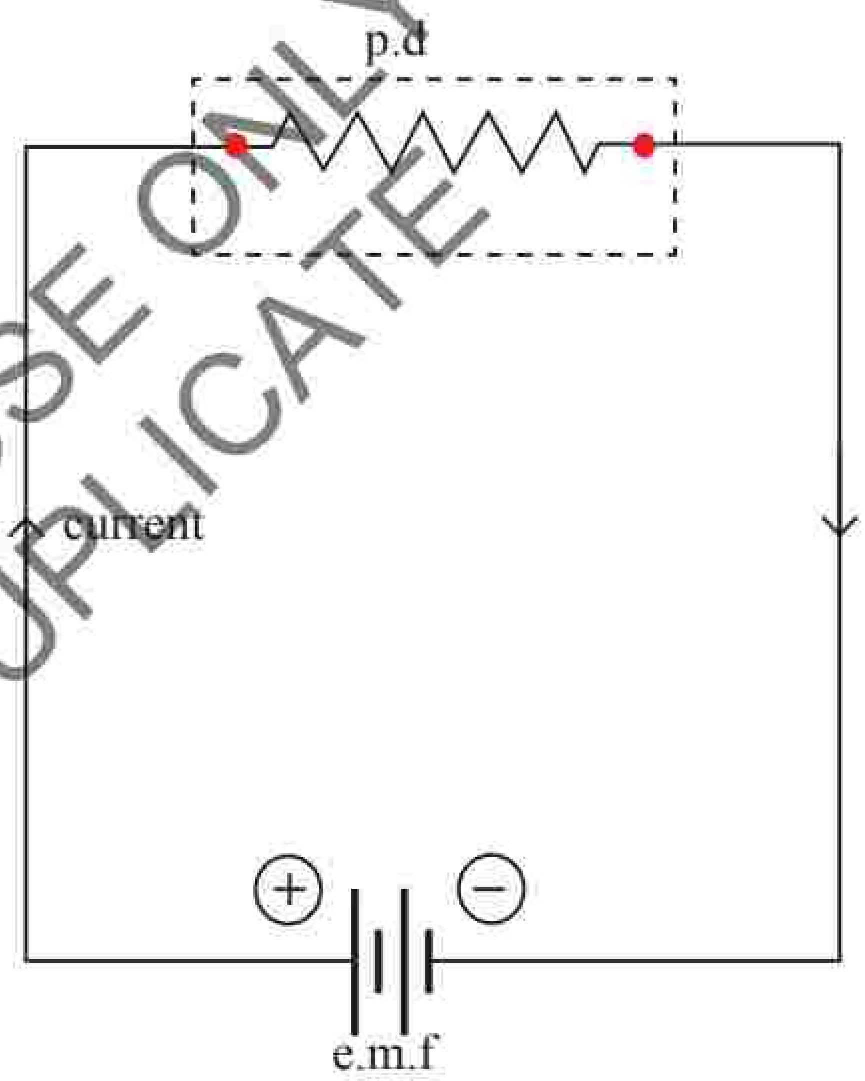


Figure 9.2: *Electromotive force (e.m.f) and potential difference (p.d)*

Measurement of e.m.f of a cell and p.d across a conductor

The e.m.f of a cell and p.d are measured by using a high resistance device called a voltmeter. Voltmeter measures the difference in potential between two points. It is always connected in parallel with the cell or the load because the e.m.f of a cell is compared to the p.d across the voltmeter's terminals. Normally, the positive terminal of the voltmeter is connected to the positive terminal of a cell, whereas the negative terminal of the voltmeter is connected to the negative terminal of a cell to measure correctly the e.m.f of the cell.

Activity 9.1

Aim: To measure the e.m.f of a cell and potential difference of a conductor

Materials: dry cell (1.5 V), voltmeter, bulb, two switches, connecting wires

Procedure

1. Connect the circuit as shown in Figure 9.3.

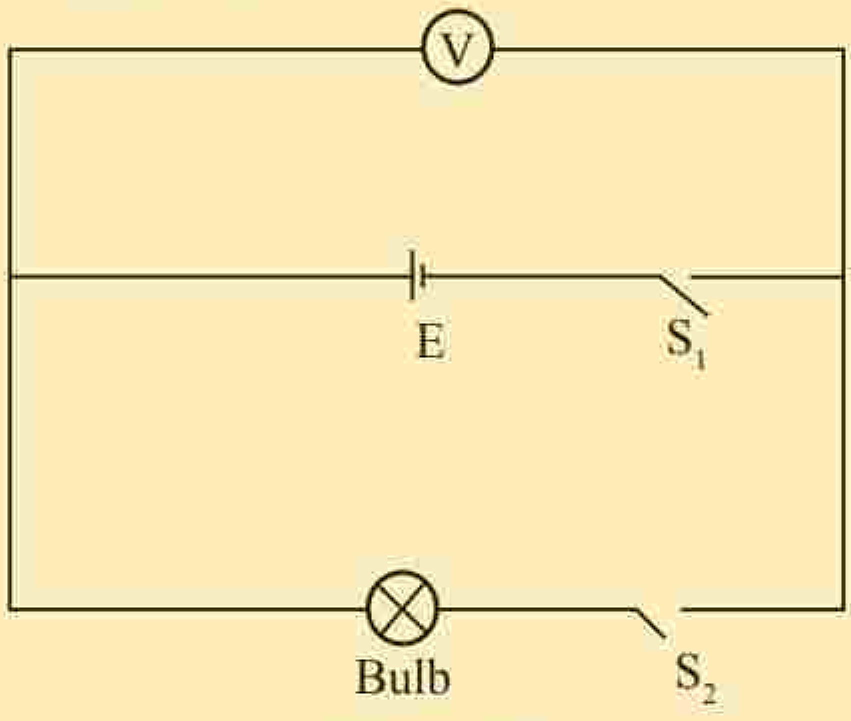


Figure 9.3

2. Close switch S_1 then note the voltmeter reading.
3. Close both switches (S_1 and S_2) and record the voltmeter reading.

Questions

(a) Compare the voltmeter reading when only S_1 is closed and when both S_1 and S_2 are closed. Explain your results.

(b) What is the e.m.f of the cell?

(c) What is the p.d across the bulb?

The voltmeter reading when the current is flowing through the bulb (when both switches are closed) is less than the reading when no current flows through the bulb (only switch S_1 is closed). The voltmeter reading when no current flows out of the cell or battery is the cell's e.m.f. The voltmeter reading when both switches are closed and the current flows through the circuit is the p.d across the bulb.

Electric current in a conductor

When an e.m.f source is connected to a conductor to form a complete circuit, charges flow from one terminal to another through the materials that connects the two terminals. The flow of charges forms a stream of charged particles moving through the circuit. This stream is known as an electric current, normally denoted by the letter *I*. The charged particles that form electric current can be electrons, holes or ions. The moving charged particles are called charge carriers. The type of charge carrier depends on the material through which charges flow. In normal conductor, charge carriers are electrons. In semiconductors, charge carriers can be electrons or holes. In an electrolyte, the charge carriers are ions, while in plasma, charge carriers are both ions and electrons.

Electric current flowing through a circuit is measured by using a device called an ammeter. The SI unit of electric current is ampere, which is the flow of electric charges per unit time. That is, an ampere is the flow of one coulomb of charges per second. The ampere is denoted by the letter A.

Electric charge and time

The electric current, I, flowing through a circuit can be determined by the amount of charges flowing at a point in the circuit at a given time. That is,

$$I = \frac{charge\ (Q)}{time\ (t)}$$

But, $Q = ne$,

therefore,

$$I = \frac{ne}{t}$$

where n is the number of electrons and e is the charge of an electron.

Example 9.1

Given that, the charge of an electron is 1.6×10^{-19} C, find the number of electrons that pass in one second through any cross-section of a conductor in which there is a steady current of 1 ampere.

Solution

$$I = \frac{ne}{t}$$

$$n = \frac{It}{e} = \frac{1\ \text{A} \times 1\ \text{s}}{1.6 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18}$$

Therefore, the number of electrons is 6.25×10^{18}.

Resistance to an electric current

The flow of charges through a wire in a circuit can be compared to the flow of water through a pipe connecting two water reservoirs. Water flows through the pipe if there is a pressure difference between the two reservoirs. As water flows through a pipe, it loses energy due to friction between the moving water molecules and the pipe's inner walls. Similarly, electric charges will flow between two points if one point is at a higher electric potential than the other point. This difference in electric potential is the potential difference (p.d) between the points. Therefore, electric current flows when there is a p.d between points in a circuit.

The flow of charges through a material is not direct. As charges flow in a material, they encounter numerous collisions with atoms in the material, resulting in the hindrance of the flow of charges. This hindrance of charge flow by the material is known as electrical resistance, denoted by the letter R. In other words, electrical resistance is the opposition to the flow of electrical current through a material.

A material that offers low resistance to the flow of current is termed a good conductor. On the other hand, a material is said to be an insulator if it does not allow the flow of electric current through it. Some materials have resistance in between that of conductors and insulators; these materials are known as semiconductors. The SI unit of electrical resistance is ohm, denoted by a symbol Ω. It is worth noting that electrical current loses its energy as it flows through a conductor.

Exercise 9.1

1. Briefly explain the diffences between electromotive force and potential difference.
2. How is the voltage measured in an electrical circuit?
3. Explain what is meant by an electric current.
4. State the SI unit of an electric current.
5. Briefly explain how the voltmeter is protected against overloads.

The potential difference across a conductor, current and resistance

The relationship between p.d across a conductor, current flowing through the conductor and the resistance of the conductor can be determined by performing Activity 9.2.

Activity 9.2

Aim: To determine the relationship between potential difference, current and resistance

Materials: a d.c power supply or 1.5 V cell, ammeter, bulb, switch, voltmeter, rheostat, connecting wires

Procedure

1. Use the apparatus to make an electric circuit as shown in Figure 9.4.

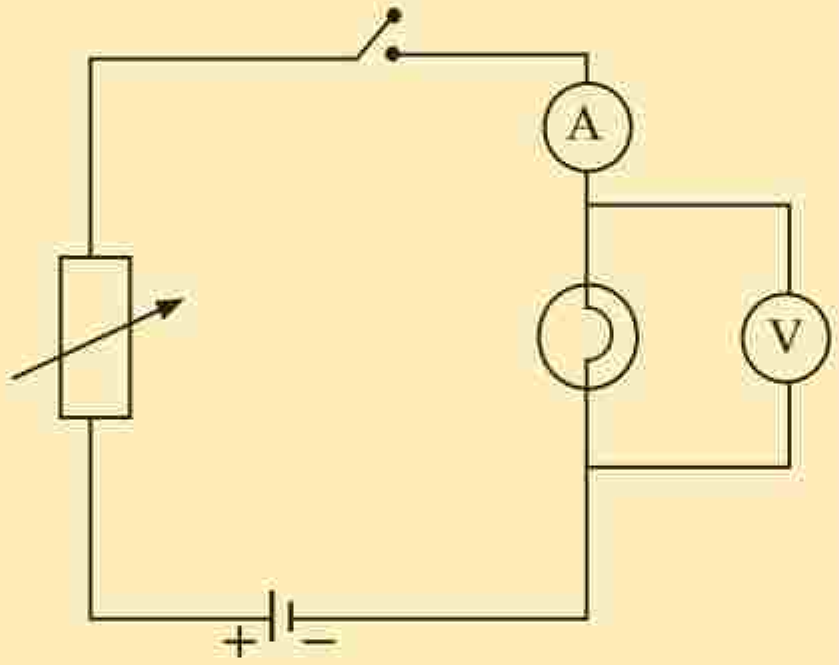


Figure 9.4

2. With the switch open, measure and record the potential difference across the bulb and the current through the circuit by using the voltmeter and ammeter, respectively.
3. Close the switch and adjust the rheostat until the voltmeter reads 0.2 V.
4. Measure and record the current through the bulb. Observe the brightness of the bulb.
5. Continue increasing the p.d. at intervals of 0.2 V up to 1.2 V by adjusting the rheostat. Read the corresponding current value in each case.
6. Record the results as shown in Table 9.1.

Table 9.1

P.d. across the bulb (V)	Current through the bulb (A)
0.2	
0.4	
0.6	
0.8	
1.0	
1.2	

Questions

(a) What happened to the bulb's brightness as the p.d. was increased?

(b) Plot a graph of p.d against the current.

(c) Explain the shape of the graph. What does the nature of this graph imply?

Current flows when a cell is connected across the ends of a conductor. For a metal conductor such as copper wire, the current flowing through it is directly proportional to the potential difference across the ends of the conductor. This complies with Ohm's law, which states that, "*The current passing through a conductor at constant temperature is proportional to the potential difference between its ends*."

That is,

$$I \propto V \text{ or } I = kV$$

where k is the constant of proportionality called conductance of a conductor, denoted by G. Hence,

$$G = \frac{I}{V};\ \frac{1}{G} = \frac{V}{I}$$

Nevertheless, the reciprocal of conductance is the resistance of the conductor. That is, $\frac{1}{G} = R$. Therefore, Ohm's law can be expressed as:

$$R = \frac{V}{I} \text{ or } V = IR$$

The ohm can be defined as the resistance of a conductor such that, when a potential difference of 1 volt is applied to two points of a conductor, a current of 1 ampere flows through it. Therefore,

$$ohm = \frac{volt}{ampere}$$

By keeping temperature and other physical properties constant, the resistance (R) of a conductor remains constant. This relationship enables us to determine currents and voltages in electric circuits.

Activity 9.3

Aim: To determine the value of unknown resistance by using Ohm´s law

Materials: battery (E), ammeter (A), unknown resistance (R), switch (S), voltmeter (V), rheostat (R_h) of at least 500 Ω , connecting wires

Procedure

1. Connect an electric circuit as shown in Figure 9.5.

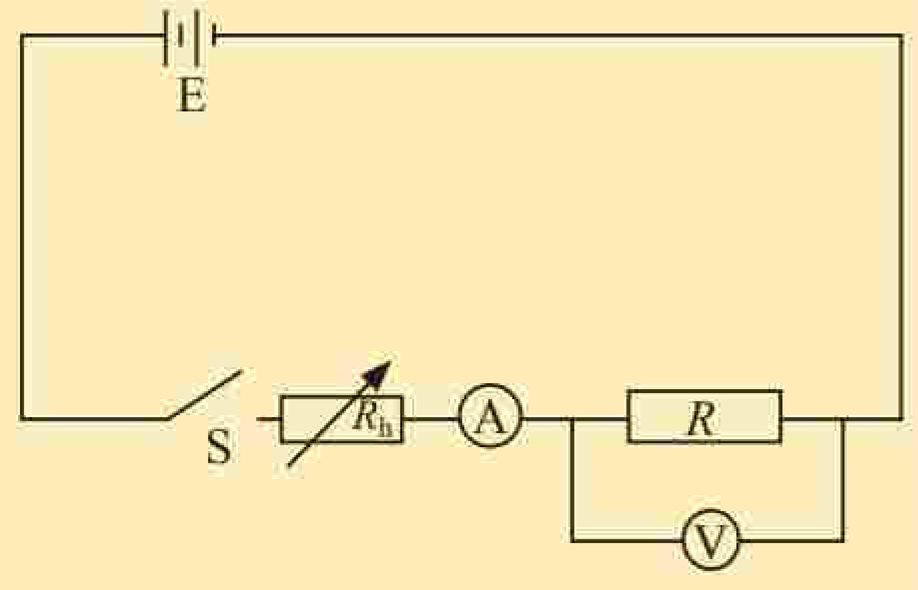


Figure 9.5

2. Close switch S and adjust the rheostat, R_h so that a current of 0.1 A passes through the unknown resistor, *R*.
3. Record the current, *I* and p.d, *V*.
4. Adjust the R_h to get a current of 0.2 A passing through *R*. Record *I* and *V* as in step 3.
5. Repeat step 4 to obtain at least five more readings.
6. Record the results in the table similar to Table 9.2.

Table 9.2

Current, I (ampere)	p.d. V (volt)
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	

Questions

(a) Plot the graph of p.d, *V* against the current, *I*.

(b) What does the nature of the graph imply?

(c) Deduce the value of the unknown resistor, *R*.

Example 9.2

A cell develops a potential difference of 2 V across a resistor of 4 Ω. What is the current in the resistor?

Solution

Using Ohm's law,

$$V = IR; \ I = \frac{V}{R}$$

$$I = \frac{2 \text{ V}}{4 \ \Omega} = 0.5 \text{ A}$$

The current in the resistance is 0.5 A.

Factors that determine the resistance of a conductor

The property of a conductor to resist the flow of electric current is called resistance. The resistance of a conductor is determined by its temperature, length, cross-sectional area and type of material.

Temperature

When the temperature of a conductor increases, atoms of the conductor vibrate more vigorously. This increases the number of collisions between the charge carriers and the atoms in the conductor, as a result, resistance increases. In some materials, the resistance varies almost linearly with temperature. For instance, the resistance of copper (Cu) varies approximately linearly with temperature. For some other materials, resistance does not vary linearly with temperature law. However, the effect of temperature on the resistance of some alloys, such as constantan and manganin, is very small. Note that, when the temperature of a conductor increases, the length of the conductor also increases due to linear expansion.

Length of conductor

When the length of a conductor is increased, while other factors are kept constant, the resistance of the conductor also increases. This is because charges collide with more atoms in a long conductor than in a short conductor. This means that, the resistance (*R*) of the wire is proportional to the length (*l*) of the wire. That is,

$$R \propto l$$

Cross-sectional area

Consider two conductors, sucha that one is longer than the other. If other factors remain constant, their resistance increases with the increase of length. A conductor with a larger cross-section area has more charge carriers to carry the electrical current than that of a small cross-section. This means, the resistance *R* of a conductor is inversely proportional to the cross-sectional area, *A* of the conductor, that is,

$$R \propto \frac{1}{A}$$

Nature of material

Resistance of a conductor also depends on the material used to make the conductor. For example, a conductor made from steel will have higher resistance than one made of copper of identical dimensions at the same temperature. For example, steel has a higher resistivity than copper. A material's property that resist current flow is known as the material's resistivity, denoted by ρ.

Since $R \propto l$ and $R \propto \frac{1}{A}$, then,

$$R \propto \frac{l}{A}$$

Hence,

$$R = \frac{\rho l}{A}$$

where ρ is the constant of proportionality. This constant of proportionality is the resistivity of the material, given as:

$$\rho = \frac{RA}{l}$$

Resistivity, is therefore, the measure of the ability of a material to oppose the flow of an electric current. The SI unit of resistivity is the ohm-metre (Ωm).

Activity 9.4

Aim: To determine the resistivity of constantan wire

Materials: battery, ammeter, switch, voltmeter, rheostat, connecting wires, micrometer screw gauge, a constantan wire of length 20 cm

Procedure

1. Connect an electric circuit as shown in Figure 9.6.

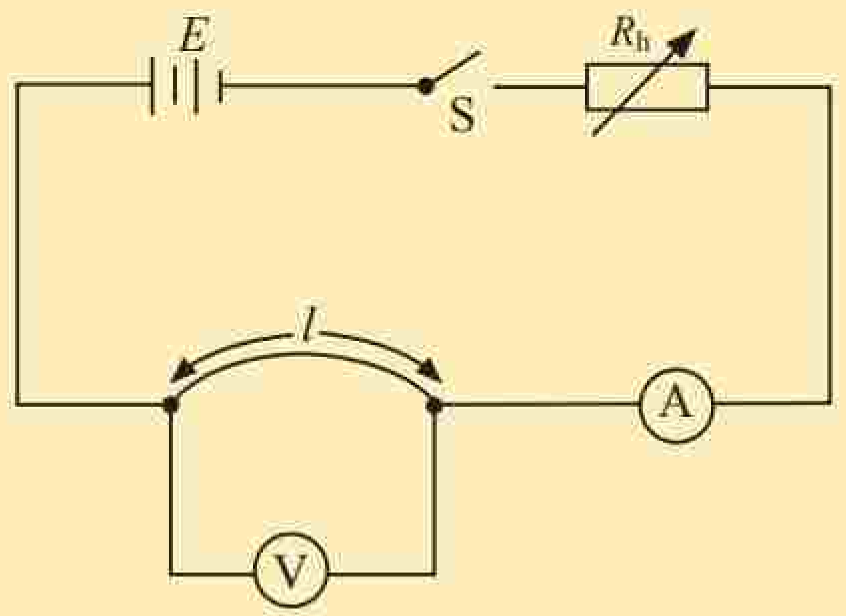


Figure 9.6

2. Close the switch S and adjust the rheostat so that the ammeter reads 0.2 A.
3. Record the current, I and potential difference, V.
4. Repeat step 2 and 3 with current, I = 0.3 A, 0.4 A and 0.5 A.
5. Record the results in the format shown in Table 9.3.

Table 9.3

Current, I (amperes)	p.d., V (volt)
0.2	
0.3	
0.4	
0.5	

6. Measure the diameter, D, and length l, of the constantan wire. Calculate the cross-sectional area, A.

Questions

(a) What happened to the potential difference as the current increased?

(b) Plot the graph of p.d against current, I and determine the slope of the graph.

(c) Deduce the resistivity of the constantan wire.

Different materials have different values of resistivity. The resistivities of some materials are given in Table 9.4.

Table 9.4: Resistivities of some materials

Material	Resistivity in Ωm (at 20 °C)
Silver	1.6×10^{-8}
Copper	1.68×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Steel	1.05×10^{-7}
Platinum	1.06×10^{-7}
Chromium	1.3×10^{-7}
Manganin	4.8×10^{-7}
Constantan	4.9×10^{-7}
Lead	2.1×10^{-7}
Mercury	9.8×10^{-7}
Nichrome	1.0×10^{-6}
Glass	$1 \times 10^{9} - 1 \times 10^{13}$
Rubber	$1 \times 10^{13} - 1 \times 10^{15}$
Quartz	7.5×10^{17}

Example 9.3

What is the resistance of a copper wire of length 20 m and a diameter of 0.080 cm? (Resistivity of copper $\rho_{cu} = 1.68 \times 10^{-8}$ Ωm).

Solution

Given that

$l = 20$ m, $\rho_{cu} = 1.68 \times 10^{-8}$ Ωm)for copper

$$R = \frac{\rho l}{A}$$

But $A = \pi r^2$

$$= 3.14 \times (4 \times 10^{-4}\,\text{m})^2$$

$$= 5.024 \times 10^{-7}\ \text{m}^2$$

Therefore,

$$R = \frac{\rho l}{A}$$

$$= \frac{1.68 \times 10^{-8}\ \Omega\text{m} \times 20\ \text{m}}{5.024 \times 10^{-7}\ \text{m}^2}$$

$$= 0.67\ \Omega$$

The resistance of the wire is $0.67\ \Omega$.

Example 9.4

A nichrome wire has a cross-sectional area of $4 \times 10^{-8}\text{m}^2$ and a resistivity of $1.0 \times 10^{-6}\Omega\text{m}$. If a resistor of resistance 11 Ω is to be made from this wire, calculate the length of the required wire.

Solution

Given that, $A = 4 \times 10^{-8}\text{m}^2$,

$\rho = 1.0 \times 10^{-6}\ \Omega\text{m}$, $R = 11\ \Omega$

Therefore,

$$l = \frac{AR}{\rho}$$

$$l = \frac{4 \times 10^{-8}\ \text{m}^2 \times 11\ \Omega}{1.0 \times 10^{-6}\ \Omega\text{m}} = 0.44\ \text{m}$$

The length of the wire is 0.44 m.

Example 9.5

A constantan wire has a length of 45 cm, a diameter of 0.37 mm and resistivity of $4.9 \times 10^{-7}\,\Omega$m.

(a) What is the resistance of the wire?

(b) What will be the current in the wire if it is connected to a 1.5 V cell?

Solution

(a) $\rho = 4.9 \times 10^{-7}\ \Omega\text{m}$, $l = 0.45$ m

But $R = \frac{\rho l}{A}$

and $A = \pi r^2$, where

$$r = \frac{0.37\ \text{mm}}{2} = 0.185\,\text{mm} = 1.85 \times 10^{-4}\ \text{m}$$

$$A = 3.14 \times \left(1.85 \times 10^{-4}\ \text{m}\right)^2 = 1.075 \times 10^{-7}\ \text{m}^2$$

Therefore,

$$R = \frac{\rho l}{A}$$

$$= \frac{4.9 \times 10^{-7}\ \Omega\text{m} \times 0.45\ \text{m}}{1.075 \times 10^{-7}\ \text{m}^2} = 2.05\ \Omega$$

The resistance of the wire is $2.05\ \Omega$.

(b) Given, V = 1.5 V and R = 2.05 Ω, then,

$$I = \frac{V}{R}$$

$$= \frac{1.5\ \text{V}}{2.05\ \Omega} = 0.73\ \text{A}$$

The current in the wire will be 0.73 A.

Example 9.6

A wire of length 9 m and radius 0.05 cm has a resistance of 5 Ω. Calculate the resistivity of the wire.

Solution

Given that

$l = 9\text{ m}, r = 0.05\text{ cm} = 5\times10^{-4}\text{ m}, R = 5\ \Omega$

Resistivity of the wire is given by,

$$\rho = \frac{RA}{l}$$

$$= \frac{5\Omega \times 3.14 \times (5\times10^{-4})^2\text{ m}^2}{9\text{ m}} = 4.36\times10^{-7}\ \Omega\text{m}$$

The resistivity of the wire is $4.36\times10^{-7}\ \Omega\text{m}$.

Types of resistors

A resistor is an electrical component with two terminals. Resistors are used for restricting the flow of electric current in a circuit. That is, resistors control the magnitude of current in a circuit in accordance with Ohm's law. There are two basic types of resistors depending on their applications and characteristics. These are fixed resistors and variable resistors.

Fixed resistors

The most commonly used type of resistor is the fixed resistor. A fixed resistor has a constant value of resistance. Applications of fixed resistors include protecting electrical components from excess current, dividing voltage in a circuit, and controlling the working of some of the circuits. In a circuit, the fixed resistor is represented by the symbols shown in Figure 9.7. Diverse resistor materials are used to fabricate fixed resistors. These materials influence the resistor's properties such as tolerance and noise. Fixed resistors can be made of carbon, wound wire, metal oxide and metal films.

Figure 9.7: *Circuit symbol for fixed resistors*

Carbon resistors: Carbon, which has ability to withstand high energy pulses, is mixed with clay as a binder and molded into a cylinder to form carbon resistor. These resistors are very useful in most electronic circuits, usually with very large resistances ranging from 1 Ω to 10 MΩ. Figure 9.8 shows a carbon resistor, which is also called an electronic resistor.

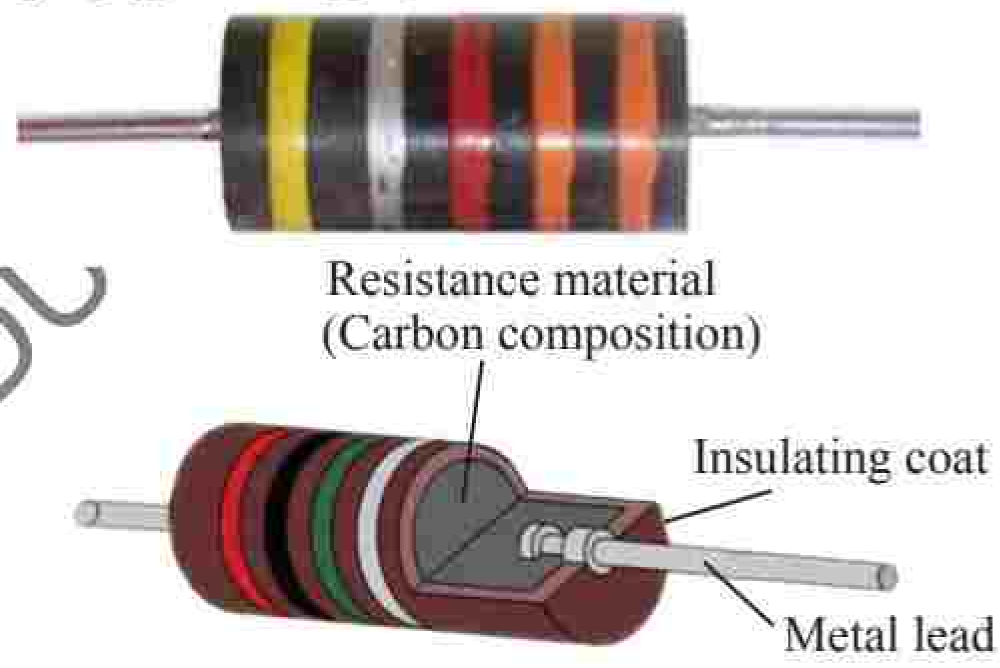


Figure 9.8: *A carbon resistor*

Wire-wound resistors: These are resistors made by winding wires of certain metallic alloys around spools as shown in Figure 9.9. Materials used to make the wire-wound resistors have a high resistivity. Such materials include nichrome and manganin. In a wire-wound resistor, a large current can pass through it without overheating. Wire-wound resistors are typically used in high power circuits and power supply systems. They are generally used where high sensitivity accurate measurement and balanced current control is required.

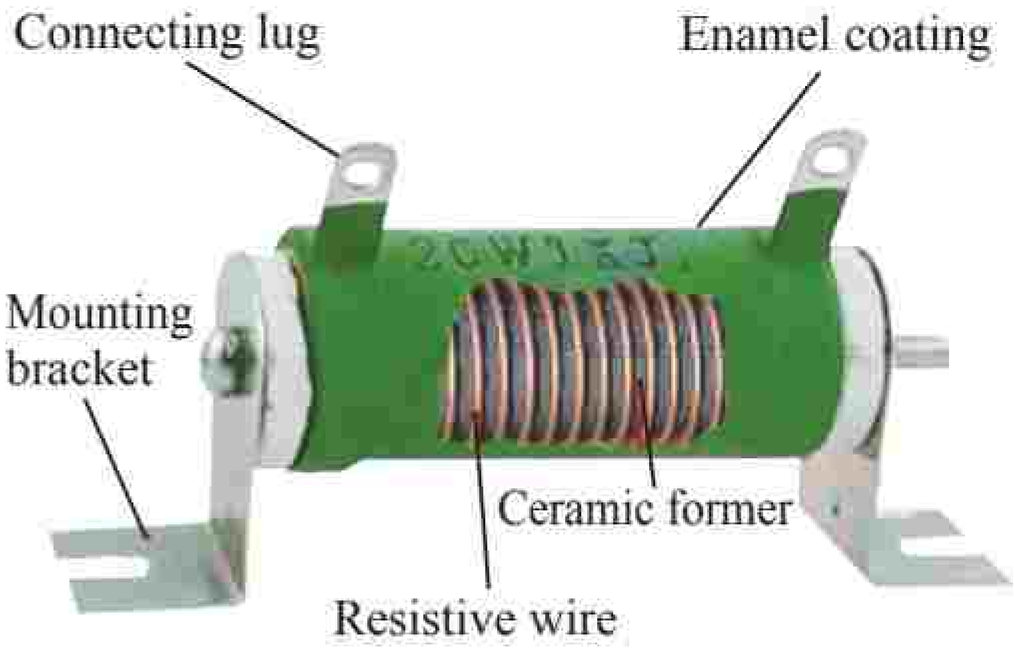


Figure 9.9: *A wire-wound resistor*

Metal film resistors: Metal film resistors, some of which are shown in Figure 9.10, are usually made up of a stable ceramic core, coated with metal alloys, such as nickel-chromium alloy. These resistors have a thin metal layer as a resistive element in a non-conducting material. Metal film resistors have higher accuracy than carbon resistors. They have good characteristics for tolerance and stability.

Figure 9.10: *Metal film resistors*

Metal oxide film resistors: Metal oxide resistors are made up of a ceramic core coated with metal oxide, such as tin oxide, as shown in Figure 9.11. Metal oxide film resistors perform much better than metal film and carbon resistors as they can operate at high temperatures.

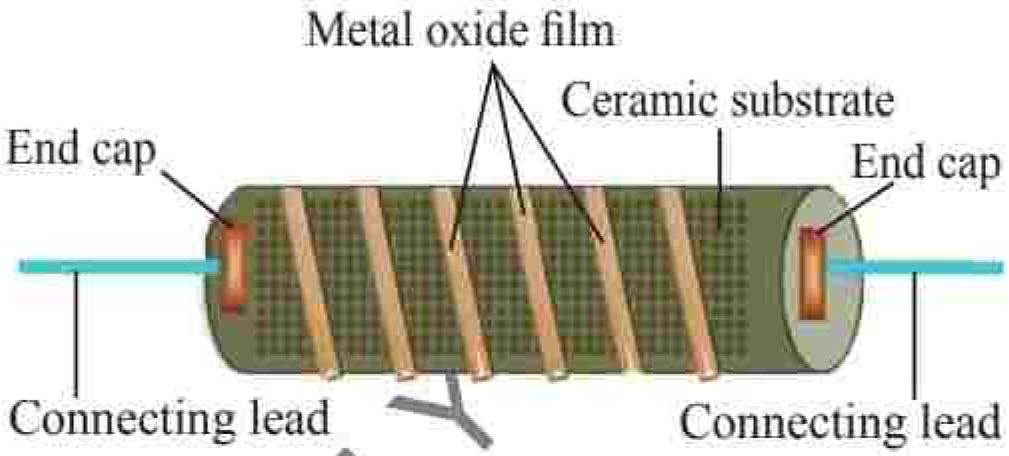
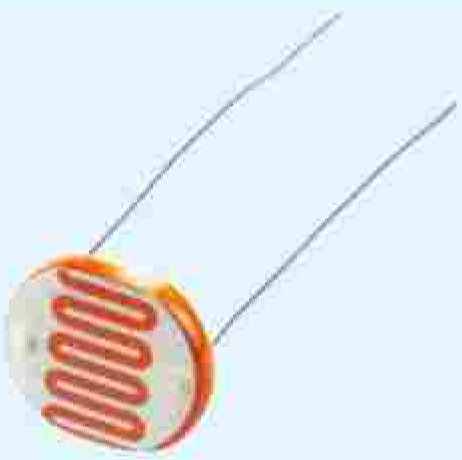
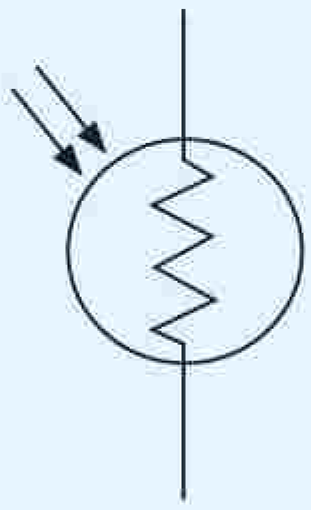


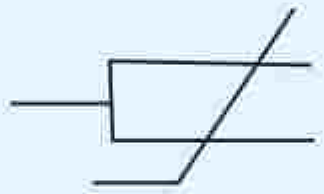
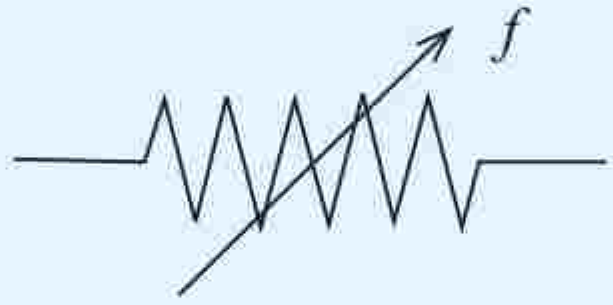
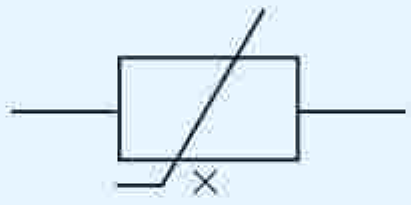
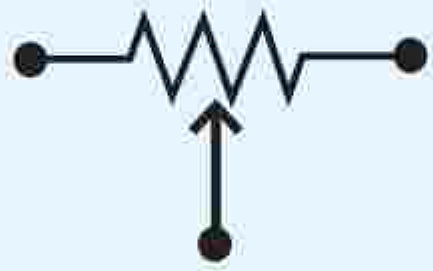
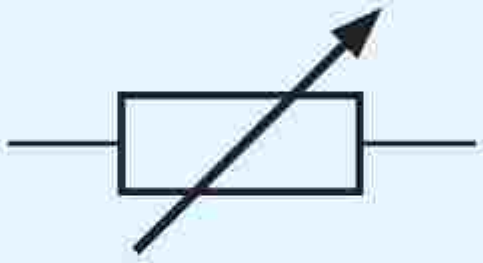
Figure 9.11: *Metal-oxide film resistors*

Variable resistors

These are resistors whose electric resistance can be adjusted to suit the requirements of a circuit. Its resistance can be adjusted from zero to the maximum value (the value proposed by the manufacturer). Examples of variable resistors include rheostat, voltage dividers, thermistors, photoresistors and magneto-resistors. Some types of variable resistors and their circuit symbols are shown in Table 9.5.

Table 9.5: Variable resistors and their symbols

Appearance of a resistor	Symbol
 Photoresistor	

Appearance of a resistor	Symbol
 Thermistor	
 Force resistor	
 Magneto-resistor	
 Potentiometer	
 Rheostat	

Rheostat

In Activity 9.2, the amount of current flowing in the circuit was controlled by a rheostat. This section introduces you to the physics of a rheostat (Figure 9.12). A rheostat, has three-terminals. When used as a current limiter, terminals A and C in Figure 9.12 (b) are fixed points whereas terminal B is the adjustable terminal to control the amount of current passing in a circuit. The circuit is composed of the terminal A connected to terminal B through the wiper or slider in direct contact with the resistive wire and resistance rod.

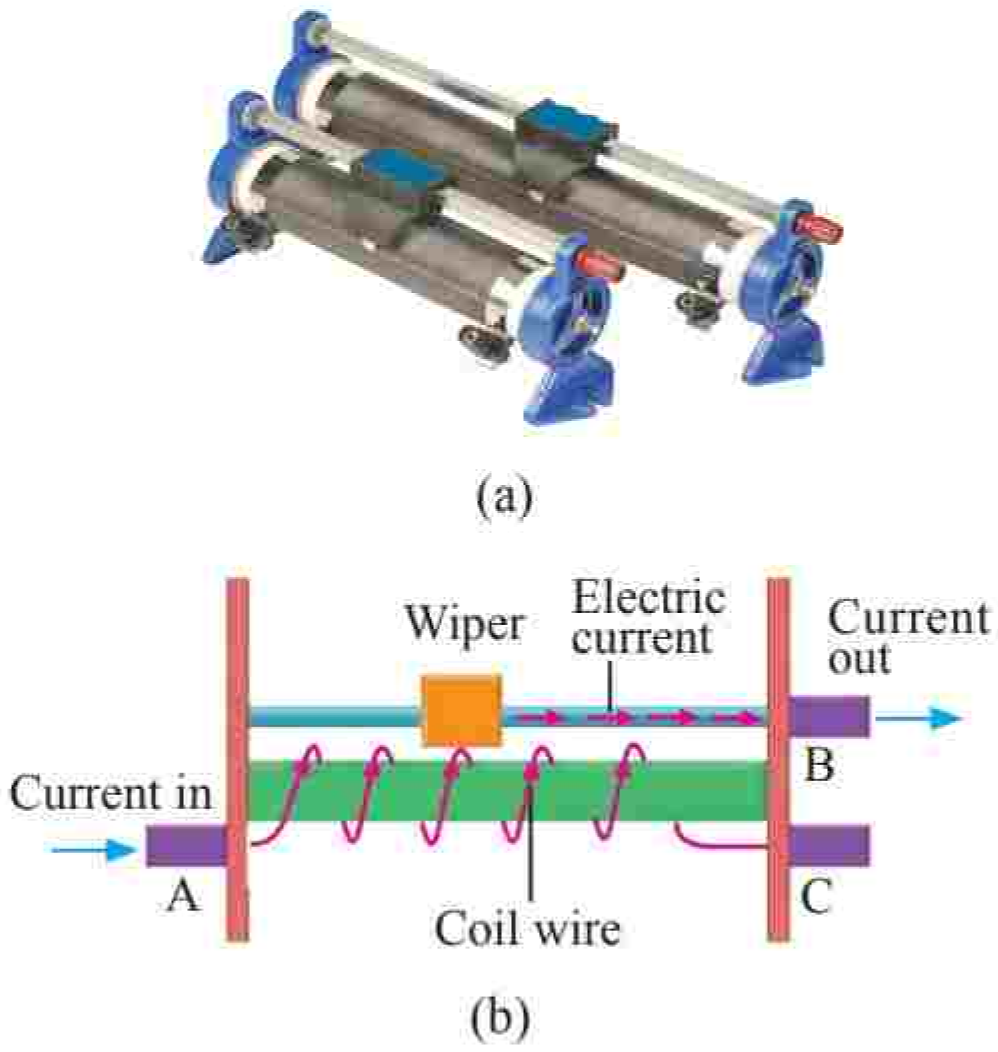


Figure 9.12: *A rheostat*

From the previous section, you learned that, $\rho = \frac{RA}{l}$. For constant resistivity and cross-sectional area *A*, the resistance, *R,* of a conductor depends only on its length. Thus, if the slider or wiper is moved close to point A, rheostat will offer minimum resistance. This is because the length of the resistive wire is reduced, then more current will flow through the circuit. On the other hand, if the wiper is close to point B, the length of the resistive wire is at maximum, thus, less current will flow in the circuit as the unit offers maximum resistance. It is also possible to connect points B and C. Here, minimum resistance will be offered when slider is close to B, and maximum when close to A. Since it is possible to control the current flowing in the circuit, then, the rheostat can be used to control light dimmers, speed of motors, temperature of electrical heaters and ovens.

Voltage dividers

For a particular application such as tuning a radio or increasing the volume of a radio, the circuit is designed such that, the portion of its input voltage is used as an output (V_{out}). This means that there is a division of the input voltage (V_{in}) in the circuit (Figure 9.13 (a)). This can be realized by using a voltage divider.

A voltage divider has resistor arrangement in three terminals. Two terminals are fixed, and one terminal is connected to a sliding contact. There are two basic types of voltage dividers, namely, rotary Figure 9.13 (b) and Figure 9.13 (c), and linear (Figure 9.13 (d)).

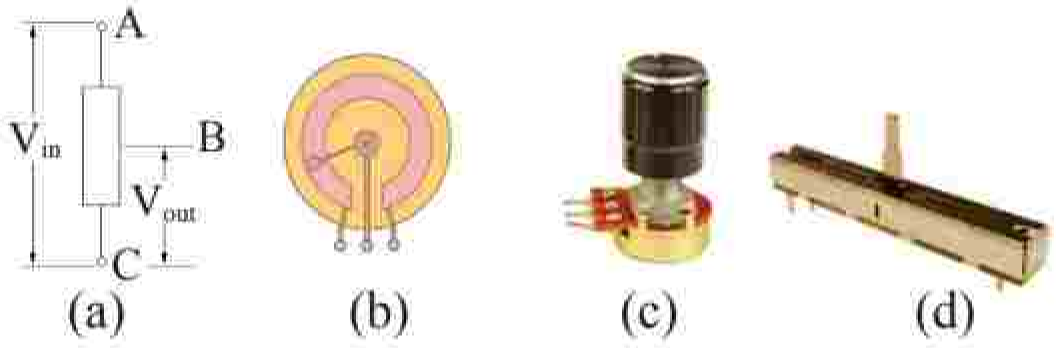


Figure 9.13: *Rotary and linear potentiometers*

In a physics laboratory, a rheostat (Figure 9.12) can be used as a voltage divider by making use of all its three terminals. The e.m.f source is connected to terminals A and C and the output is taken at terminal B. When the slider is at point A, the voltage divider offers minimum potential difference. On the other hand, if it is closed to C, it gives maximum voltage. Since the voltage divider controls the voltage, it can be used in controlling the volume of radio and other electronic appliances. It can also be used in tuning signals in radios.

Thermistors

A thermistor is a variable resistor whose resistance changes when the surrounding temperature changes. There are two types of thermistors; negative temperature coefficient and positive temperature coefficient thermistors. For the positive temperature coefficient thermistor, the resistance increases when temperature increases, whereas, for the negative one, resistance decreases with an increase in temperature. Thermistors are used in sensing circuits such as the one used to control cooling fan's speed. The appearance and symbol of thermistors are shown in Table 9.5.

Magneto-resistor

In a magneto-resistor, the resistance changes with the strength and direction of a magnetic field. The resistance of magnetic field increases when the magnetic strength applied to a magneto-resistor increases and decreases when the strength of the magnetic field decreases.

Magneto-resistors are sensors which measure the presence, strength and direction of magnetic field. Appearance and symbol of magneto-resistor are shown in Table 9.5.

Light-dependent resistor

A light-dependent resistor (LDR) is a variable resistor in which resistance changes when light energy is applied to the resistor. The resistance decreases when the intensity of the light increases. They are also called photoresistors. LDR are used in photographic light meter, fire smoke or burglar alarms. They are also used to control street lights. The appearance and symbol for LDR are shown in Table 9.5.

Equivalent resistors

Depending on the purpose, the fixed resistor can be connected in either parallel or series configuration. The resultant resistance in the circuit is called *total resistance or effective resistance, or equivalent resistance.*

Resistors in series

When two or more resistors are connected end to end consecutively, and the same current flows through each resistor, such a connection is said to be a series connection. It is a single-loop circuit combination of resistors and battery as shown in Figure 9.14.

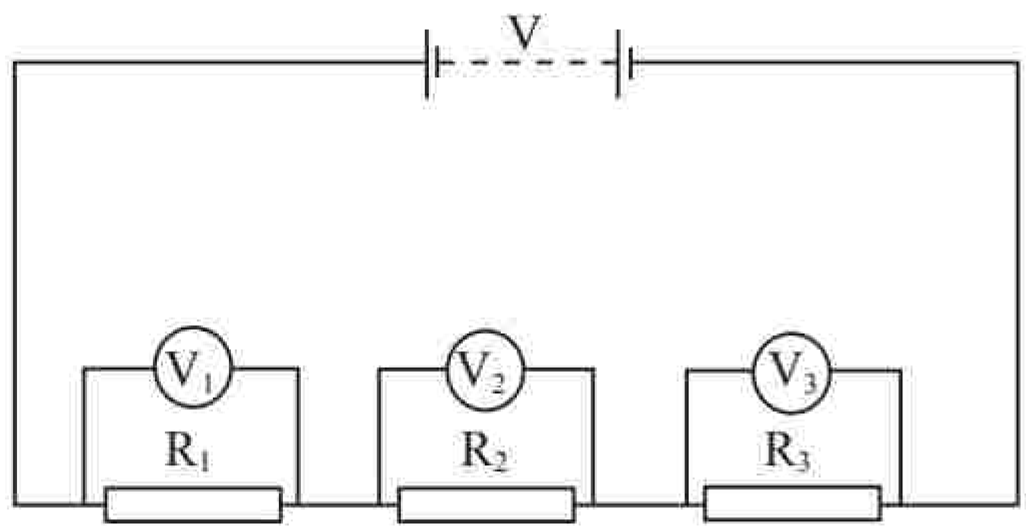


Figure 9.14: *Resistors connected in series*

The current flowing in the circuit is the same at all points. The sum of the p.d in the external circuit is equal to the potential difference across the battery. Therefore,

$$V = V_1 + V_2 + V_3$$

From Ohm's law, the current, *I,* in the circuit is given by,

$$I = \frac{V}{R_T}$$

where R_T is the total resistance, thus,

$$V = IR_T$$

$$IR_T = IR_1 + IR_2 + IR_3$$

$$IR_T = I(R_1 + R_2 + R_3)$$

$$R_T = R_1 + R_2 + R_3$$

When resistors are connected in series, the total resistance in the circuit is the sum of the individual resistances. Therefore, when there are n resistors connected in series, the total or equivalent resistance is given by,

$$R_T = R_1 + R_2 + R_3 + ... + R_n$$

Note that, when resistors are identical, the total resistance is given by $R_T = nR$, where R is the resistance of individual resistor and n is the number of identical resistors.

Resistors in parallel

Resistors are said to be in parallel connection when two or more resistors are placed side by side with their corresponding ends joined together, such that, the same p.d is applied to each resistor. Figure 9.15 shows the parallel connection of resistors.

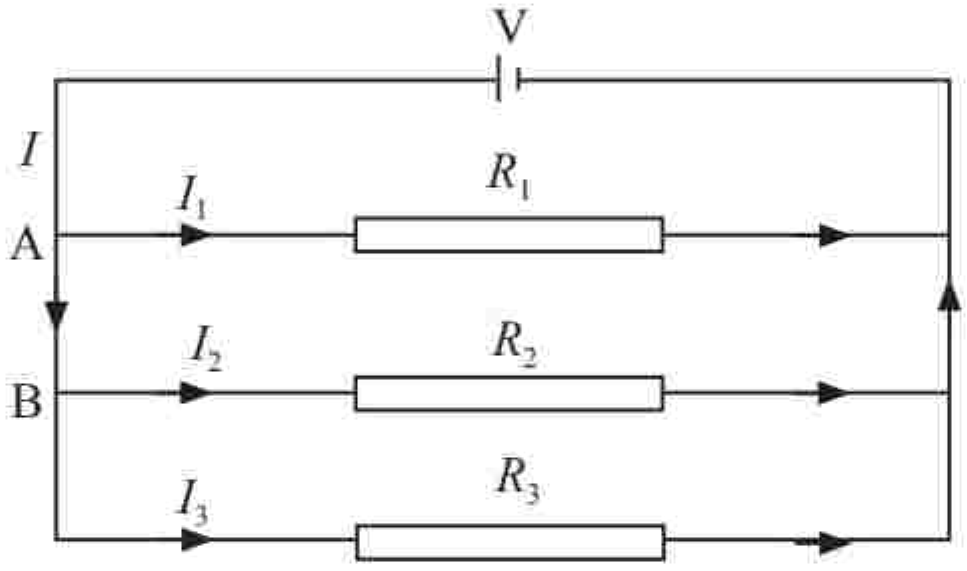


Figure 9.15: *Resistors connected in parallel*

When resistors are connected in parallel, the p.d. across each resistor is the same as the total voltage in the circuit. Though the voltage is the same for all branches, the currents flowing in each branch are not equal. Thus, Ohm's law can be applied in each branch. To find the different currents, one considers the current through each junction. The sum of the currents in each resistor in Figure 9.15 is equal to the total current in the circuit. That is,

$$I = I_1 + I_2 + I_3$$

From Ohm's law, the current, *I,* in the circuit is given by,

$$I = \frac{V}{R_T}$$

where R_T is the equivalent resistance. Applying Ohm's law to each resistor gives:

$$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{V}{R_T} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Therefore,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

When resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

Generally, when *n* resistors are connected in parallel, the reciprocal of equivalent resistance is given by,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

For identical *n* resistors connected in parallel, $R_1 = R_2 = R_3 = ... = R_n = R$. The reciprocal of total resistance is given by $\frac{1}{R_T} = \frac{n}{R}$. Therefore, the total resistance is given by $R_T = \frac{R}{n}$.

It should be noted that, when resistors are connected in series, their total resistance is higher than that of individual resistors. However, for resistors connected in parallel, the total resistance is lower than that of individual resistors.

Activity 9.5

Aim: To investigate the effective resistance for resistors in series and in parallel

Materials: two dry cells of 1.5 V each, connecting wires, bulbs

Procedure

1. Connect two 1.5 V cells in series and then connect them to one of the bulbs, A as shown in Figure 9.16.

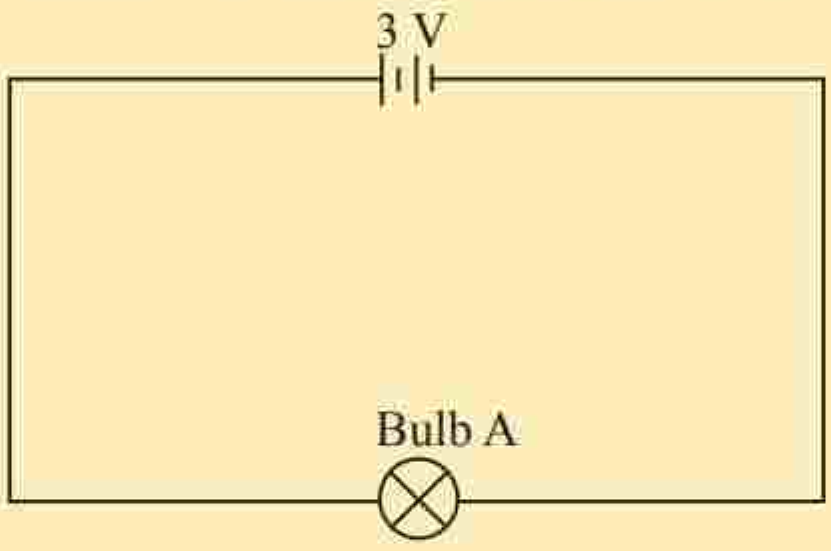


Figure 9.16

Observe the brightness of the bulb.

2. Connect another bulb, B in series with bulb A as shown in Figure 9.17.

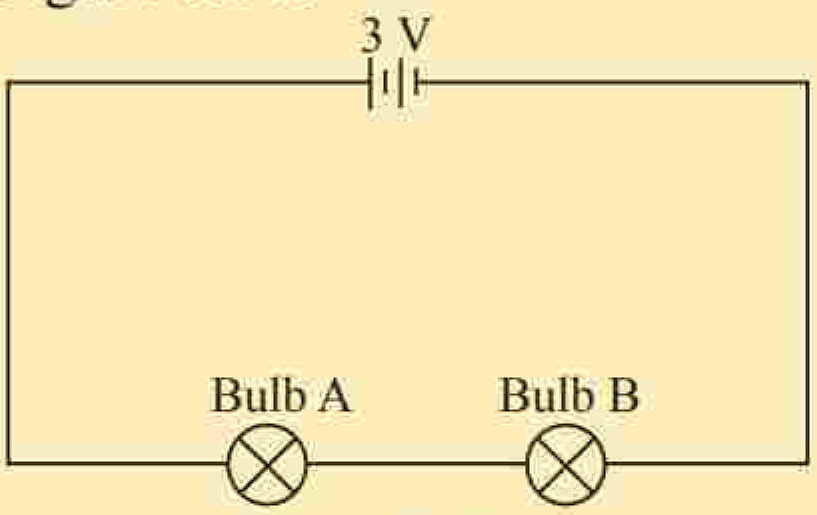


Figure 9.17

Observe the brightness of the bulbs.

3. Connect another bulb, C, in series with bulbs, A and B as shown in Figure 9.18. Observe how the brightness of bulbs A and B changes.

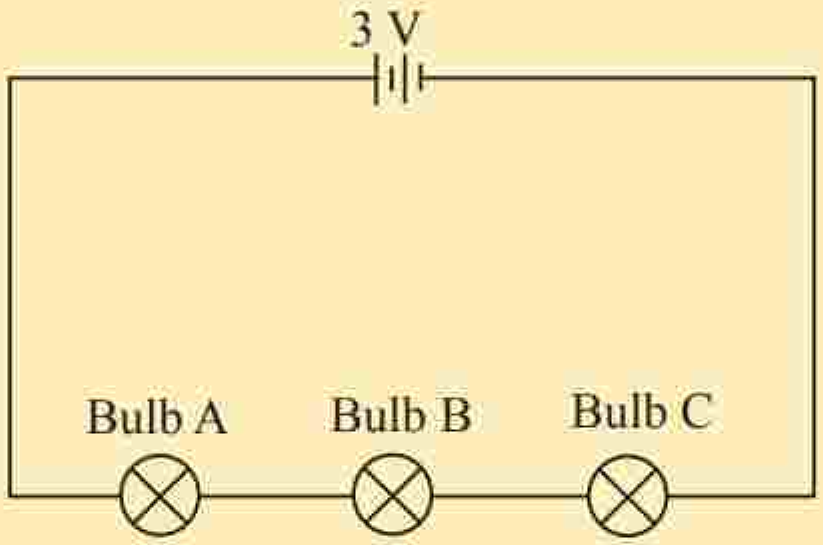


Figure 9.18

4. Connect bulb B in parallel with bulb A as shown in Figure 9.19. Observe the brightness of the bulbs compared to the initial brightness of bulb, A.

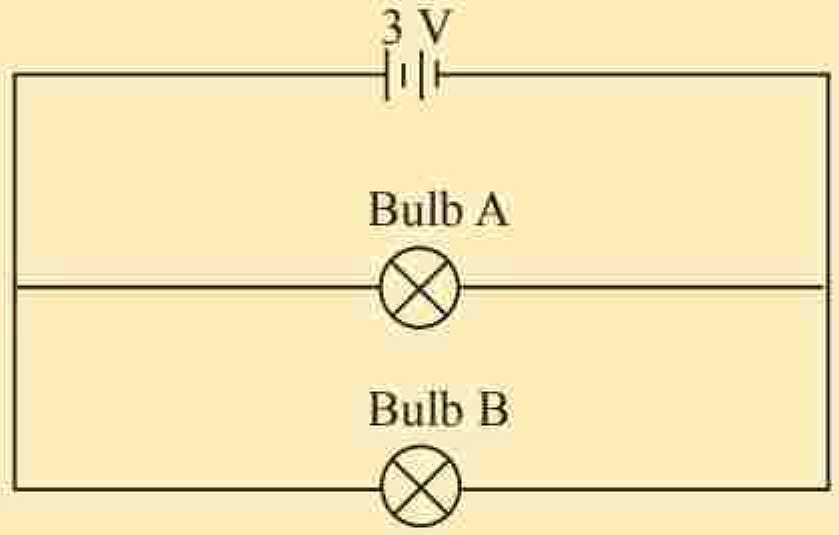


Figure 9.19

5. Connect bulb C in parallel with bulbs A and B as in circuit shown in Figure 9.20. Observe how the brightness of the bulbs changes. Disconnect the circuit.

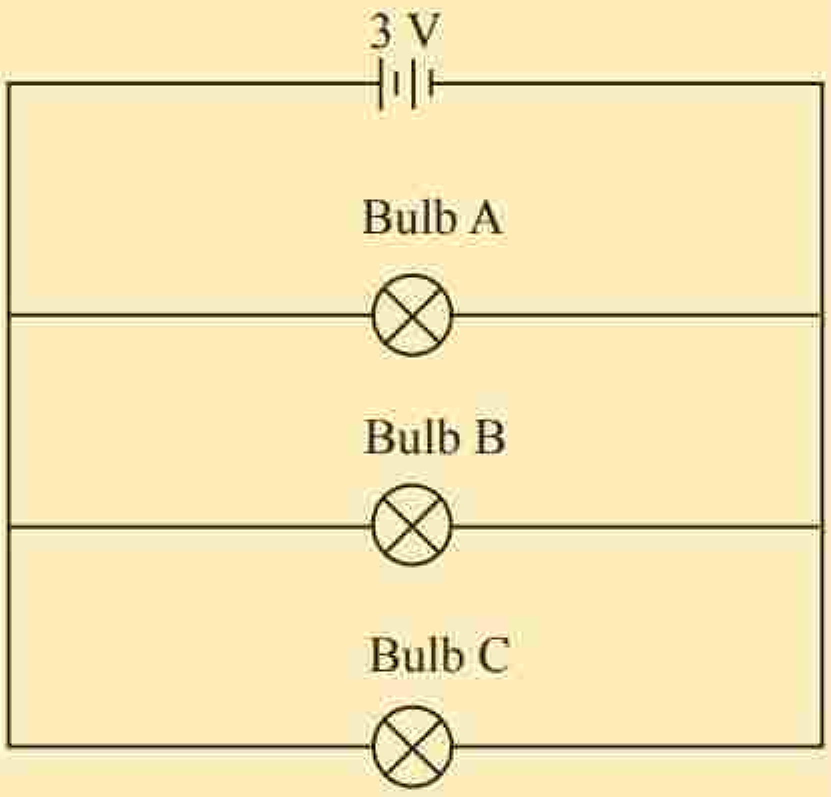


Figure 9.20

Questions

(a) What happens to the brightness of the bulbs as more bulbs are added in series?

(b) What happens to the brightness of the bulbs as more bulbs are added in parallel?

The brightness of the bulbs was reduced as more bulbs were added in series. This is because the total resistance increases when more bulbs (resistors) are added and connected in series. When bulbs are connected in parallel, the brightness of the bulbs was increased with an increase in the number of bulbs in the circuit. This is because the effective resistance in the circuit decreases with an increase in the number of bulbs. The effective resistance in parallel connection is always lower than that of the individual bulbs.

Example 9.7

Figure 9.21 shows an electric circuit.

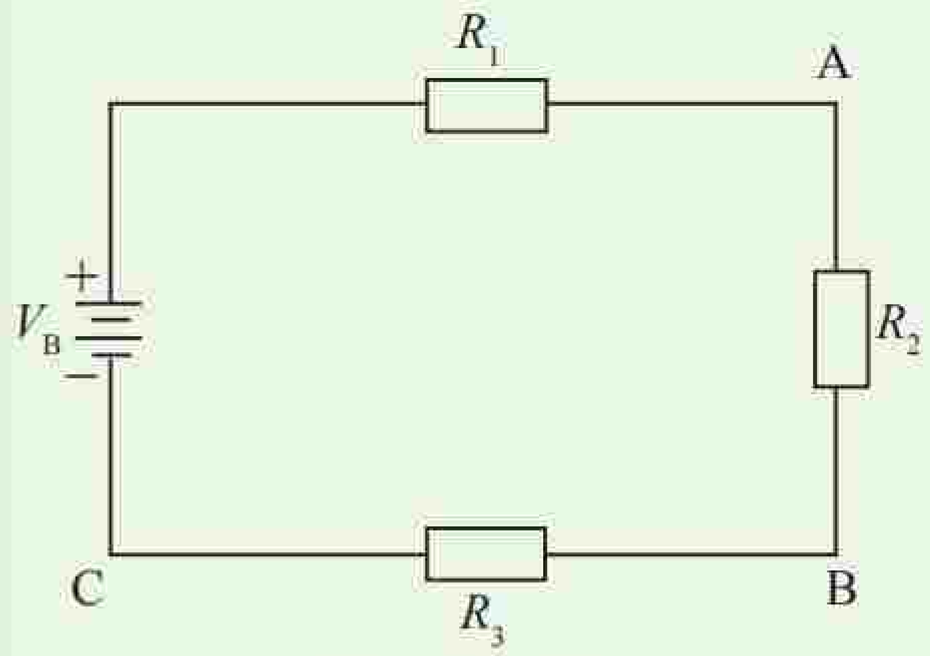


Figure 9.21

Given that, the battery has a voltage (V_B) = 9 V, R_1 = 4 Ω, R_2 = 5 Ω, R_3 = 6 Ω:

(a) What is the total resistance of the circuit?

(b) What current flows in the circuit?

(c) What is the potential drop across each resistor?

(d) What is the electric potential at point A ?

Solution

(a) $R_{total} = R_1 + R_2 + R_3$

$= 4\ \Omega + 5\ \Omega + 6\ \Omega$

$= 15\ \Omega$

(b) $I = \dfrac{V}{R_{total}}$

$= \dfrac{9\ \text{V}}{15\ \Omega}$

$= 0.6$ A

(c) p.d across R_1

$V_{R_1} = IR_1$

$= 0.6\ \text{A} \times 4\ \Omega = 2.4$ V

p.d across R_2

$V_{R_2} = IR_2$

$= 0.6\ \text{A} \times 5\ \Omega = 3.0$ V

p.d across R_3

$V_{R_3} = IR_3$

$= 0.6\ \text{A} \times 6\ \Omega = 3.60$ V

(d) To find the electric potential at point A, take note that, the potential is measured with reference to the negative terminal of the source. This becomes our reference, 0 V. Therefore, we start at the negative terminal of the battery where the potential is 0 V. Travelling around the circuit in the direction of electron flow, we pass through R_3 where there is a voltage drop of 3.6 V, then pass R_2 where the p.d.

is 3.0 V up to point A. Therefore, the potential at point A is,

$$V_A = V_{R_3} + V_{R_2}$$
$$= 3.6 \text{ V} + 3.0 \text{ V}$$
$$= 6.6 \text{ V}$$

Example 9.8

In the circuit shown in Figure 9.22, the battery has a voltage $V = 9$ V, $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 6\ \Omega$.

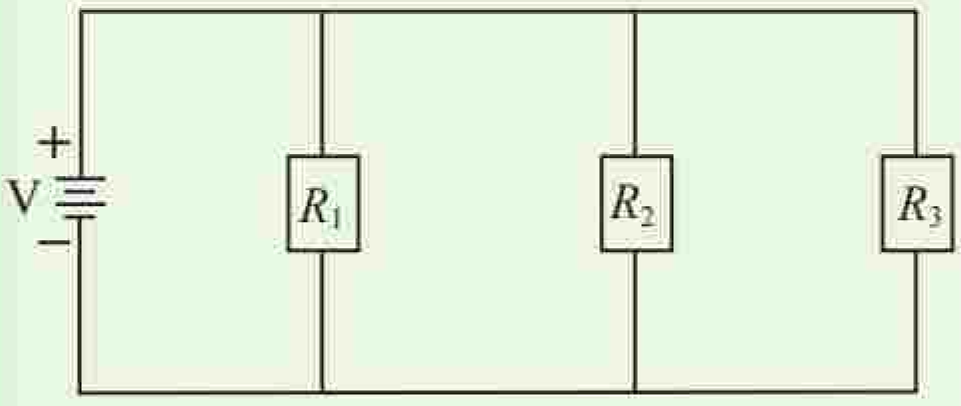


Figure 9.22

From the given circuit, find the,

(a) effective resistance and total current flowing.
(b) current flowing through each resistor.
(c) sum of currents flowing through the circuit.

Solution

Given $R_1 = 4\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 6\ \Omega$; required to find the total resistance R_T.

Using

(a) $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{4\Omega} + \frac{1}{5\Omega} + \frac{1}{6\Omega} = \frac{37}{60\Omega}$$

$$R_T = 1.62\,\Omega$$

Total current, I is given by

$$I = \frac{V}{R_T} = \frac{9 \text{ V}}{1.62\ \Omega} = 5.56 \text{ A}$$

The total current flowing in the circuit is 5.56 A.

(b) The current flowing through each resistor is given by;

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2} \text{ and } I_3 = \frac{V}{R_3}$$

$$I_1 = \frac{9 \text{ V}}{4\ \Omega} = 2.25 \text{ A};\ I_2 = \frac{9\ V}{5\ \Omega} = 1.8 \text{ A}$$

$$\text{and } I_3 = \frac{9\ V}{6\ \Omega} = 1.5 \text{ A}$$

The current through R_1 is 2.3 A, R_2 is 1.8 A and R_3 is 1.5 A.

(c) The total current flowing through the circuit is,

$$I = I_1 + I_2 + I_3$$
$$= 2.25 \text{ A} + 1.8 \text{ A} + 1.5 \text{ A} = 5.55 \text{ A}$$

The total current flowing through the circuit is 5.55 A.

Example 9.9

Determine the current reading on the ammeter in the circuit shown in Figure 9.23.

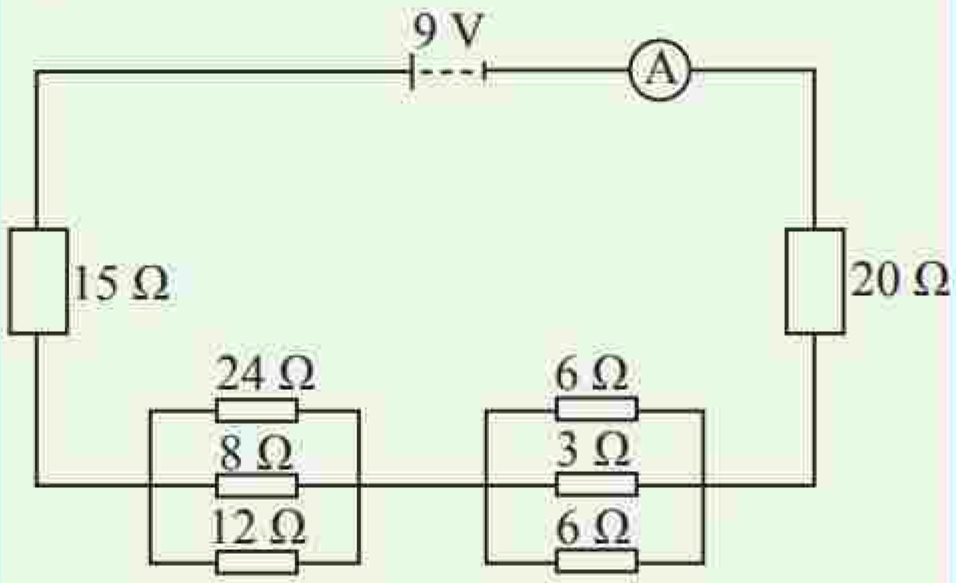


Figure 9.23

Solution

For three resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For the first set of three resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{24\ \Omega} + \frac{1}{8\ \Omega} + \frac{1}{12\ \Omega} = \frac{1}{4\ \Omega}$$

$$R_T = 4\ \Omega$$

For the second set of resistors in parallel,

$$\frac{1}{R_T} = \frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}$$

$$\frac{1}{R_T} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = \frac{2}{3\ \Omega}$$

$$R_T = 1.5\ \Omega$$

The effective resistance of the resistors in parallel, i.e., 4 Ω and 1.5 Ω, are now in series with resistors of 15 Ω and 20 Ω. Therefore, the total resistance R_T is given by,

$$R_T = 4\ \Omega + 1.5\ \Omega + 20\ \Omega + 15\ \Omega = 40.5\ \Omega$$

The current is obtained through,

$$I = \frac{V}{R_T} = \frac{9\text{ V}}{40.5\ \Omega} = 0.22\text{ A}$$

Therefore, the current reading on the ammeter in the circuit is 0.22 A.

The internal resistance of a cell

A cell always offers resistance to the passage of current in a circuit. This resistance is called internal resistance. The internal resistance of a cell acts as if it is a resistor connected in series with the cell so that when current flows through the cell, there is a potential drop across this resistor. Consider a cell of e.m.f, *E* and internal resistance, *r* connected across an external resistor, *R* as shown in Figure 9.24 (a).

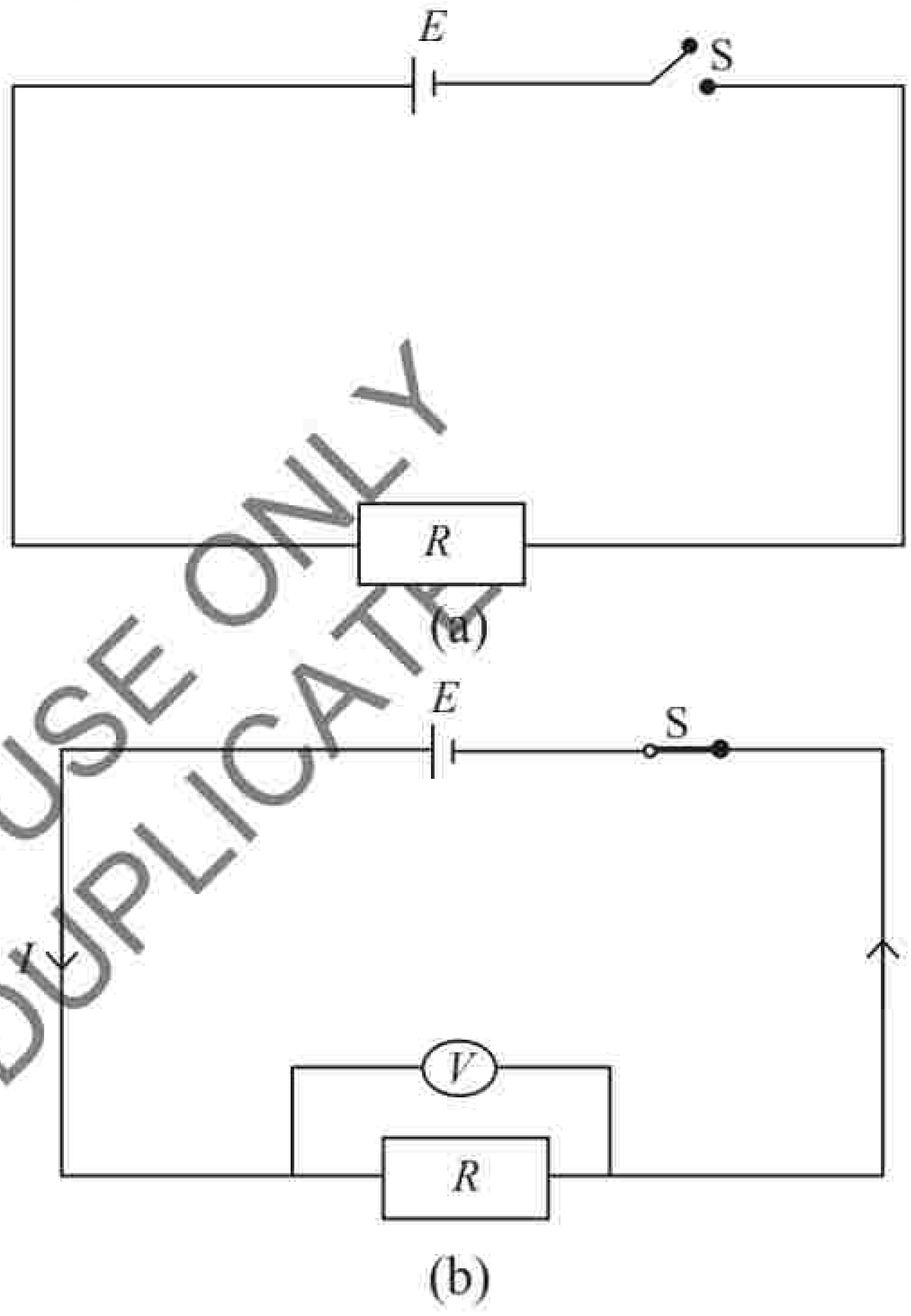


Figure 9.24: *A simple circuit*

Suppose the switch is closed as shown in Figure 9.24 (b), current, *I,* flows through the circuit. From Ohm's law, the p.d, *V*, across the resistor, *R,* is given by,

$$V = IR$$

The potential drop (V_1) caused by the cell's internal resistance is given by,

$$V_1 = Ir$$

The total voltage in the circuit is the sum of the potential drop due to the cell's internal resistance and that of the resistor,

R. That is,

$$E = V + Ir$$

When no current is drawn in the circuit, e.m.f is equal to p.d across the cell $E = V$. Generally, the total e.m.f of the cell (s) is given by,

$$E = V + V_1$$

$$= IR + Ir$$

$$E = I(R + r)$$

The equation $E = I(R + r)$ can be used to determine the internal resistance of a cell and the e.m.f of the cell experimentally. From Figure 9.24, different $R\,(\Omega)$ values can be used to give different values of current, I (A), and the two sets of values are plotted as shown in Figure 9.25 based on the equation:

$$R = E\frac{1}{I} - r$$

With R in the vertical axis and reciprocal of current in the horizontal axis.

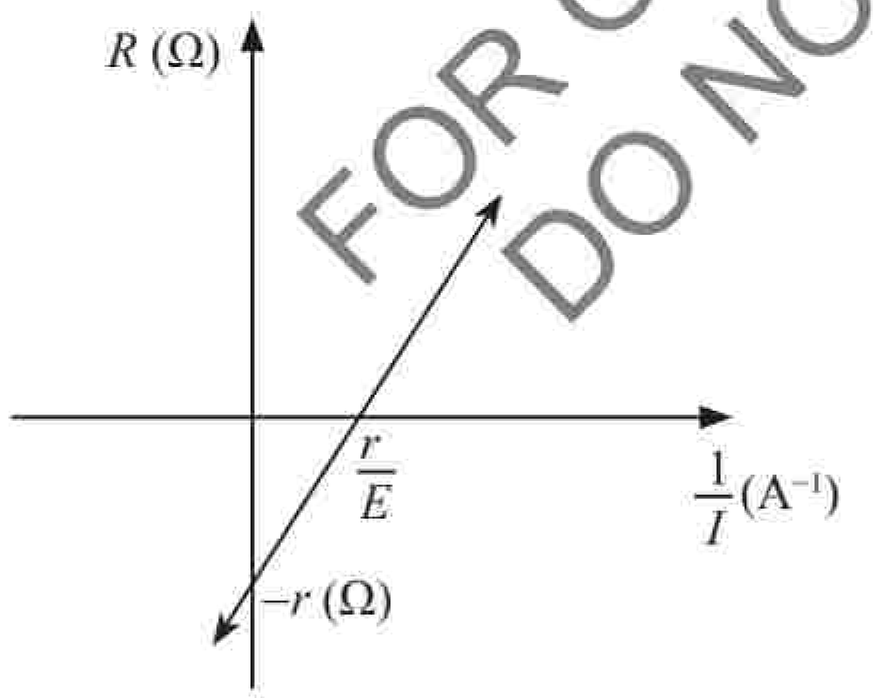


Figure 9.25: *The graph of resistance R against reciprocal of current,* $\frac{1}{I}$

The numerical value of e.m.f and r can be determined from the graph. The slope of the graph represents the value of e.m.f, E and the vertical intercept represents the value of internal resistance, r, of the cell. Similarly, when the axes of the variables are interchanged, the equation becomes:

$$\frac{1}{I} = \frac{1}{E}R + \frac{r}{E}$$

Therefore, the slope of this graph (Figure 9.26) represents the reciprocal of the e.m.f of the cell and the vertical intercept represents the product of slope and internal resistance of the cell.

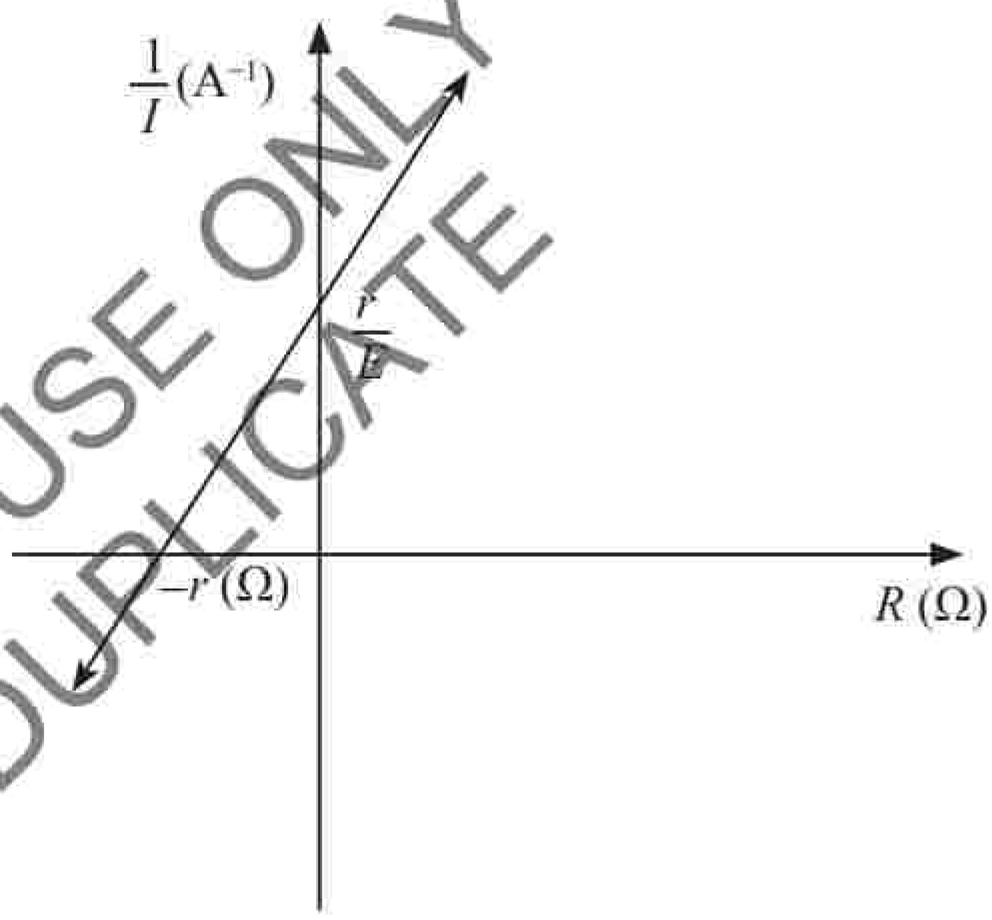


Figure 9.26: *The graph of reciprocal of current,* $\frac{1}{I}$ *against resistance, R*

Activity 9.6

Aim: To determine the electromotive force, *E,* and internal resistance, *r,* of a cell

Materials: a cell, set of standard resistors (2-10 Ω), ammeter, switch

Procedure

1. Connect the circuit as shown in Figure 9.27.

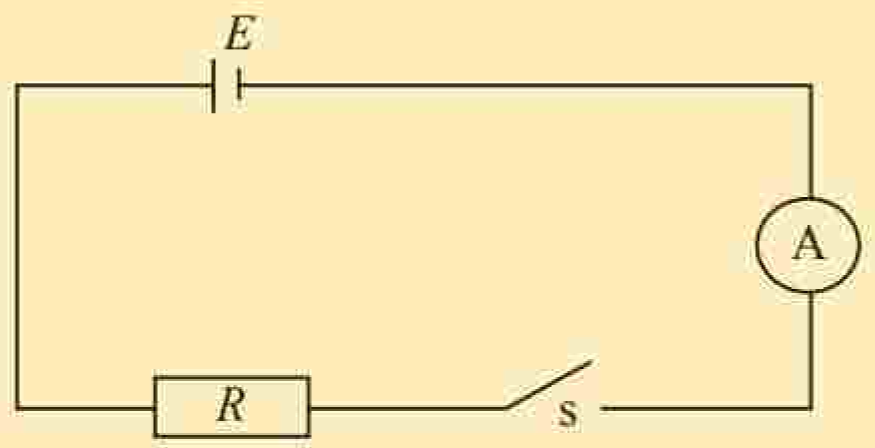


Figure 9.27

2. Set R at 10 Ω, close switch S and read the ammeter and record current, I.
3. Repeat step 2 for R = 8 Ω, 6 Ω, 4 Ω and 2 Ω.
4. Record results as shown in Table 9.6.

Table 9.6

Resistance, R (Ω)	Current, I (A)	$\frac{1}{I}$ (A^{-1})
10		
8		
6		
4		
2		

Questions

(a) Plot a graph of R against $\frac{1}{I}$.

(b) Determine the vertical intercept.

(c) What is the value of r?

(d) Determine the slope of the graph.

(e) What is the value of E?

Example 9.10

An electric cell with minimal voltage, E, has a resistance of 3 Ω connected across it. If the voltage falls to $0.6E$, calculate the internal resistance of this cell.

Solution

Referring the circuit of Figure 9.24 (b) with switch S closed:

$$E = E,\ R = 3\ \Omega,\ V = 0.6\ E$$

From

$$E = I(R + r)$$

$$E = IR + Ir$$

$$E = 0.6E + Ir$$

$$Ir = E - 0.6E$$

$$Ir = 0.4E \qquad (1)$$

From Ohms' law

$$V = IR$$

$$3I = 0.6E$$

$$I = \frac{0.6E}{3} \qquad (2)$$

Substituting equation (2) into (1) and solving for r, you get $r = 2\ \Omega$.

Wheatstone bridge

From Activity 9.3, Ohm's law was used to determine the value of the unknown resistance of a conductor. A similar approach cannot be applied to accurately measure a very low value of resistance down to the milli-Ohms (mΩ) ranges. However, when resistors are connected in series-parallel arrangement as shown in Figure 9.28, the value of unknown resistance of a conductor even in mΩ ranges can be measured. This diamond-like arrangement is called the Wheatstone bridge circuit. It consists of mainly four resistors known as the bridge arms and a galvanometer in between them.

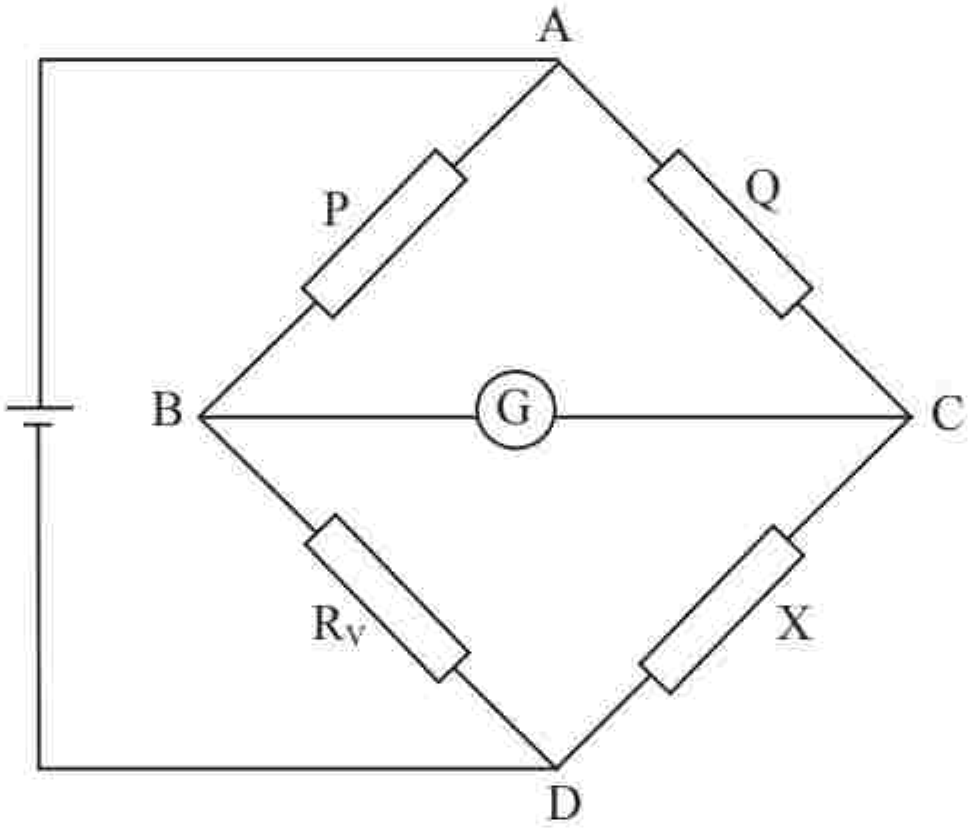


Figure 9.28: *The Wheatstone bridge*

Working principle of a Wheatstone bridge

To understand how the Wheatstone bridge works, let us consider a bridge circuit in Figure 9.29.

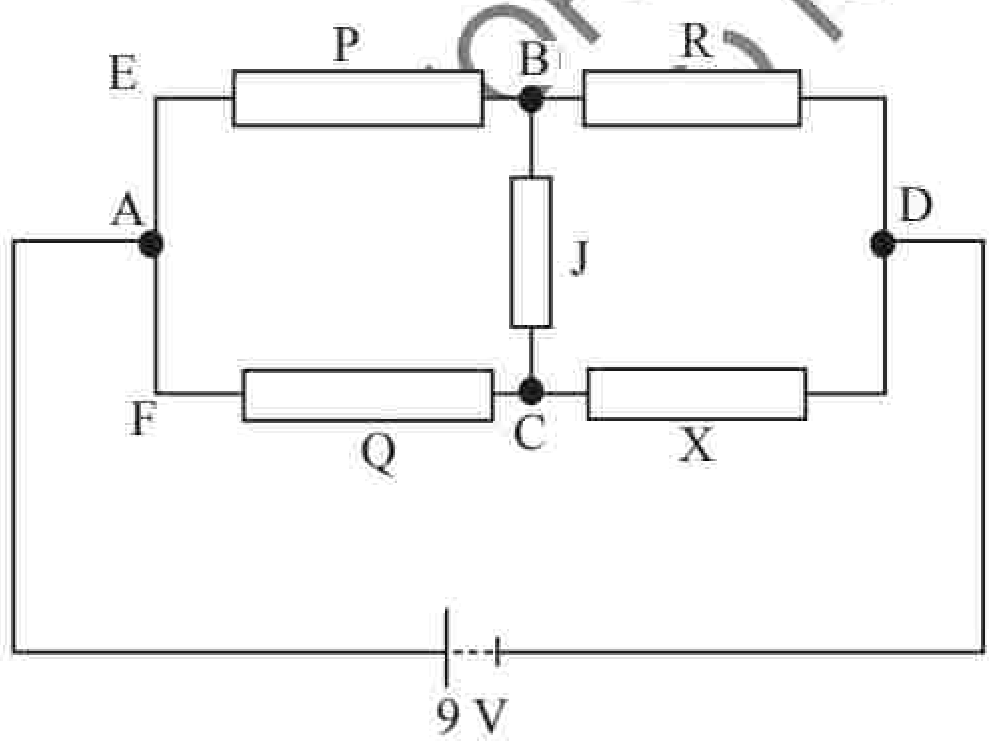


Figure 9.29: *Wheatstone circuit*

If the resistors, P = 5 Ω, Q = 1 Ω, R = 10 Ω, X = 2 Ω and J = 6 Ω, what is the current flowing through the circuit?

The circuit in Figure 9.29 does not have any resistors arranged in series or in parallel. Thus, one cannot reduce the circuit to an equivalent resistance using parallel and series formulae. However, if you observe the circuit carefully, you realize that, the resistors on one side of the joint resistor, J, are in the same ratio as the resistor on the other side of J. That

is, $\frac{P}{Q}=\frac{5\ \Omega}{1\ \Omega}=5$ and $\frac{R}{X}=\frac{10\ \Omega}{2\ \Omega}=5$.

Therefore, $\frac{P}{Q}=\frac{R}{X}$. A bridge circuit that has this property is said to be a balanced Wheatstone bridge.

Now, suppose the resistor, J, is removed from the circuit so that resistor, P, is in series with resistor, R and resistor, Q is in series with resistor, X. Moreover, the resistors, P and R are in parallel with resistors, Q and X. Therefore, the voltages at points E and F are the same. Since,

$\frac{P}{Q}=\frac{R}{X}$, the voltage drop at P is the same as the voltage drop at Q, meaning that voltage at point B (V_B) is the same as voltage at point C (V_C). This means the potential difference between points B and C is zero. Thus, when the resistor, J is connected, current through it is zero. That is, when the bridge circuit is balanced, $I_J = 0$ A.

Now, suppose a resistor J is replaced with a galvanometer, G as shown in Figure 9.28. The variable resistor, R_V, can be varied to a point where the current through the galvanometer, $I_G = 0$ A. At this point, the bridge circuit is balanced and therefore,

$$\frac{P}{Q} = \frac{R_v}{X},$$

which means,

$$X = \frac{QR_v}{P}$$

This formula gives the value of an unknown resistor, *X*. Hence, the Wheatstone bridge can be used in this way to determine the value of an unknown resistor, *X*.

Activity 9.7

Aim: To determine the resistance of an unknown resistor, *X* using a Wheatstone bridge

Materials: zero centred galvanometer, a dry cell, decade resistance box R_v, known resistors P and Q (ranging from 1 Ω to10 Ω), the unknown resistor, *X* whose values range from 4 Ω to 10 Ω, a switch, connecting wires.

Procedure

1. Use the materials provided to set up the Wheatstone bridge circuit as shown in Figure 9.28.
2. Switch ON the circuit to allow current to flow through the circuit. Ensure that the amount of current flowing through the galvanometer does not deflect it out of range.
3. Adjust the resistance, R_v until the reading on the galvanometer, G is zero. At this point the bridge is balanced.
4. Determine the value of the unknown resistor, *X* using the formula:

$$X = \frac{QR_v}{P}$$

5. Repeat steps 3 and 4 by using different values of P and Q to obtain several values of *X*. Calculate the average value of the unknown resistor, *X*.

Questions

(a) What is the value of the unknown resistor, *X*?

(b) What are the possible sources of errors in this activity?

Slide wire bridge (Metre bridge)

A slide wire bridge (Metre bridge) is a modified Wheatstone bridge. It is made of one known resistor, *R*, an unknown resistor, R_X, and a wire of uniform cross-section area, as shown in Figure 9.30.

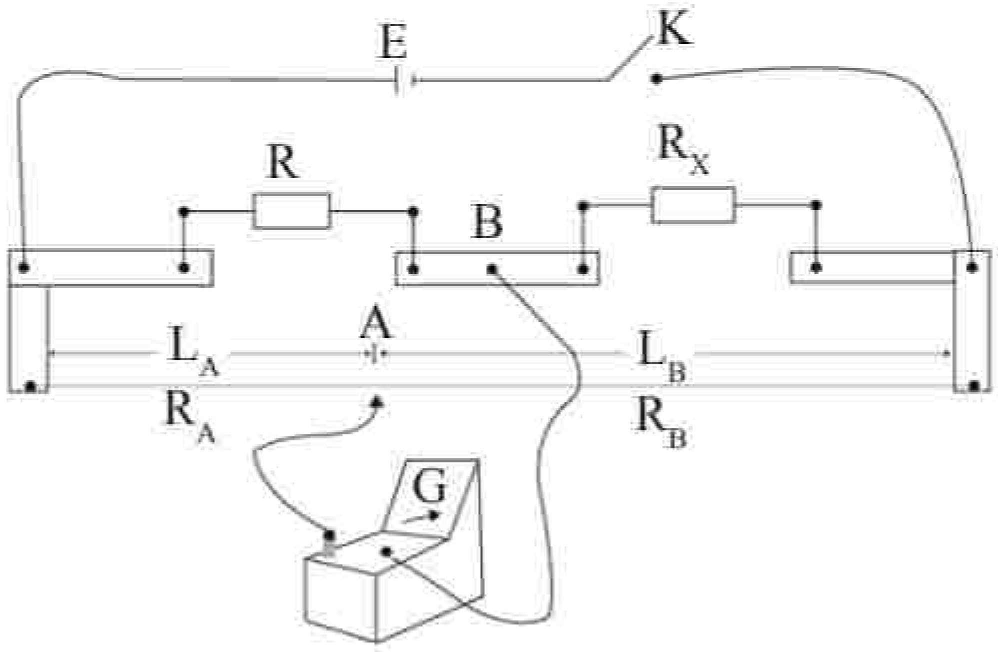


Figure 9.30: *Metre bridge*

The ratio $\frac{R_B}{R_A}$ is adjusted by sliding a jockey along the length of the wire. Because the wire has a uniform cross-sectional area, the ratio $\frac{R_B}{R_A}$ is equivalent to respective lengths L_B and L_A. That is,

$$\frac{R_B}{R_A} = \frac{\frac{\rho L_B}{A}}{\frac{\rho L_A}{A}} = \frac{L_B}{L_A}$$

But

$$\frac{R_B}{R_A} = \frac{R_X}{R} \Rightarrow \frac{L_B}{L_A} = \frac{R_X}{R}.$$

This expression can therefore be written as,

$$R_X = \frac{L_B}{L_A} R$$

This expression can be used to determine the value of the unknown resistor.

Activity 9.8

Aim: To determine the resistance of an unknown resistor using the metre bridge

Materials: a resistor with unknown resistance, a cell, galvanometer, metre bridge, decade resistance box, switch, jockey, connecting wires

Procedure

1. Set up the metre bridge circuit as shown in Figure 9.31. Let the resistance box be R.

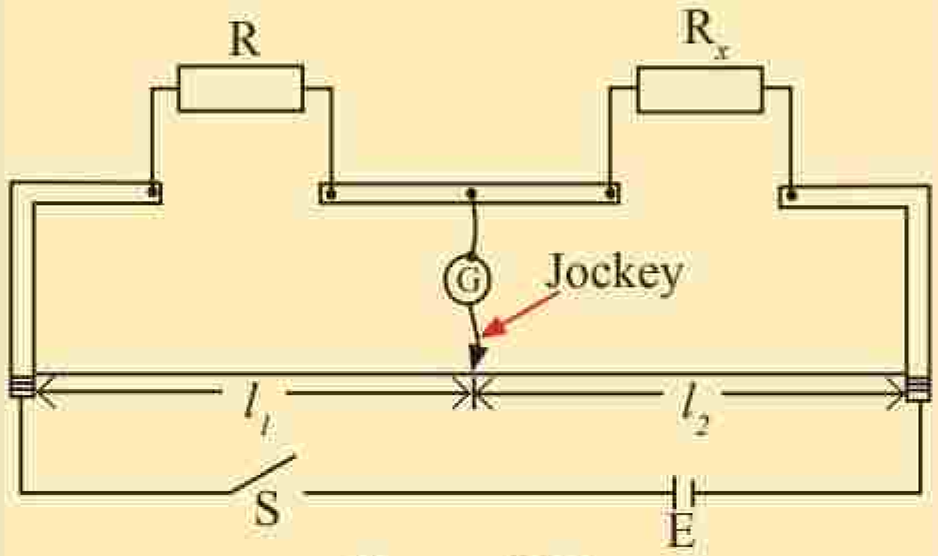


Figure 9.31

2. Set R to 1 Ω, then close switch, S and slide the jockey over the metre bridge wire until the galvanometer reads zero. Record length l_1 and l_2.
3. Repeat step 2 for R = 2 Ω, 3 Ω, 4 Ω and 5 Ω. Read and record the value of l_1 and l_2 in each case.
4. Record results using Table 9.7.

Table 9.7

Resistor, R (Ω)	l_1 (cm)	l_2 (cm)	$\frac{l_1}{l_2}$
1			
2			
3			
4			
5			

Questions

(a) Plot a graph of R against $\frac{l_1}{l_2}$.

(b) Deduce the value of unknown resistance, R_X.

Example 9.11

A metre bridge is set up as shown in Figure 9.32 using a standard 10 Ω resistor. The galvanometer shows zero deflection when the Jockey contact is at 48 cm from end A. Determine the resistance of a resistor, X.

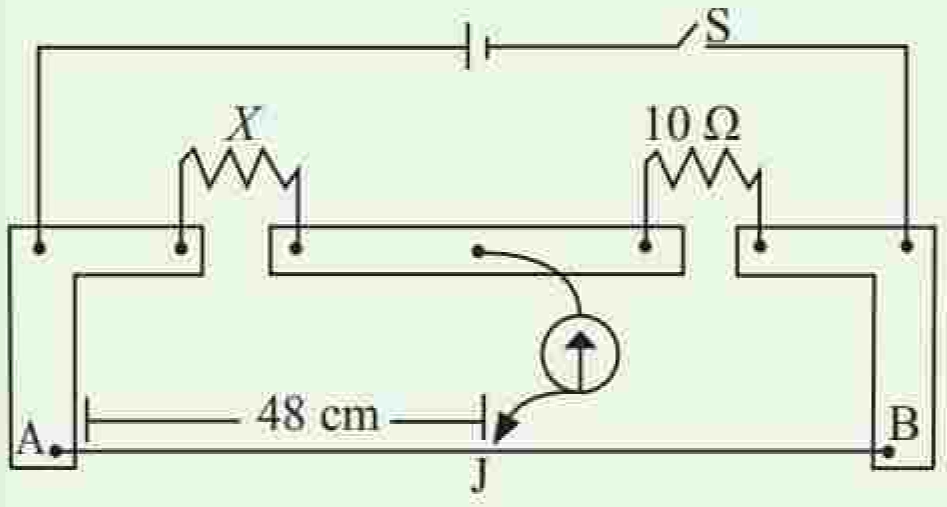


Figure 9.32

Solution

From $R_X = \frac{L_A}{L_B} R$, where

$R_X = X, R = 10\ \Omega, L_B = 52$ cm

and, $L_A = 48$ cm . Therefore,

$$R_X = \frac{L_A}{L_B} R$$

$$= \frac{48 \text{ cm}}{52 \text{ cm}} \times 10\ \Omega = 9.23\ \Omega$$

Therefore, the resistance X is 9.23 Ω.

The potentiometer

When a voltmeter is connected directly across the poles of a cell to measure its e.m.f, very small current passes through it. The voltmeter does not give the e.m.f to a high degree of accuracy since the e.m.f is the p.d between the terminals when no current flows or when the cell is 'an open circuit'. Another disadvantage of the voltmeter is that, it requires the use of a pointer, which might result in inaccurate readings.

A potentiometer is a reliable tool for measuring e.m.f. It consists of a length AB of uniform resistance wire, that is, a wire with a constant diameter, with a steady current flowing through it from an accumulator C (Figure 9.33). Any length from A, is read from a metre rule AB, and the length of AB may be one metre.

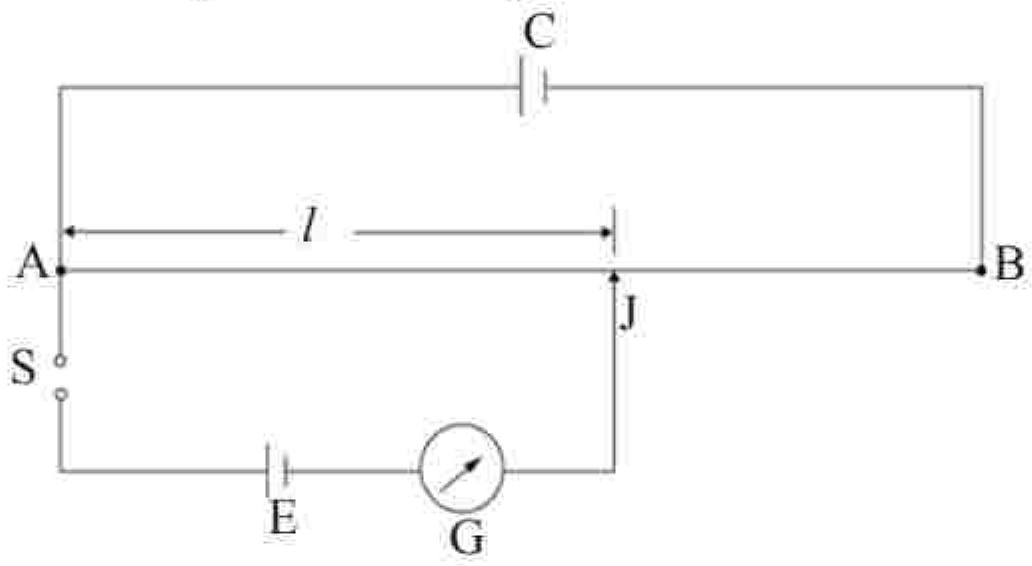


Figure 9.33: *A potentiometer*

On closing the switch, S, there will be a fall of potential across AB and hence there will be a potential difference between point A and any other point on AB. The fall of potential across AB produces a current in the galvanometer in the direction AGJ so that, whatever is the position of J on AB, the galvanometer will always be deflected. Suppose a second cell, D, is introduced in series with the galvanometer, the current due to the cell *E* in the direction AGJ, will now be opposed by the current due to the cell D flowing in the direction JGA. There will now be two p.d between A and J. Now it should be possible to find a position, J, of the jockey on AB for which the p.d across AJ due to *E* balances the p.d across AJ due to D.

Activity 9.9

Aim: To compare the e.m.f of two cells using a potentiometer

Materials: a slide-wire potentiometer, galvanometer, 2 dry cells, jockey, connecting wires, switch, accumulator

Procedure

1. Set up the potentiometer circuit as shown in Figure 9.34 where C is an accumulator, E_1 and E_2 are the two cells for comparison.

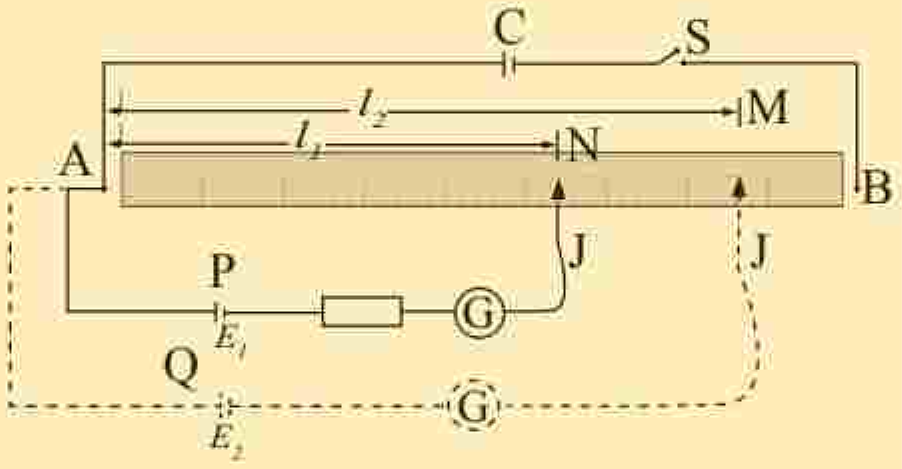


Figure 9.34

2. Join the +ve terminal of P to A, the same terminal to which the +ve terminal of C is joined.
3. Join the -ve terminal of P to a slider, J, through a galvanometer, G.
4. Gently tap on the wire with J, find the point N (balance point) when no current flows in G. Measure the balance length AN or l_1 from the metre rule.
5. Disconnect the cell P (E_1) from the circuit, and replace it with cell Q (E_2), the other cell for comparison. As before, obtain the new balance point M and read off the length AM or l_2. Measure the balance length l_2.

Questions

(a) Suppose the value of E_1 is known, find the value of E_2.

(b) It is clear from this activity that, as the e.m.f increases, the length of the potentiometer wire increases as well. Why does the length increase with an increase of e.m.f?

Theoretically, the e.m.f E_1 of cell P balances the p.d. V_1 between A and N on the wire because no current flows in G at the balance point N. As a result, $E_1 = V_1$. Similarly, if E_2 is the e.m.f of cell Q and V_2 is the p.d. between A and M, $E_2 = V_2$. As a result, $\frac{E_1}{E_2} = \frac{V_1}{V_2}$. The p.d, V across any length, l, is directly proportional to l if the wire carries a constant current. As a result, $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

Thus, the e.m.f of the two cells can be compared.

Example 9.12

The balance length of a potentiometer wire for a cell of e.m.f, $E_1 = 1.63\text{ V}$ is 85 cm. If the cell is replaced by another one of e.m.f, $E_2 = 1.07\text{ V}$, calculate the new balance length.

Solution

Given

$E_1 = 1.63\text{ V}, \ l_1 = 85\text{ cm}, E_2 = 1.07\text{ V}$

Then, the new length l_2 is obtained from a formula:

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Therefore

$$l_2 = \frac{E_2 l_1}{E_1} = \frac{1.07\text{ V} \times 85\text{ cm}}{1.63\text{ V}} = 55.79\text{ cm}$$

The new balance length is 55.79 cm.

Exercise 9.2

Answer all questions

1. A wire of length 1.2 m and diameter 0.64 mm has a resistance of 2.4 Ω. Calculate the resistance of a wire of length 0.80 m and diameter 0.32 mm of the same material.
2. A wire of length 2 m and a cross-sectional area of 0.5 mm^2, has a resistance of 2.2 Ω. Calculate the resistivity of the material making up the wire.
3. A wire of length 0.40 m and a diameter of 0.60 mm has a resistance of 1.5 Ω. What is the resistivity of the material from which the wire is made?
4. What length of wire of diameter 0.6 mm and resistivity 1.1×10^{-6} Ωm would make a resistor with a resistance of 44 Ω?
5. A battery of e.m.f 12 V and internal resistance 1.5 Ω is connected to a 4 Ω resistor. Calculate the:
   (a) total resistance of the circuit
   (b) current through the battery
   (c) p.d across the cell terminals
6. The p.d across the terminals of a cell is 1.1 V when a current of 0.20 A is being drawn from the cell. If the p.d across the cell is 1.3 V when a current of 0.10 A is being drawn, determine the:
   (a) internal resistance of the cell
   (b) cell's e.m.f
7. Differentiate between variable resistors and fixed resistors. Give three examples of each.
8. Distinguish between electric current and the convectional current.
9. Briefly explain what is meant by a coulomb of electricity.
10. A battery of unknown e.m.f, *E*, and internal resistance, *r*, is connected in series with an ammeter and a variable resistor, *R*. The current in the circuit is found to be 2.0 A when the value of R is 0 Ω and 1.5 A when the value of R is 6.0 Ω. Determine the values of *E* and *r*.
11. Two batteries, V_1 and V_2, are connected in series with an 8 Ω resistor as shown in Figure 9.35.

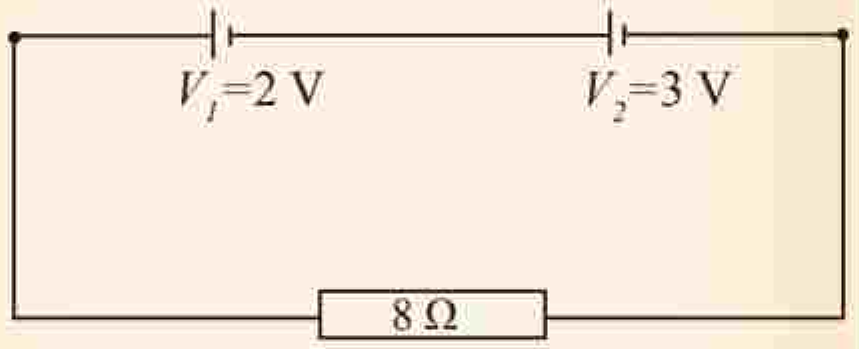


Figure 9.35

Given that, the internal resistances of the two batteries are 2 Ω and 1 Ω, respectively, determine the p.d across the 8 Ω resistor.

Heating effect of an electric current

When an electric current flows through a conductor, the electrical energy gets converted into thermal energy and the conductor heats up. This is a demonstration of the heating effect of an electric current. In an electric iron, the electrical energy is converted to heat.

The heat produced helps in removing wrinkles and shrinkage in our clothes. In this process, the electric iron becomes hot. This is a primary example of heating effect of electric current. Other electrical appliances that work on heating effect of electric current include immersion water heaters, hot plates, electric kettles and filament lamps.

Mechanism of heating of electric current

If a potential difference is applied at the ends of a conductor, an electric field is set up in the conductor. Free electrons in a conductor are accelerated and hence they gain kinetic energy. When this energy is imparted on atoms, ions and other electrons, it causes them to vibrate resulting to thermal energy. As these electrons move through the conductor, they collide with ions and atoms of the conductor and hence slowed down or lose kinetic energy. Hence, the temperature of conductor increases.

Note that, a conductor of higher resistivity results in more heating for the same applied potential difference.

Activity 9.10

Aim: To investigate the heating effect of an electric current

Materials: battery, thermometer, resistive wire, ammeter, voltmeter, rheostat, stopwatch, calorimeter with jacket, connecting wires, water, switch

Procedure

1. Determine the resistance, *R*, of the resistive wire using a meter bridge procedure.
2. Set up the apparatus as shown in Figure 9.36.

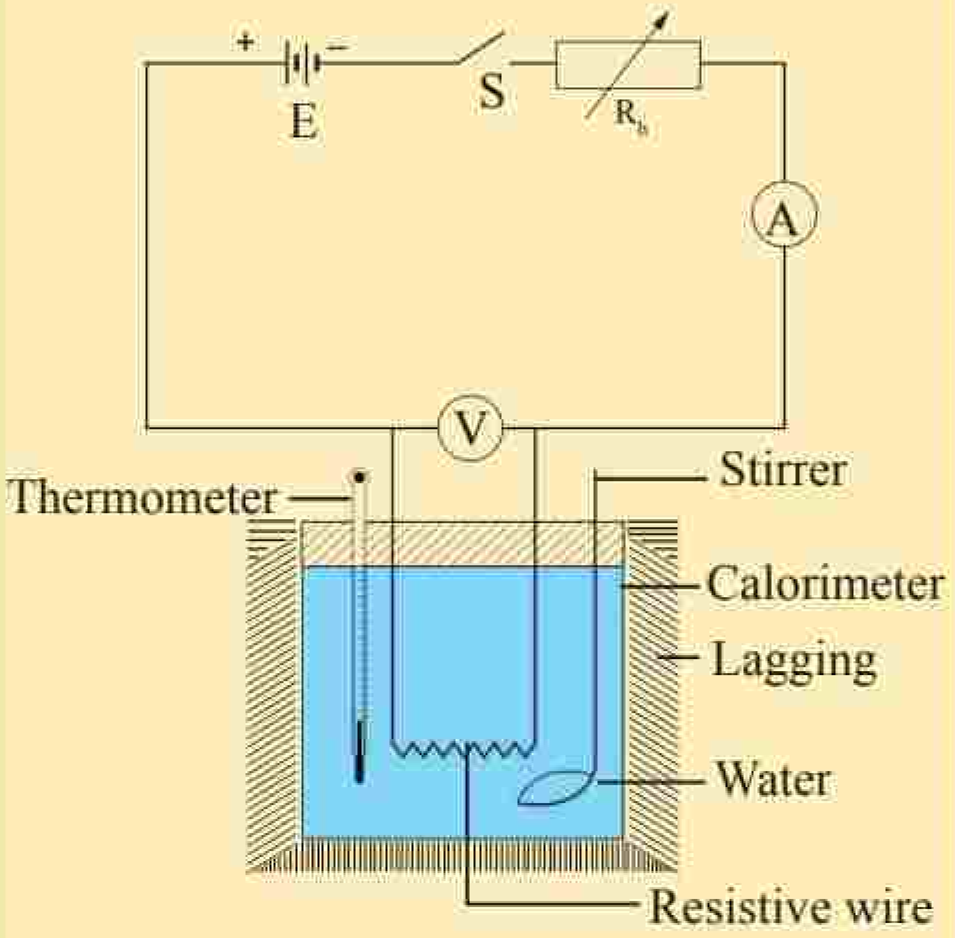


Figure 9.36

3. Close switch S and adjust the rheostat, R_h, to give appropriate current, *I*, and voltage, *V*.
4. Record the initial temperature θ_1 of water and leave the circuit ON for a known length of time, *t*, which is determined by a stopwatch.
5. After time, *t*, record the final temperature θ_2. The change in temperature will be $\theta = \theta_2 - \theta_1$.
6. Adjust R_h to obtain different values of I and V and repeat steps 4 and 5, to obtain at least two or more sets of *I*, *V* and θ.

7. Record results as shown in Table 9.8.

Table 9.8

Current, I (A)	Voltage, V (V)	Temperature change, θ (°C)	Square of current, I^2 (A^2)

Questions

(a) Plot a graph of change in temperature, θ, against the square of current, I^2.

(b) What is the nature of the graph? Explain its physical meaning.

The amount of heat, H, absorbed by the water is proportional to the increase in its temperature. The plot of θ against I^2 indicates that, $\theta \propto I^2$. Since $H \propto \theta$, therefore, $H \propto I^2$.

Activity 9.11

Aim: To demonstrate the effect of time on heating effect of an electric current

Materials: battery, calorimeter with jacket, thermometer, resistive wire, ammeter, voltmeter, rheostat, stopwatch, switch, stirrer, water, connecting wires

Procedure

1. Since the resistance of the resistive wire is known, connect the apparatus as shown in Figure 9.36. Close switch S and adjust the rheostat to obtain suitable current, *I*, and voltage, *V*.
2. Record an initial temperature of the water together with *I* and *V*. Leave the circuit ON for a known length of time, t.
3. At the end of a time, t record the value of change in temperature. Leave the circuit ON as in step 2, then record the change in temperature with corresponding time.
4. Repeat step 3 and obtain at least 2 more values of change in temperature with respect to heating time.

5. Record results as shown in Table 9.9.

Table 9.9

Time, t (min)				
Temperature change, θ (°C)				

Questions

(a) Plot the graph of temperature, θ, against time, t.

(b) Explain the nature of a graph.

Heat absorbed by water is proportional to the increase of heating time. The graph of θ against t shows that,

$\theta \propto t$, thus, $H \propto t$.

Activity 9.12

Aim: To observe the effect of resistance on the heating effect of an electric current

Materials: battery, calorimeter with jacket, thermometer, resistive wire, ammeter, voltmeter, rheostat, stopwatch, water, switch, stirrer, connecting wires

Procedure

1. Set up an arrangement as shown in Figure 9.36.
2. Close switch S and adjust a rheostat to obtain suitable values of I and V. Record the initial temperature of water.
3. Leave a current ON for a known length of time and record the value of change in temperature.
4. Change the resistance of a resistive wire by varying its length, then, repeat steps 2 and 3.
5. Repeat step 4 and obtain at least 2 more readings of change of temperature with the corresponding resistance of wire.
6. Determine the resistance of each length of the wire and record the results as shown in Table 9.10.

Table 9.10

Length of the wire, l (cm)	Resistance of the wire, R (Ω)	Temperature change, θ (°C)

Questions

(a) Plot a graph of change in temperature, θ, against resistance, R.

(b) Comment on the nature of the graph.

The heat absorbed by water is proportional to the increase in temperature. From the plot of change in temperature against resistance, it is observed that, θ, is proportional to the resistance of the wire, thus, $\theta \propto R$. Hence, $H \propto R$.

Therefore, from Activities 9.10 – 9.12, it can be concluded that,

$$H \propto I^2 Rt$$

Hence,

$$H = kI^2 Rt$$

If the current of 1 A flows through a resistor of 1 Ω in 1 second, then the heat generated in water is 1 J. Thus, $k = 1$. Hence,

$$H = I^2 Rt$$

This equation gives the relationship between the resistance of a conductor, current passing through a conductor, time a current flows and electrical energy generated. The electrical energy generated is measured in Joules. Thus, this equation is referred to as Joule's law of heating.

The law states that, *"When an electric current is passed through a conductor, the heat generated in a given time is directly proportional to the resistance of the conductor in ohms and the square of current in amperes"*.

Since the unit of electrical energy is joule, then the Joule is defined as the work done when a charge of one coulomb flows through a conductor with a p.d of 1 volt across it in one second.

By Ohm's law, $V = IR$. Then,

$$H = \frac{V^2}{R} t$$

Also, $R = \frac{V}{I}$

thus,

$$H = IVt$$

These are other forms of stating up Joule's law.

Example 9.13

A resistor of 100 Ω is connected across a battery of 12 V. How much heat is dissipated across the resistor in 5 s? (Ignore the internal resistance of the battery).

Solution

Given: Voltage, $V = 12$ V, Resistance, $R = 100$ Ω, Time, $t = 5$ s,

From,

$$H = \frac{V^2}{R} t$$

$$= \frac{12^2 \text{ V}^2}{100\ \Omega} \times 5 \text{ s} = 7.2 \text{ J}$$

The energy dissipated across the resistor is 7.2 J.

Example 9.14

A bulb draws a current of 0.5 A from a 240 V source. Calculate the energy dissipated in 10 minutes.

Solution

Given: I = 0.5 A, V = 240 V,

$$t = 10 \times 60 \text{ s} = 600 \text{ s}$$

Using the equation

$$H = IVt$$

$$= 0.5\,\text{A} \times 240\,\text{V} \times 600\text{ s}$$

$$= 72000\text{ J}$$

The energy dissipated is 72000 J or 72 kJ.

Example 9.15

Three resistors with resistances 9 Ω, 12 Ω and 15 Ω are connected in series across a 12 V battery. Calculate the energy dissipated by the 12 Ω resistor in 10 s. Internal resistance of the battery is negligible.

Solution

Total resistance

$$R = 9\ \Omega + 12\ \Omega + 15\ \Omega = 36\ \Omega$$

Current $I = \frac{V}{R} = \frac{12\text{ V}}{36\ \Omega} = 0.33\text{ A}$

If $t = 10$ s, then

Heat energy can be calculated as,

$$H = I^2Rt$$

$$= 0.33^2\text{ A}^2 \times 12\ \Omega \times 10\text{ s}$$

$$= 13.07\text{ J}$$

The energy dissipated by the 12 Ω resistor in 10 s is 13.07 J.

Electric power

In many circuits, it is essential to know the rate at which electrical energy is transferred into other forms of energy. The rate of doing work or the rate at which energy is dissipated is called power.

$$\text{electric power} = \frac{\text{electrical energy dissipated}}{\text{time taken}}$$

$$= \frac{\text{energy transfer}}{\text{time taken}}$$

$$P = \frac{I^2Rt}{t} = I^2R$$

By Ohm's law $IR = V$

Therefore,

$$P = IV$$

Using $I = \frac{V}{R}$;

$$P = \frac{V^2}{R}$$

The SI unit of power is joule per second or watt (W). The power is assumed to be 1 watt if 1 ampere current flows in it against a potential difference of 1 volt, that is 1 watt = 1 ampere × 1 volt . Likewise, if 1 joule of energy is being spent per second, the power consumed by an electrical appliance is said to be 1 watt (1 watt = 1 joule/second).

Example 9.16

If the lamp on a 240 V supply draws a current of 0.25 A, calculate the power the lamp uses to transfer electrical energy into heat and light energy.

Solution

$$P = IV$$

$$P = 240\text{ V} \times 0.25\text{ A} = 60\text{ W}$$

The lamp transfers 60 J of electrical energy into heat and light energy each second.

Power rating of electrical appliances

Every electrical appliance should carry a label stating the potential difference for which it has been designed and the power an appliance can convert when operating at the stated potential difference. The rate at which an appliance dissipates energy is called the rating of an appliance and is usually marked on the body of an appliance.

For example, an appliance marked 3000 W, 240 V, dissipates energy at the rate of 3000 joules per second when connected to a 240 V supply. Figure 9.37 shows an electric iron marked 220-240 V, 1000 W, indicating that, the appliance dissipates energy at the rate of 1000 joules per second when connected to a 220 – 240 V supply.

It is worth noting that, if the appliance is connected to a higher voltage than the one indicated, it will blow, and if connected to a lower voltage it will not function properly.

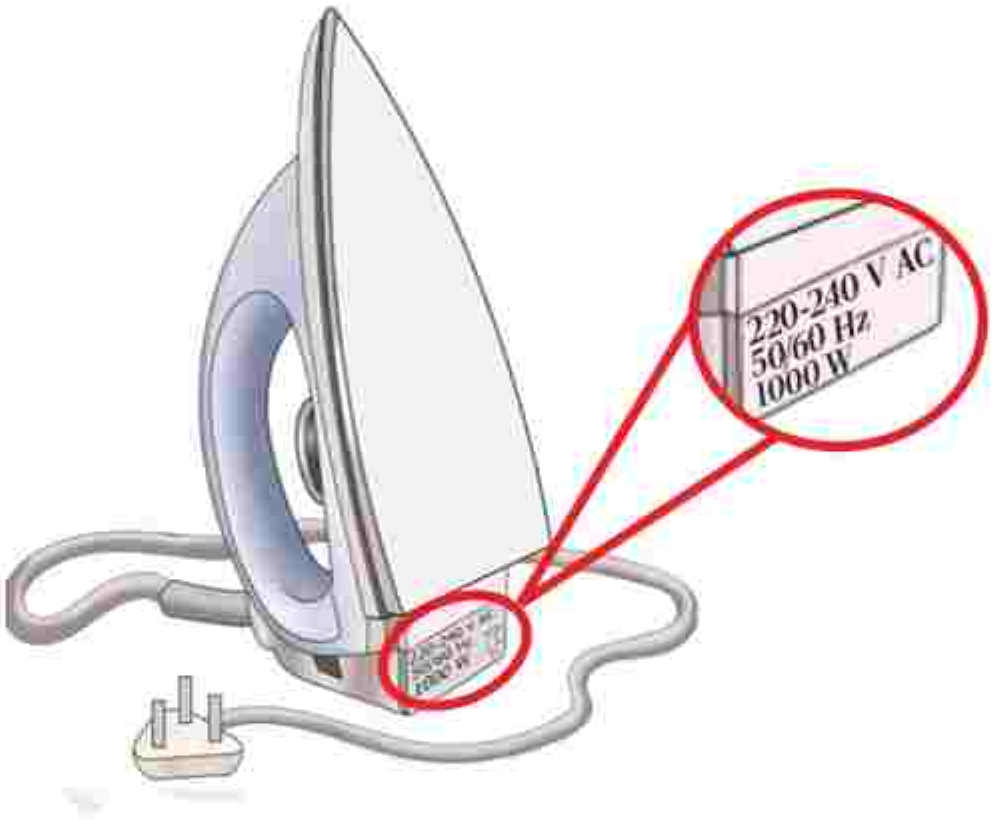


Figure 9.37: *An electric iron*

Task 9.1

(a) Identify the electrical appliances found at your home; list them and discuss their power ratings.

(b) Suppose the appliances you listed are used for 1 hour, what will be the total energy used?

Commercial unit of electrical energy

You learnt that, the unit of electrical power is watt, and thus, unit of electrical energy should be watt-second. However, watt-second unit is too small as an electrical energy unit for everyday commercial purposes.

Thus, to resolve this challenge, a bigger unit, the kilowatt-hour (kWh), is used. One kWh is the energy consumed in one hour by an appliance working at the rate of 1 kW. Commercially, 1 kWh is named as a unit of electricity. For example, if the user consumes 5 units of electricity, it means that, electrical energy of 5 kWh has been consumed.

$1 \text{ kWh} = 1 \text{ killowatt} \times 1 \text{ hour}$

$= 1000 \text{ W} \times 3600 \text{ s}$

$= 3600000 \text{ watt-second or } 3.6 \times 10^6 \text{ J}$

$= 3.6 \text{ MJ}$

The unit of kWh is used to determine the cost of electrical energy consumed by electricity users. Normally, electricity cost is calculated by taking the price of a unit of electricity times the consumed electrical energy in kWh (total units). That is;

$$\text{cost} = \text{price} \times \text{total units}$$

In Tanzania, electricity users buy units of electricity before using the energy. Therefore, the Tanzania Electric Supply Company Limited (TANESCO) distributes special electricity meters that enable one to pay before consuming. These meters are famously known as "*Lipa Umeme Kadiri Utumiavyo*" abbreviated as LUKU. Figure 9.38 shows a picture of LUKU meter distributed by TANESCO.

Figure 9.38: *An example of a LUKU meter with 3.35* kWh *(units)*

Example 9.17

A television set (TV) rated 40 W is switched ON for 5 hours every day.

(a) How much electrical energy in kWh does this TV consume in 30 days?

(b) If the price of a unit of electricity is Tsh 229.60, what is the total electricity cost of watching this TV in 30 days?

Solution

(a) Energy = power × time

$$= 40\ \text{W} \times \frac{5\ \text{h}}{\text{days}} \times 30\ \text{days}$$

$$= 6000\ \text{Wh} = 6\ \text{kWh}$$

Therefore, the electrical energy consumed for 30 days is 6 kWh.

(b) To determine the electricity cost of watching this TV, we use the formula;

cost (Tsh) = price × total units

Therefore,

$$\text{cost (Tsh)} = \frac{\text{Tsh } 229.60}{1\ \text{kWh}} \times 6\ \text{kWh}$$

$$= \text{Tsh}1377.60$$

Hence, the coast of watching this TV for 30 days is Tanzanian shillings 1377.60.

Example 9.18

An electric heater is constructed by applying a potential difference of 120 V across a nichrome wire that has a total resistance of 8 Ω. Find the current in the wire and the power rating of the heater.

Solution

Given

$$V = 120\ \text{V}$$
$$R = 8\ \Omega$$

Then, current, $I = \frac{V}{R}$

$$I = \frac{120\ \text{V}}{8\ \Omega} = 15\ \text{A}$$

Power rating, $P = I^2R$

$$= 15^2\ A^2 \times 8\ \Omega = 1800\ W$$

Therefore, power rating of the device is 1800 watts.

Exercise 9.3

1. A 120 V circuit is equipped with a 20 A fuse. What is the least resistance that can be plugged into the circuit without causing the fuse element to melt?
2. A car's headlights consume a power of 40 W when on low beam and 50 W when on high beam. Given that, the headlights are connected to a 12 V battery, determine the:
   (a) current that flows through the headlight in each case.
   (b) resistance of the headlight in each case.
3. A generator at a power station has an output of 100 MW at 132 kV.
   (a) What is the current in the generator?
   (b) What is the daily energy output of the generator in joules and in kilowatt-hours?
4. An electric motor draws a current of 10 A when connected to a 120 V supply. Calculate the:
   (a) motor's power consumption.
   (b) energy (in joules and in kilowatt-hour) that the motor uses during 5 hours of operation.
5. An electric heater of resistance 10 Ω is connected to a 120 V mains supply. How much energy does it use when it is in operation for 2 hours?
6. Two 8 Ω resistors and a battery of e.m.f 12 V and internal resistance 2 Ω are connected in series with each other.
   (a) Draw a circuit diagram for the connection.
   (b) Calculate the power delivered to each of the external resistors.
   (c) Determine the power wasted due to an internal resistance of the battery.
7. A house has five rooms, each with a 60 W, 240 V bulb. If the bulbs are switched ON from 7.00 p.m. to 10.30 p.m., determine the power (in kilowatt-hour) consumed per day.
8. A hair dryer of resistance 100 Ω is connected to a 240 V mains supply. Determine the:
   (a) current that flows through the hair dryer.
   (b) power rating of the dryer.
   (c) energy that the dryer consumes for 8 hours.
9. Calculate the energy consumed (a) in joules (b) in kilowatt-hours, when a 2 kW oven is ON for 1·5 hours.
10. A light bulb has a rating of 240 V, 60 W. Determine the:
    (a) amount of current a bulb filament would carry when switched ON.
    (b) resistance of a filament.

Electrical installations

Electricity is an extremely convenient and clean source of power. Electricity is supplied in houses by low resistance wires (copper or aluminium) insulated with rubber. The wires are rated according to the maximum current they can carry. Due to certain faults in the electric generator or due to some faults in the wiring circuit of an appliance, the voltage or current flowing in an appliance may rise beyond a rated value. This could burn or damage, in some way, the circuit of an appliance if it is not properly protected. Protection of appliances against possible damages like these is made with the help of a fuse or circuit breaker.

Live, Neutral and Earth

Domestic electricity is supplied through two cables: the live cable (L), coloured brown or red, and the neutral cable (N), usually coloured blue or black. For a single-phase system, the live cable is at a potential of 240 V relative to the neutral line. The current in the cable alternates 60 times a second (60 Hz). The neutral cable is earthed, that is, it is connected to the ground at the power station. This ensures that, even though current flows through this cable, it remains at a zero potential. In this situation, it cannot give an electric shock on touching. To provide extra safety especially in electrical appliances, a third cable called earth (E), coloured yellow or green, is also provided and is earthed by connecting it to the ground via a thick copper rod. This cable connects the metal body of an electrical appliance to the ground through a three-pin plug. A three-pin plug and electrical wiring cables are connected as shown in Figure 9.39.

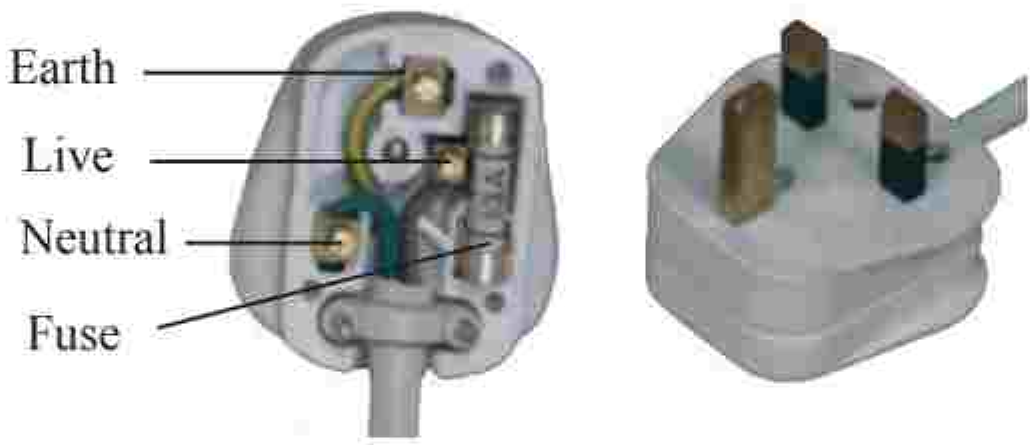


Figure 9.39: *A three-pin plug*

A fuse is connected to the live cable. The fuse should not be connected to the neutral cable, because if a fault occurs and the fuse blows, the body of an appliance remains live and can give an electric shock to the user.

The earth on the plug pin is generally longer than the other two pins, this enables it to open the protective cover of the socket. It also ensures that, an appliance is earthed before being connected to power source. When connecting a three-pin plug, the colour code mentioned must be strictly adhered to in order to avoid electric shock. The switch must be in the OFF position when pushing the plug into the socket.

The two-pin plug does not have a fuse and an earth pin. Thus, an appliance using a two-pin plug does not have its body connected to earth. Figure 9.40 shows a two-pin-plug. The connections for this plug follow the same colour code as the three-pin plug.

Figure 9.40: *A two-pin plug*

When wiring a plug, always check that the plug is correctly wired. All connections should be tight. The individual wires are supposed to be cut to a length such that, if the cable is pulled out of the plug, the live will come out before the earth and the neutral. The cable should be firmly clamped without damaging the insulation, and a fuse of the correct rating for the appliance is fitted.

A fuse and a circuit breaker

When an electric current exceeds a rated value of an electrical appliance, it causes damage to the device. To protect an appliance against this damage, a fuse or a circuit breaker is used. A fuse consists of an element, usually a piece of copper or tin-lead alloy wire that melts when current through it exceeds a specific predetermined value. The element is contained in a suitable casing and placed in series with the circuit to be protected. A fuse automatically melts and breaks a circuit when overloaded. It must be replaced before a circuit works again.

Fuses are categorised into two types, namely; rewireable fuses and cartridge fuses. Rewireable fuse is a type of fuse, whose fuse element is carried in a removable fuse link. The fuse link may be made of porcelain or other suitable insulating material. This ensures that, there is no danger to an operator when removing fuse link. Figure 9.41 shows a rewireable fuse.

Figure 9.41: *Rewireable fuse*

Cartridge fuses consists of a porcelain tube with metal end caps to which the fuse element is attached. Figure 9.42 shows a cartridge fuse.

Figure 9.42: *Cartridge fuse*

Fuse element passes through the tube from cap to cap and is welded or soldered to the inside of the caps. In installation (during wiring), fuses should be made accessible and be fitted on the front of a switch board or in a protecting case. In most cases, fuses are fitted on a distribution board.

Action of a fuse

Blowing (melting) occurs when a circuit is overloaded or when a short circuit occurs. A short-circuit fault is an unwanted connection which allows current to flow along a path which is not part of a circuit design. The new path might re-route some or all of the current from the designed path. Short-circuit faults can be identified due to the fact that, the circuit or that section of the circuit draws an unusual high current (leading to fuse blowing). The potential difference between two points joined by a short circuit is zero or much lower than expected.

Activity 9.13

Aim: To demonstrate the melting of a fuse

Materials: fuse wire 3 A, 3 dry cells size-D, rheostat, ammeter (0 – 5 A), two 2.5 V torch bulbs with bulb holders, connecting wires, switch, crocodile clips

Procedure

1. Connect the apparatus to make a circuit as shown in Figure 9.43.

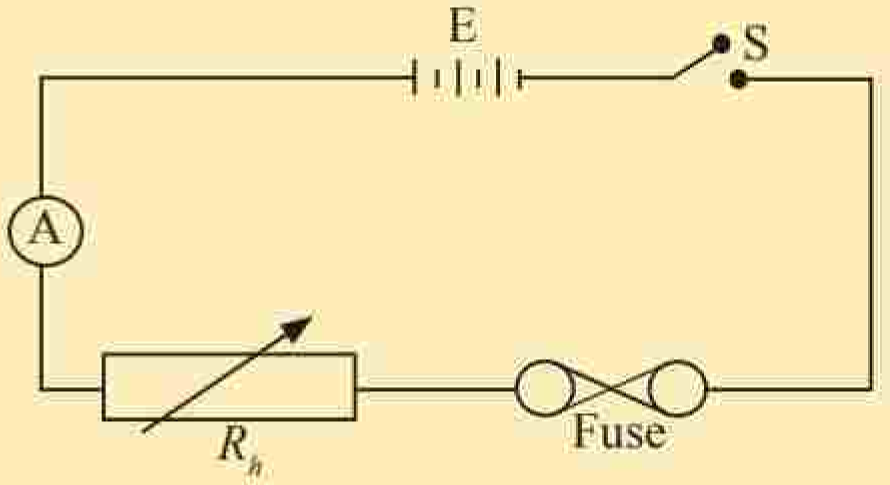


Figure 9.43

2. Start with the rheostat set at maximum resistance.
3. Close the switch and adjust the rheostat to raise the current. Observe both the ammeter and the fuse wire. Continue adjusting until the fuse wire melts. Open the switch.
4. Now disconnect the rheostat from the circuit and replace it with the two 2.5 V torch bulbs as shown in the circuit diagram in Figure 9.44.

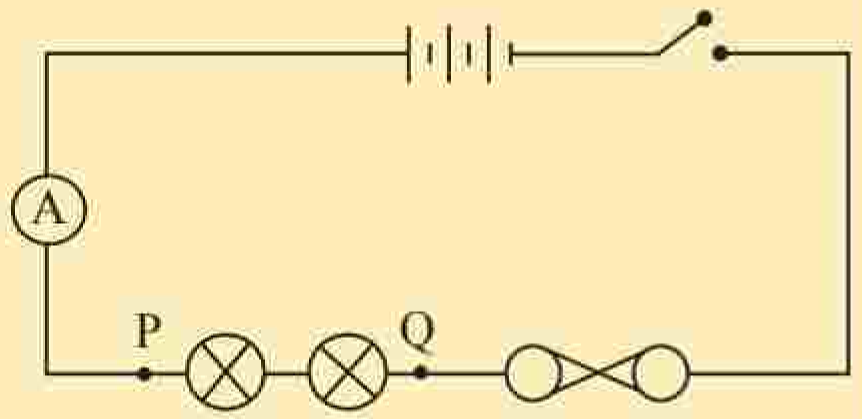


Figure 9.44

5. Close the switch and observe the ammeter reading and the brightness of the bulbs.
6. Short-circuit the bulbs by connecting a lead wire across PQ with crocodile clips. Observe the ammeter reading, the brightness of the bulbs and the fuse wire.

Questions

(a) Determine the current required to melt the fuse in step 2.

(b) What was your observation on the ammeter reading and the bulbs' brightness when the switch was closed in step 3?

(c) What was the result of short-circuiting in step 6?

When a current in the circuit exceeds a current rating of a fuse wire, a fuse wire melts. Putting a lead wire across PQ causes an increase in the ammeter reading. It also causes bulbs to go off. These two observations are an indication of a short circuit. The lead wire across PQ is an unwanted connection which diverts current through a different path. The lead wire has much lower resistance than bulbs. This results in a large current flow, hence, melting of a fuse wire.

A circuit-breaker is a type of a switch that cuts off the flow of current when current in a circuit exceeds a specific value. The circuit-breaker operates by opening the circuit in the event of excess current. Unlike a fuse, a circuit breaker can be reset once the current in a circuit has returned to normal. However, just like a fuse, a circuit breaker is connected in series with the circuit it controls.

In a circuit-breaker, switch contacts are held closed by a latch mechanism that releases the contacts quickly to open the circuit. This happens whenever there is an excess current in a circuit. Common circuit-breakers are either thermal or magnetic devices that incorporate a bimetallic strip or an electromagnet to open and close the switch respectively. Figure 9.45 illustrates an operation of a circuit breaker.

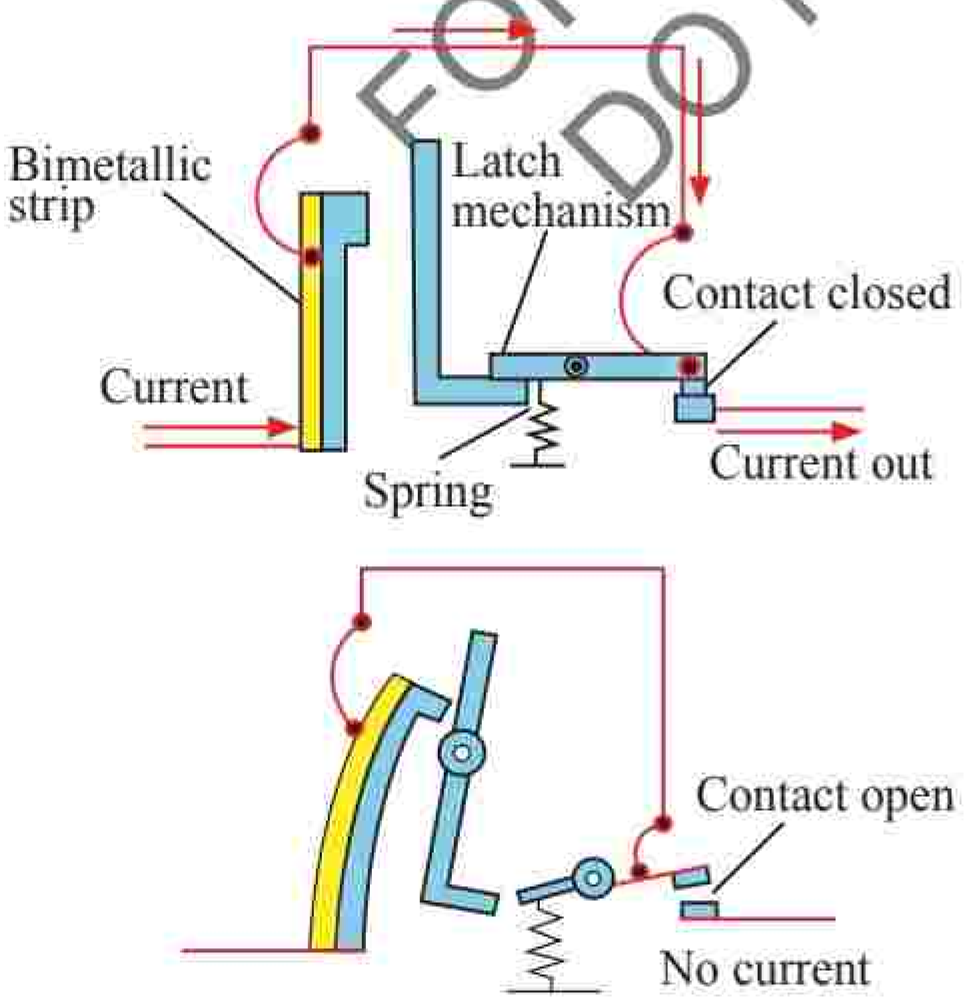


Figure 9.45: *A thermal circuit-breaker*

Magnetic circuit-breakers use an electromagnet whose pulling force increases with current. When the current increases beyond the rating of the circuit-breaker, the electromagnetic pull releases the latch which allows the switch to open by spring mechanism. Figure 9.46 shows typical circuit breakers.

Figure 9.46: *Examples of circuit-breakers*

Domestic wiring

The power company connects power to a house up to the meter and leave cables to be connected to the consumer unit where the house wiring starts. Consumer unit is a type of distribution board that electricity distributes from the main supply to separate circuits. From the consumer unit, the cables branch into several parallel circuits which carry current to the various parts of the house. There are two methods of wiring a house from the main switch; these are the ring system and the tree system.

Ring main system. In many houses, the main sockets are connected to a ring main. This is a cable which begins and ends at the consumer unit. It has live, neutral and earth wires each forming a ring around the house or part of the house.

In a circuit, the current has two parts so that thin cables can be used. The sockets usually have a 30 A fuse at the consumer unit. Figure 9.47 shows a typical ring main circuit.

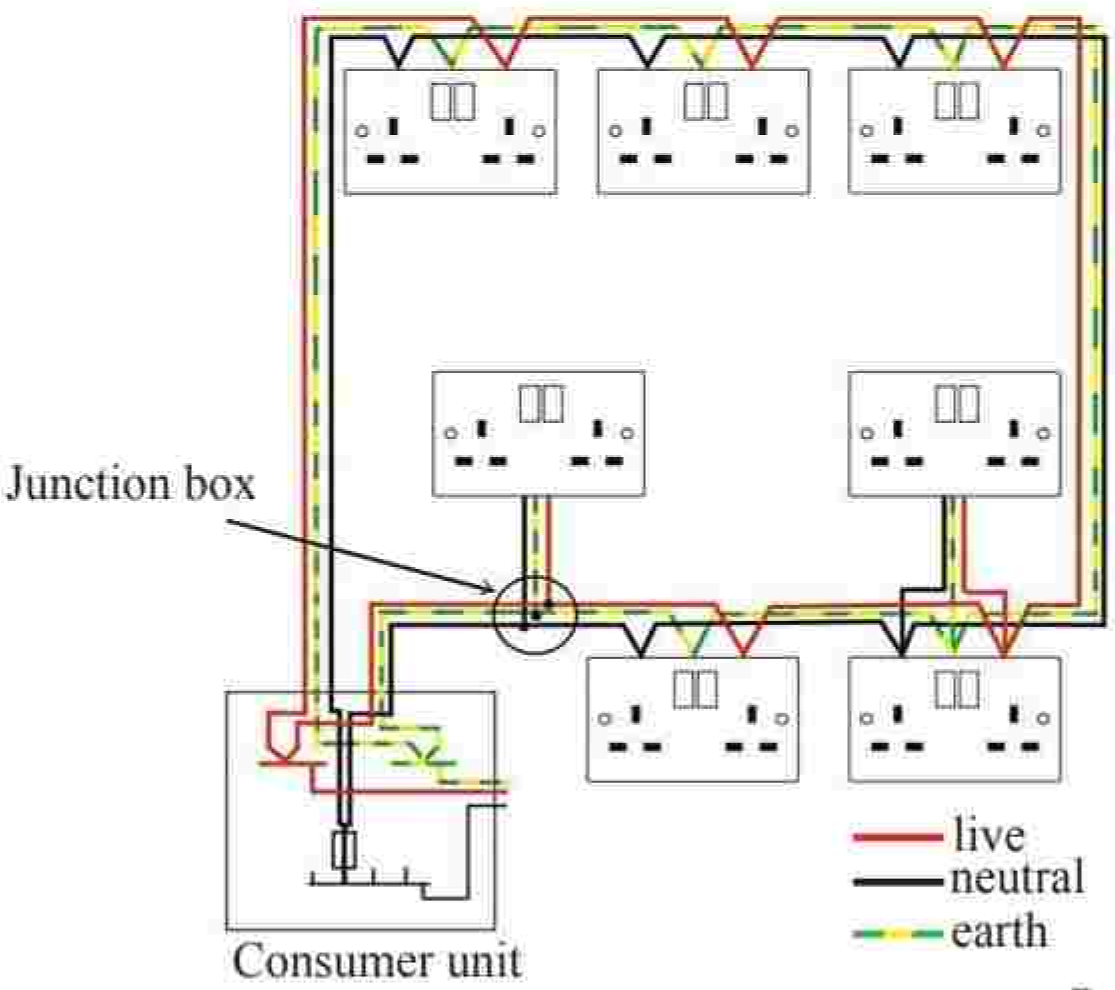


Figure 9.47: *Consumer unit and a ring main in a house*

The tree system method uses a cable from the main switch to the distribution box as shown in Figure 9.48.

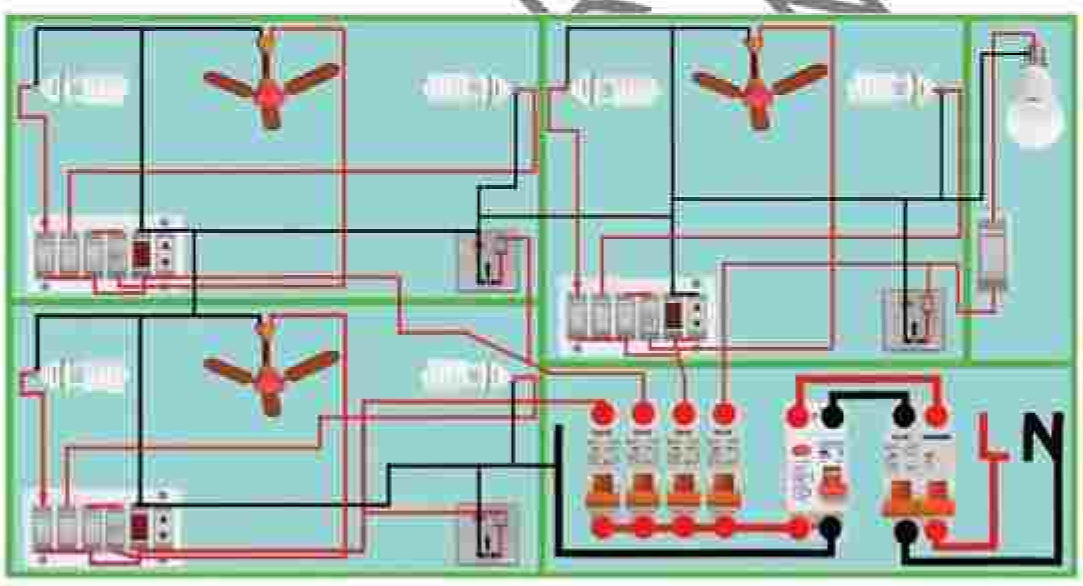


Figure 9.48: *The tree system method of house wiring*

In the distribution box, there are lighting fuses and power fuses. For example, there are separate fuses for light in corridors, bed-rooms, living room and kitchen. Most houses have light fuses rated 5 A. Also, in a separate distribution box or the same box, there are fuses for power sockets or points in a living room, bedroom, kitchen and for the hot water system. The fuses for the power supply may be rated 15 A. Hence, cables for power points are different from those for lights. In these two methods of wiring, the ring system method of wiring is more convenient for wiring modern houses than the tree system method.

Lighting circuits are used in controlling the lighting fixtures in a house. Unlike the ring circuit, the lighting circuit does not form a loop returning to the consumer unit. Instead, the consumer unit is normally connected to the first lamp, which in turn is connected to the second lamp, and so on. There are two types of lighting circuits, namely; loop-in lighting circuit and junction box lighting circuit.

In loop-in lighting circuit, the live, neutral and earth wires run from the consumer unit to each of the ceiling designs, one after the other. From each rose, another set of wires run to the switch which operates the light. Figure 9.49 illustrates the loop-in-circuit.

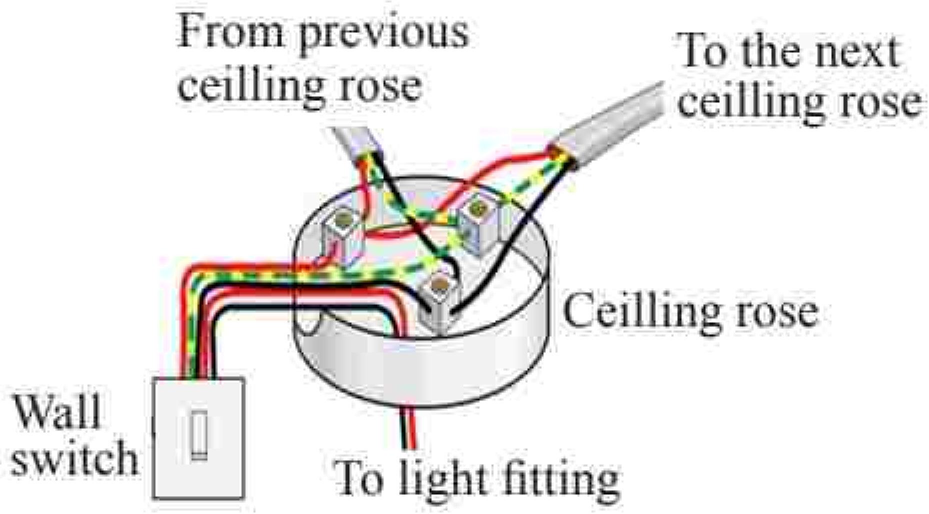


Figure 9.49: *Loop-in circuit*

In junction box lighting circuit, neutral and earth wires run to one junction box after another. From the junction box, one wire runs to the light and other runs to the switch for that light. Figure 9.50 illustrates the junction box lighting circuit.

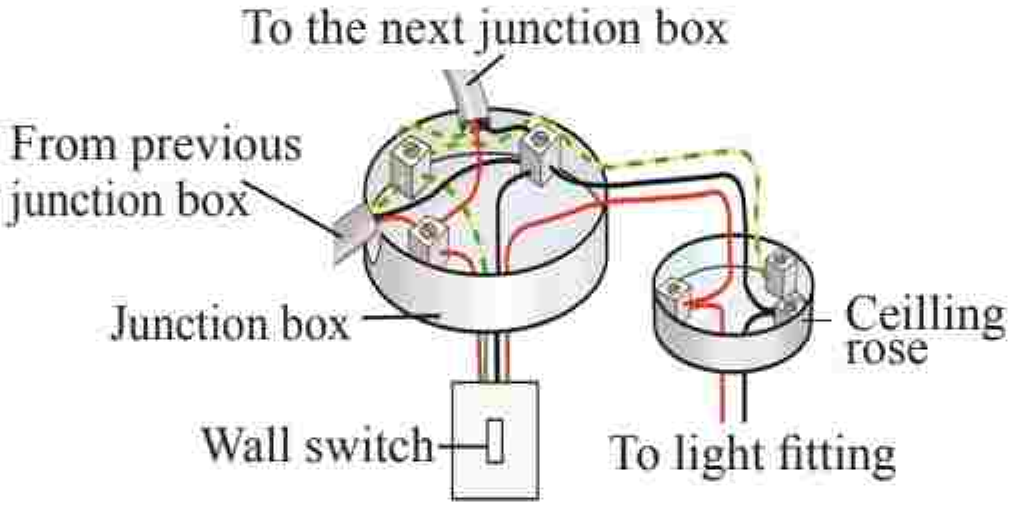


Figure 9.50: *Junction box lighting circuit*

Task 9.2

1. Open up a three-pin plug and fit in a cable and a fuse. Ask the teacher to check before you close it up.
2. Open the two-pin plug and fit in the cables as per instructions. Ask the teacher to check before you close it up.

Project 9.1

Aim: To demonstrate a ring and tree wiring on an electrical board

Materials: two switches, wire cutters, electrical board, bulb, red or brown, black or blue and yellow or green cables each about 2 metres long

Procedure

1. Draw a schematic diagram of a ring in circuit on a sheet of paper. Use the diagram to connect the circuit on the electrical board. Ask your teacher to check your connections before dismantling.
2. Draw a loop-in circuit to carry lamps. Use the diagram to connect the circuit on the electrical board. Ask your teacher to check your connection before dismantling.

Checking electrical faults in domestic appliances

Two devices are useful when checking electrical appliances faults. These are the multimeter and the live mains lead indicator. Multimeters are tool used for measuring a variety of electrical variables. The most basic parameters measured are voltage and current (both a.c and d.c.). They also have a range switch so that precise readings can be taken. There are moving coil and digital multimeters. The moving-coil multimeters (analogue) (Figure 9.51 (a)) have some disadvantages as it involves various settings when in use. The digital multimeter (Figure 9.51 (b)), has no moving parts, thus easy to use.

(a) Analogue (b) Digital

Figure 9.51: *Multimeters*

The live mains lead indicator, or commonly called tester, is made up of a form of a screwdriver with a hollow insulating handle containing a tiny neon discharge tube. One electrode of the neon tube is usually in contact with the metal probe of the screwdriver. The other is connected to a metal cap of the handle through a high carbon resistor. When one inserts the metal probe into a live socket and touches a metal cap with a finger, current leaks to the earth through the body and neon tube glows. Owing to the high resistance, this current is very little and so there is no risk of electric shock. Figure 9.52 shows a typical tester.

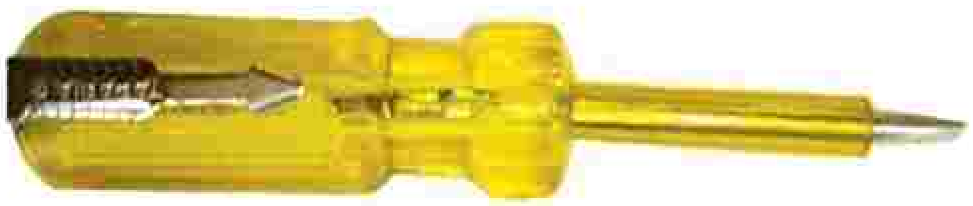


Figure 9.52: *Live mains lead indicator(tester)*

Most of the faults that occur in electrical appliances are simple and can be repaired easily. If, for example, an electric kettle fails to work, the first thing to check is, if there is power in the socket. This is done using the tester.

The next step is to check the cable from the socket to the appliance. If no fault is detected, the plug should be opened to check the fuse. Also check each cable for continuity using a multimeter. If these are in good working condition, then the next place to check for faults is in the heating element. This can be checked for continuity or short circuit using a multimeter.

If the element is faulty, then, it must be replaced as repair may not be possible. If the element is not faulty, then, look for loose connections. Connections should be made firm and/or cleaned of rust and other dirt.

Repairs in electrical appliances should be limited to connections of power supply only. Beyond that, an electrical technician should be consulted. Care should be taken to avoid electric shocks during such repairs. Simple repairs on electrical connections in a domestic system can also be carried out. When a fuse blows, it is very likely to be due to a fault in an appliance. If the fault is detected and corrected, then the fuse can be replaced (or circuit-breaker switched ON). Other faults occur due to wire cutting or joining, sockets getting dirty and switches breakage. All repairs should be done with the mains switched OFF. When replacing switches and sockets, the colour code should be followed strictly.

Exercise 9.4

1. State the properties and functions of a fuse. How does a fuse in a lighting circuit differ from that in a heating or power circuit? What is the importance of using a fuse in an electrical appliance?

2. (a) A refrigerator is marked 250 V, 400 W. Calculate the maximum current that can flow through it.

   (b) Discuss what might happen to the refrigerator if it is connected to a supply of:

   (i) 280 V.

   (ii) 110 V.

3. (a) Explain why electrical appliances have to be earthed.
   (b) State and explain the precautions to take during electrical wiring of a house.
   (c) Explain with the aid of a diagram how electricity from a fuse box is distributed to other parts of a house.
4. (a) Explain briefly why most plugs have three pins.
   (b) In a three-pin plug, we find wires labelled N, L and E:
      (i) What do these letters mean?
      (ii) At present, what is the internationally accepted colour for the wires labelled N, L and E?
5. Give the differences between the tree system and ring system methods of wiring a house.
6. Explain why the earth pin is thicker and longer in three-pin plug.

Cells

A cell is a set-up used to cause a flow of electric current in a conductor. The flow of current is caused by reactions releasing and accepting electrons at different ends of a conductor. There are two common types of electrochemical cells, namely primary cells and secondary cells.

Primary cells

Primary cells are also known as voltaic cells. A primary cell is formed by dipping two different metals into a conducting solution. The two metals are called electrodes while a solution is referred to as the electrolyte. In a primary cell, reactants are used up after some time and must be replaced. Figure 9.53 shows the features of a simple primary cell.

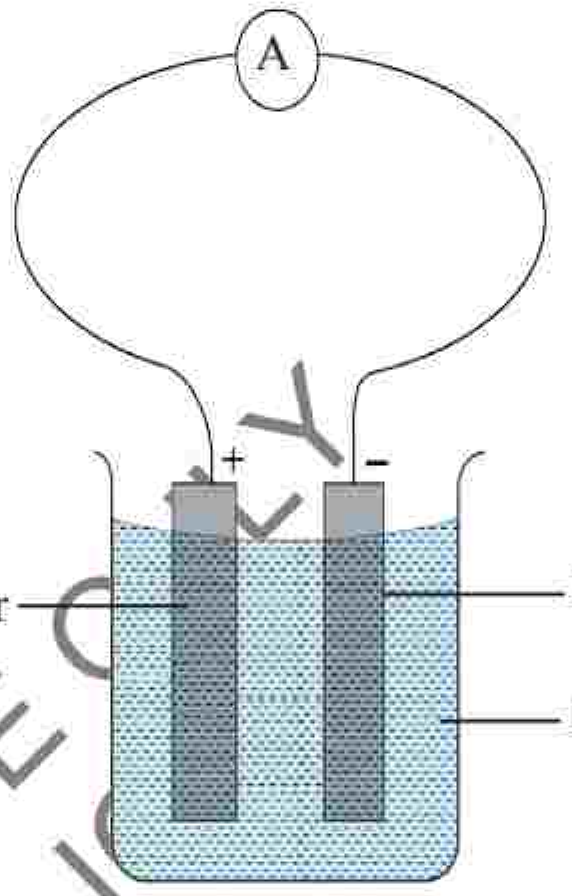


Figure 9.53: *A simple cell*

Electrodes of a simple cell are made of copper (positive) and zinc (negative) while an electrolyte is dilute sulphuric acid.

The following are the processes that occur when the cell is in operation:

The dilute sulphuric acid ionises into sulphate ions (SO_4^{2-}) and hydrogen ions (H^+):

$$H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}$$

Zinc also goes into solution as zinc ions (Zn^{2+}) releasing two electrons which travel along the wire (external circuit) to the copper electrode. The zinc ions combine with the sulphate ions to form zinc sulphate ($ZnSO_4$). The same time when zinc ions go into solution, an equivalent number of hydrogen ions move to the copper electrode where they gain electrons and are liberated as hydrogen gas (bubbles):

$$2H^{+} + 2e^{-} \rightarrow H_2(g)$$

By losing electrons, copper becomes positively charged and enables it to attract electrons from Zinc through connecting wire. This movement of electrons through the wire is called the electric current.

The conventional direction of electric current is from the positive terminal to the negative terminal of the cell, which is the opposite to the direction of flow of electrons.

Mode of action of a dry cell (Leclanché cell)

The Leclanché cell consists of carbon (positive electrode) and zinc (negative electrode). The electrolyte in this cell is ammonium chloride (NH_4Cl) **(Sal-ammoniac)** solution. Figure 9.54 shows a Leclanché cell.

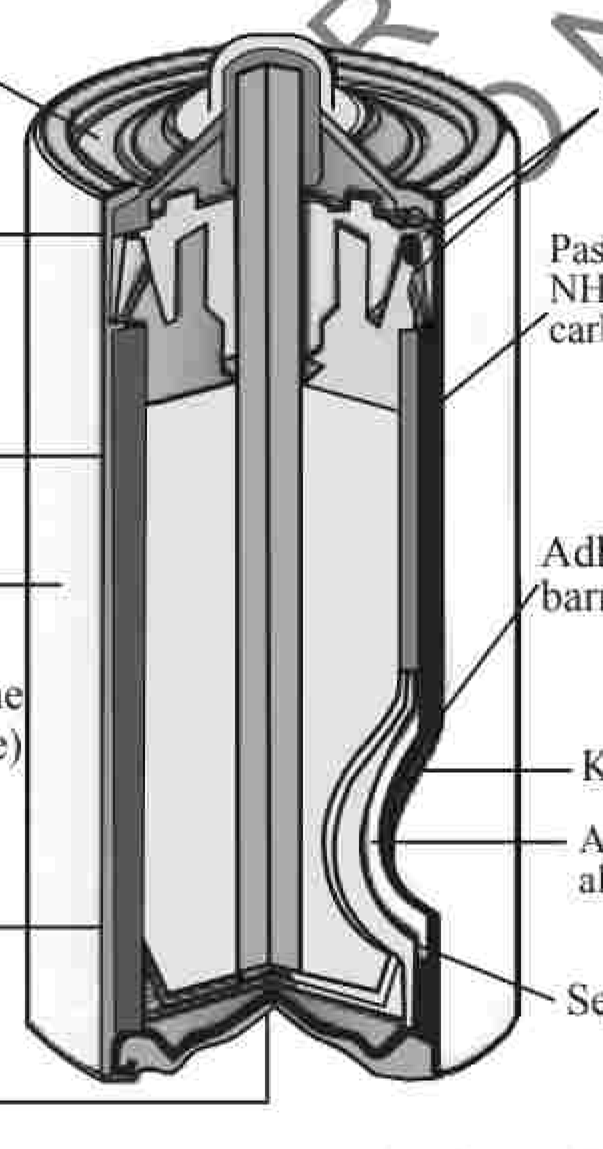


Figure 9.54: *Leclanché cell*

There is no local action in this cell. Polarization is reduced by manganese dioxide (MnO_2) which oxidizes hydrogen to water. Manganese dioxide mixed with carbon is put in a porous pot around the carbon electrode as shown in Figure 9.54. However, manganese dioxide is a slow depolarizer. When a large current is drawn from a cell, polarization takes place causing a reduction in the voltage of a cell. If allowed to rest shortly, a cell soon recovers. The dry cell is a modified Leclanché cell in which the main electrolyte can be a liquid or a paste. If the electrolyte is a liquid, the cell is said to be a wet cell. Thus, Leclanché cell is a wet cell. If the electrolyte is a paste, the cell is referred to as a dry cell. An assortment of dry cells is shown in Figure 9.55.

Figure 9.55: *Dry cells*

The Leclanché cell exists today is the 'dry' cell. Figure 9.56 shows the structure of a dry cell.

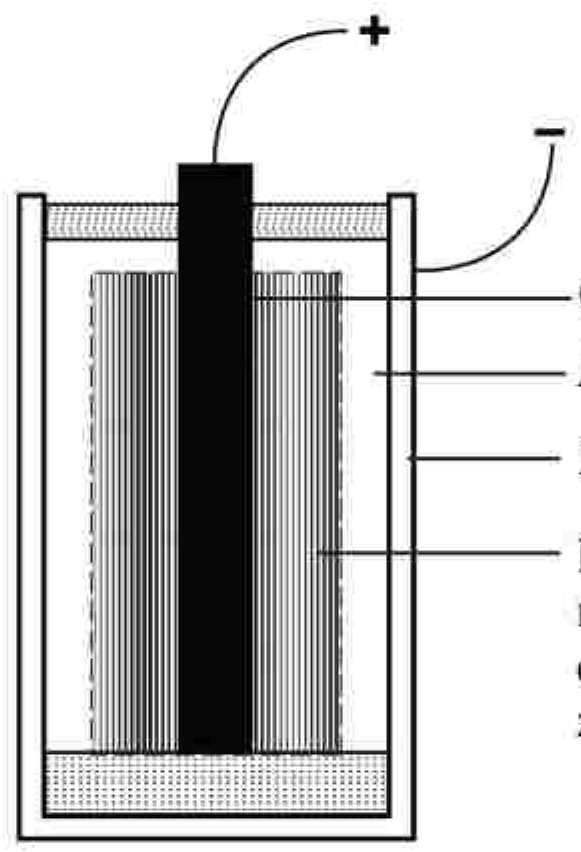


Figure 9.56: *Structure of a dry cell*

The components of a dry cell are the same as those of the Leclanché cell except that instead of ammonium chloride solution, a cell is filled with a paste of manganese dioxide, ammonium chloride and zinc chloride. The two salts (ammonium chloride and zinc chloride) act as the electrolyte. The negative electrode (zinc) is the one containing the electrolyte and depolarizer.

Cells in series and parallel

Cells are often used in combination. A battery is formed when two or more cells are connected together. Cells can be connected in either series or parallel.

Cells in series

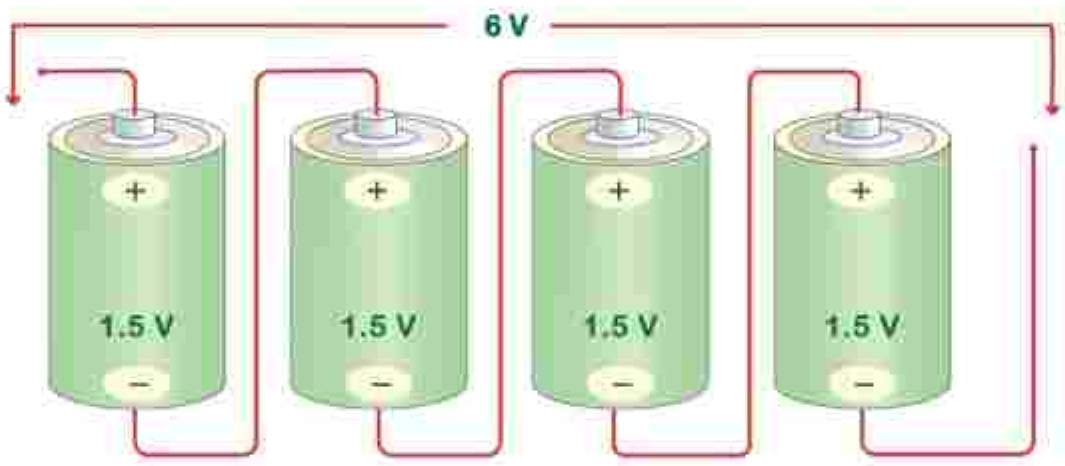
To obtain a bigger e.m.f, cells are arranged in series as shown in Figure 9.57. The current flowing through the cells is the same. The total voltage E_T across the cells is equal to the sum of the voltage of the individual cells. This is given by:

$$E_T = E_1 + E_2 + E_3 + E_4$$

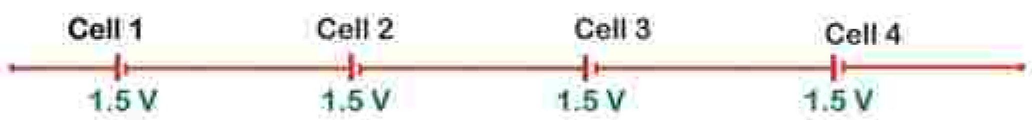
The total voltage, E_T for cells arrangements in Figure 9.57 is;

$$E_T = 1.5\text{ V} + 1.5\text{ V} + 1.5\text{ V} + 1.5\text{ V} = 6\text{ V}$$

In the case of *n* cells, each of volts *E*, $E_T = nE$.



(a) Cells in series arrangement



(b) Cells in circuit diagram

Figure 9.57: *Cells in series*

If cell 3 is reversed in Figure 9.57, then the equivalent e.m.f. is given by,

$$E_T = E_1 + E_2 + E_3 + E_4$$

$$= 1.5\text{ V} + 1.5\text{ V} - 1.5\text{ V} + 1.5\text{ V}$$

Therefore, $E_T = 3.0\text{ V}$.

The sum of individual internal resistances, r equals total internal resistance, r_T. If the four cells in Figure 9.57 are identical and have the same e.m.f and internal resistance, then,

$$r_T = r_1 + r_2 + r_3 + r_4$$

$$r = 1\ \Omega + 1\ \Omega + 1\ \Omega + 1\Omega = 4\ \Omega$$

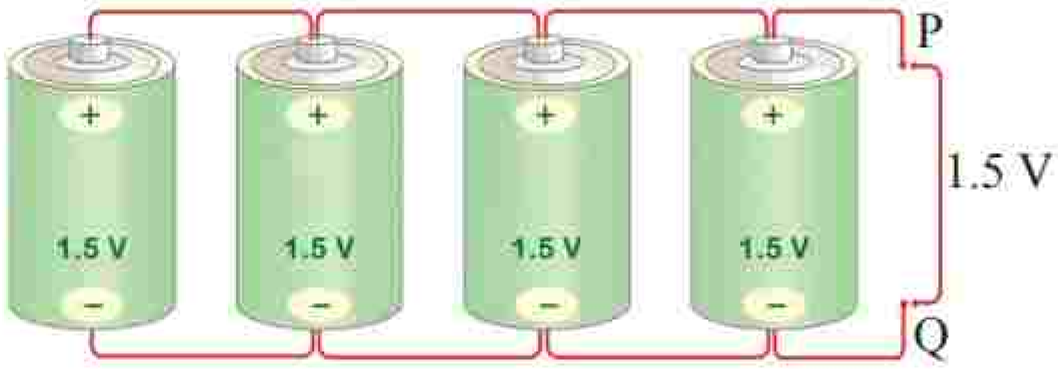
where $r_T = r_1 = r_2 = r_3 = r_4$

In the case of *n* cells:

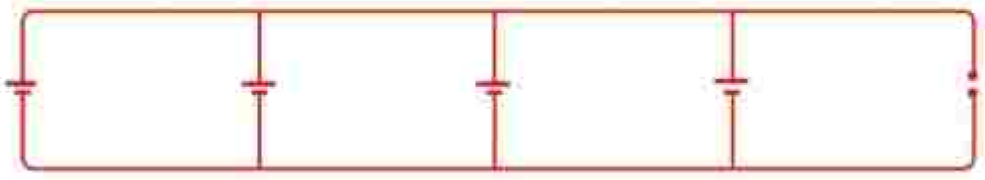
$$r_T = nr$$

Cells in parallel

Suppose four identical cells, each of e.m.f 1.5 V and internal resistance $1\,\Omega$, are arranged in parallel as shown in Figure 9.58. The total e.m.f recorded when a voltmeter is connected to terminals *P* and *Q*, is the e.m.f of one of the four cells. This is due to the fact that, the four cells operate together as a single cell whose materials are the sum of those in the individual cells. This is because the e.m.f of a cell is determined by the nature of the chemicals used and not on their amount. As a result, the e.m.f equals to that of one of the cells.



(a) Cells in parrallel arrangement



(b) Cells in circuit diagram

Figure 9.58: *Cells in parallel*

The internal resistance of the battery is different from that of a single cell. For this case, the internal resistances of four cells are parallel, as shown in Figure 9.58. As a result, the total internal resistance is given by,

$$\frac{1}{r_T} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$$

$$\frac{1}{r_T} = \frac{1}{1\,\Omega} + \frac{1}{1\,\Omega} + \frac{1}{1\,\Omega} + \frac{1}{1\,\Omega} = \frac{4}{1\,\Omega}$$

Therefore, $r_T = \frac{1}{4}\Omega$ or $0.25\ \Omega$.

Thus, the battery has an e.m.f of 1.5 V and an internal resistance of 0.25 Ω.

Example 9.19

Determine the voltage across AB in the circuit shown in Figure 5.59.

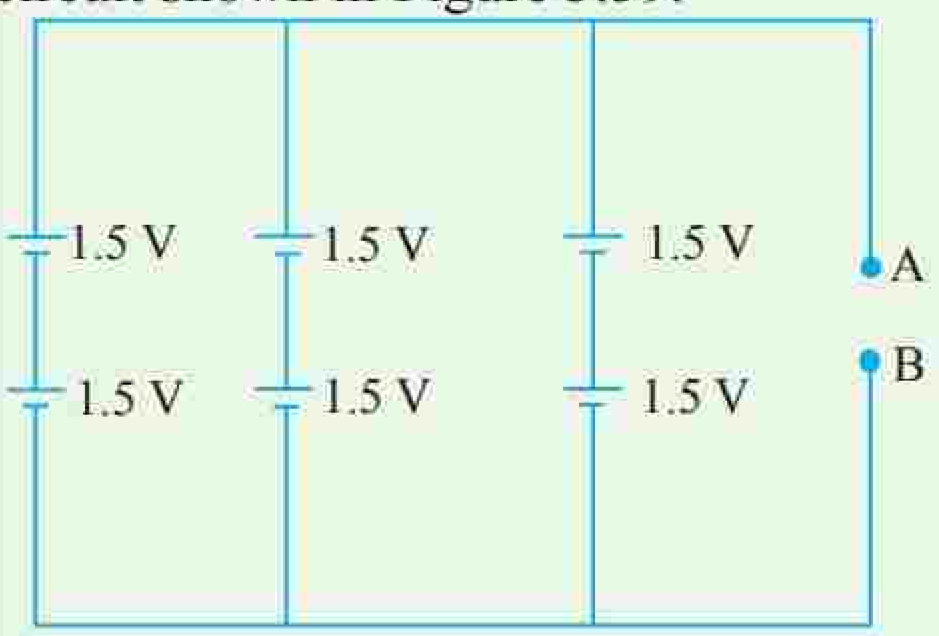


Figure 9.59

Solution

Note that, all the internal loops are connected in series while the external loops are connected in parallel.

For the internal loops, the total voltage in each loop = (1.5 + 1.5) V = 3 V. This means that, the external loop is made up of 3 batteries, each with a voltage of 3 V. The total voltage (across AB) is the same as that of a single battery (3 V).

Cell defects

When the simple cell is in operation, current drops to a very small value after some time. A simple cell has two main defects which cause the current to diminish quickly when cell is being used. These defects are local action and polarization.

Local action

The local action of a cell is the deterioration of the battery due to currents that are flowing from and to the same electrode. The deterioration is caused by embedded impurities such as iron, lead and carbon in a zinc electrode. The impurities act as positive electrode and create electric currents between zinc and this positive (impurities) electrode. Local action can be reduced by using pure zinc or by rubbing mercury on the zinc plate to form amalgam.

Polarization

Polarization is a defect that occurs in simple electric cells due to the accumulation of hydrogen gas around a positive electrode. The hydrogen gas produced in chemical reactions inside the cell can get accumulated around the electrodes resulting in an insulation. Polarization in cells can be minimized using depolarizers such as potassium dichromate or manganese oxide, which oxidizes hydrogen to water.

Simple cells, Daniel and Leclanché cells, are all called primary cells. In this type of cells, the current is produced from non-recoverable or irreversible chemical reactions. For example, when all zinc has been dissolved in a simple cell, it will never be recovered to its original form by passing a current through the cell in the opposite direction.

Secondary cells

However secondary cells can be recharged after they have run down. Examples of secondary cells are the lead-acid cell and the nickel-ferrous cell. Secondary cells are also called accumulators.

Mode of action of lead-acid accumulator

The lead-acid battery consists of a number of lead-acid cells. Each cell has two groups of lead plates. One group forms a positive terminal while the other group forms a negative terminal. All positive terminal plates are connected together with a connecting strap. This is also the case for negative terminal plates. The positive and negative terminal plates are interlaced so that they alternate. Between the plates are sheets of insulating material called separators. These are made up of porous wood or fiberglass. The separators prevent the positive and negative plates from coming into contact thereby avoiding production of a short circuit. The positive plates are made of lead dioxide while the negative plates are made of porous lead metal. The two sets of plates with the separators between them are placed in a container filled with a dilute solution of sulphuric acid. The term lead-acid refers to the lead plates and the sulphuric acid which are the main components of the battery. Figure 9.60 illustrate the components of the lead-acid accumulator.

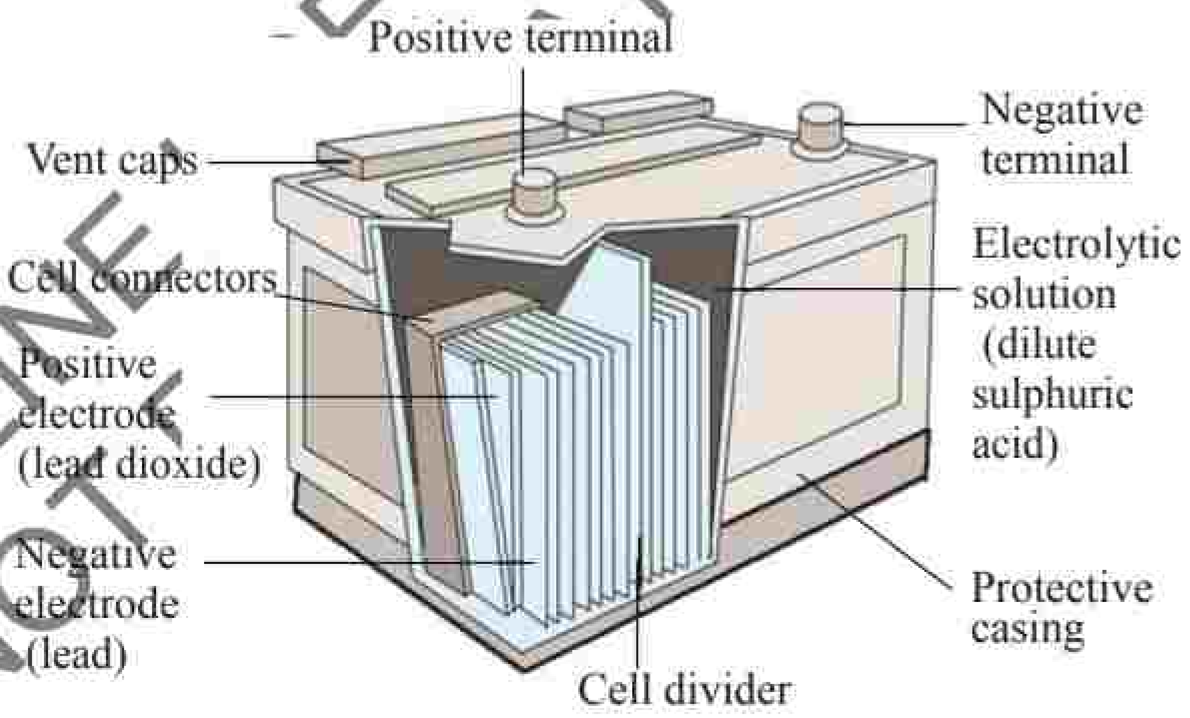
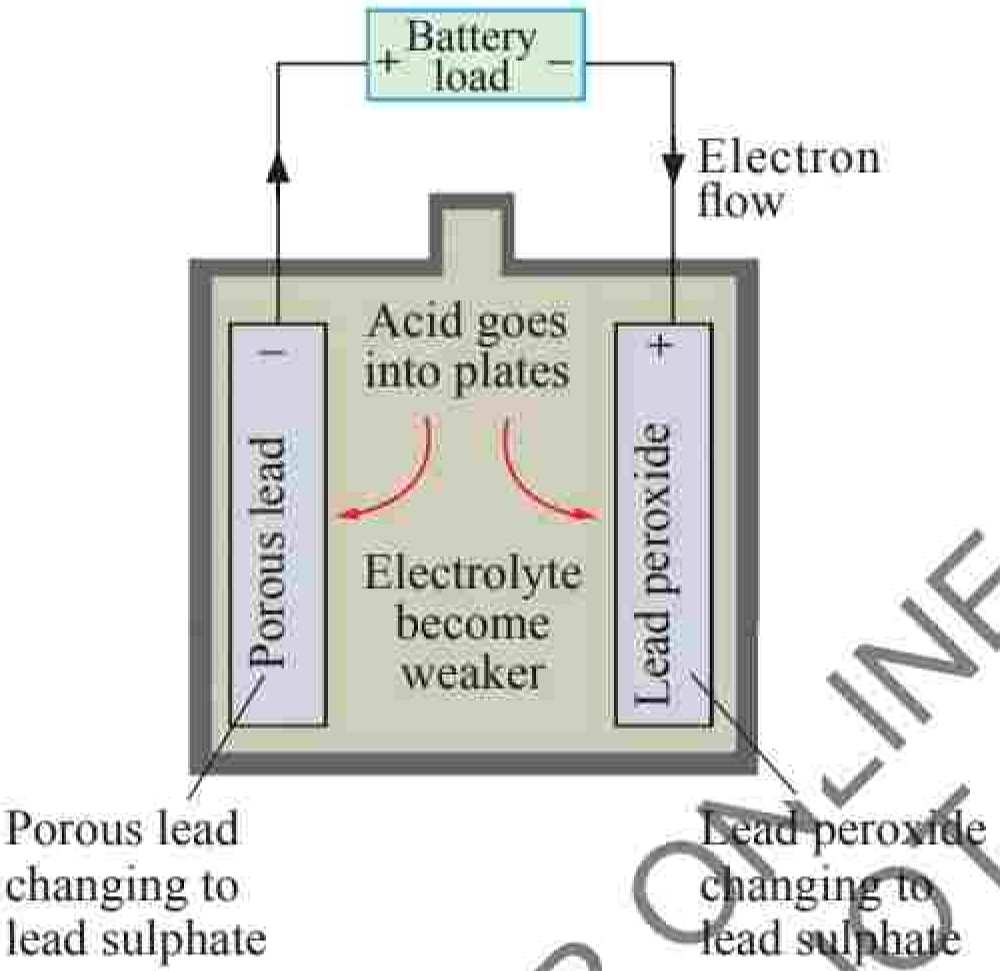


Figure 9.60: *Lead-acid accumulator*

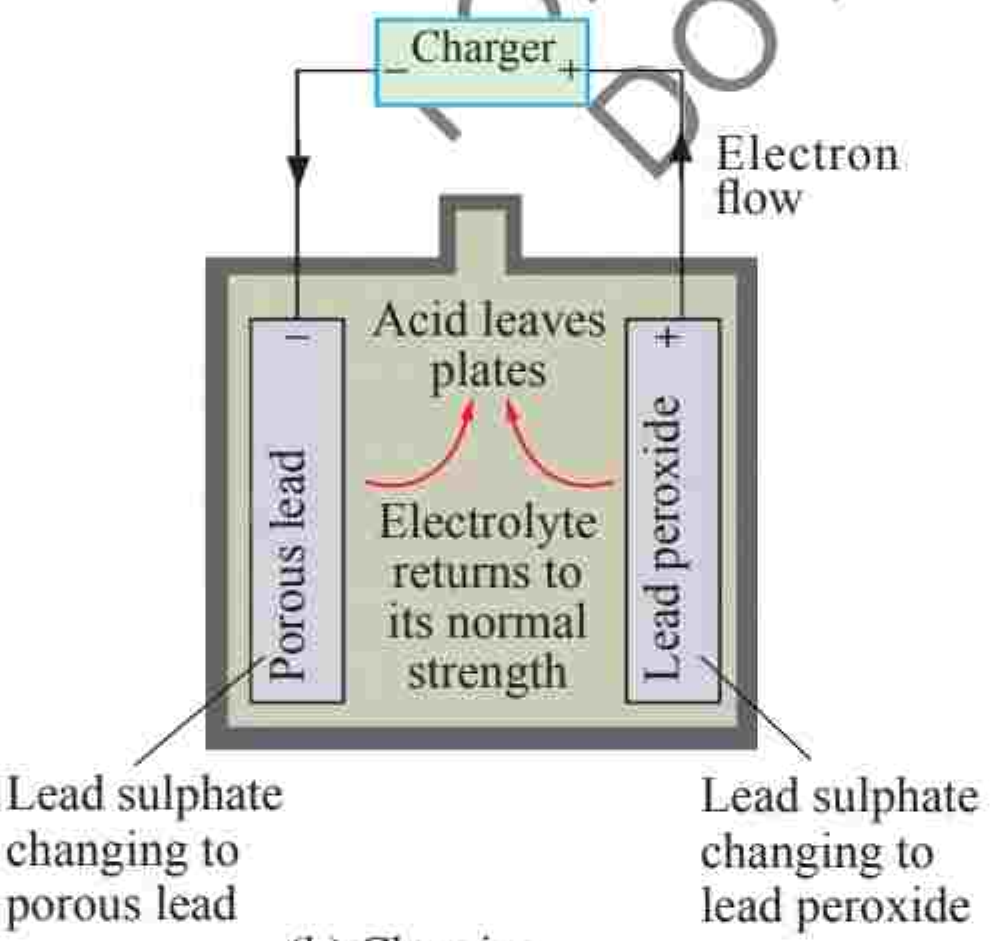
Charging and discharging the lead-acid battery

When the battery is providing energy, it is said to be discharging. The energy is produced when the acid (electrolyte) gradually combines with the active material of the electrodes. This lowers the concentration of sulphuric acid.

Charging the lead-acid battery is to drive all the acid out of the plates and return it to the electrolyte. During charging (Figure 9.61(b)), a direct current is passed through the battery in the opposite direction of that during the discharge (Figure 9.61(a)). The negative terminal of the battery charger is connected to the negative terminal of the battery, while the positive terminal of the charger is connected to the positive terminal of the battery. This reverses the action of discharge and restores the battery to its original charged condition. When the battery is fully charged, the active material of positive plates is lead peroxide while that of the negative plate is porous lead metal. The concentration of the acid is maximum in the charged state.



(a) Dis-charging



(b) Charging

Figure 9.61: *Lead-acid battery*

If the battery is only partially discharged, it can be recharged using a battery charger as shown in Figure 9.61(b). Lead-acid batteries should never be allowed to become completely discharged. The maximum recommended discharge is 75% of the maximum voltage. When the battery is completely discharged, both electrodes are completely converted to lead sulphate. In this state, the battery is said to be sulphated, meaning that it will no longer function as it cannot be recharged. The main advantage of the lead-acid accumulator is its ability to be recharged. Its major disadvantages are its large size and weight.

Activity 9.14

Aim: To charge and discharge a lead-acid cell

Materials: power pack (or a pack of 4 dry cells), two lead plates (3 cm × 5 cm each), dilute sulphuric acid, rheostat, ammeter, bulb, voltmeter, connecting wires, switch

Procedure

1. Connect a circuit as shown in Figure 9.62.

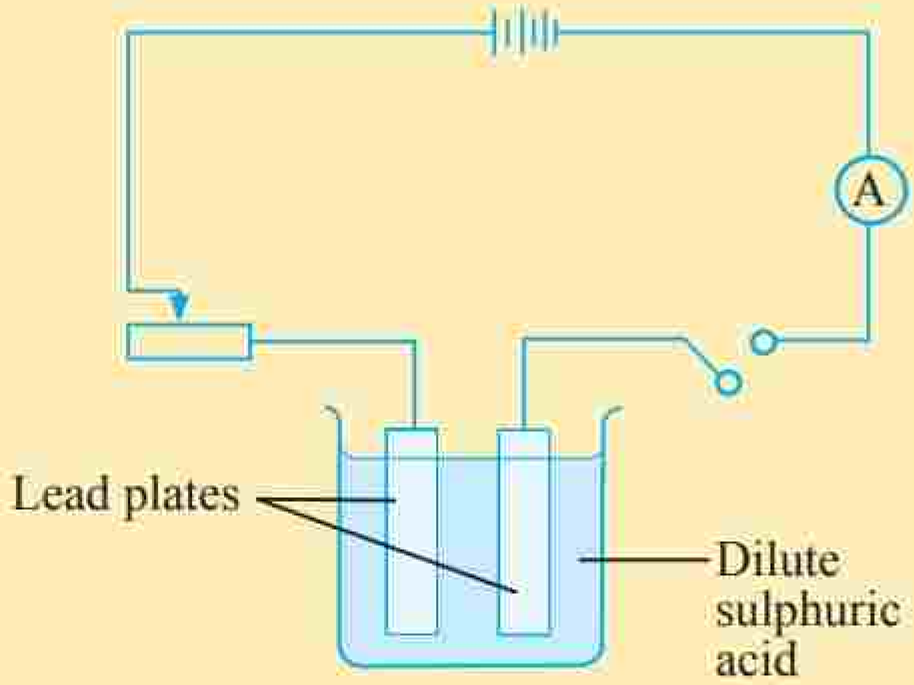


Figure 9.62

2. Close the switch and adjust the rheostat so that the reading on the ammeter is 1.5 A. Let the current flow for about one minute.
3. Open the switch and use the voltmeter to measure the potential difference across the plates.
4. Connect a 2.5 V bulb across the plates and observe its brightness.
5. Disconnect the circuit, pull out the plates and note the colour of both the positive and the negative plates.

Questions

(a) What was the voltage across the lead plates when a 1.5 A current was allowed to flow for one minute?
(b) What was the colour of each plate at the end of an experiment?

The plates produce a potential difference of approximately 2.0 V or slightly more after running a constant current for a short time. For a short period of time, this voltage causes the bulb to glow brightly. The plate that is linked to the positive terminal of a power source becomes the positive terminal of this cell. The cell is said to be charged when a current is passed through the plates. The positive plate changes its colour to brownish red. This is due to lead dioxide (or red lead) which is formed from lead during the charging process.

Lead-acid accumulators are commonly used as car batteries. Although it is rechargeable, the accumulator does not last forever. After some time, the paste falls from the frame and the cells can no longer be recharged. If well taken care off, however, lead-acid batteries can last for a long time. The following are some heads on how to care for lead-acid batteries:

1. The cells should be charged regularly and should never be left discharged. Cars have alternators that automatically charge the batteries.
2. The acid level should be maintained by adding distilled water when necessary. Never add acid.
3. The terminals should be kept clean and greased.
4. Rough handling should be avoided, for example dropping battery down.
5. The cells should not be short-circuited, that is, they should not be allowed to drive very large currents or connecting the two terminals directly to each other.
6. When charging, the rate specified by the manufacturer should not be exceeded.

Uses of cells and accumulators in daily life

The following are some of the uses of cells and lead-acid batteries:

Dry cells are used for operating radios, electronic calculators and other small electrical devices. On the other hand, accumulators offer a lot of use in our daily life as follows:

1. They are used to provide power in motor vehicles. Figure 9.63 shows a typical car battery.

Figure 9.63: *A car battery*

2. They are used to provide energy to power domestic appliances such as radio, television and lighting fixtures.
3. They are used to store electrical energy produced by solar panels.

Exercise 9.5

1. Distinguish between a:
   (a) cell and a battery.
   (b) primary cell and a secondary cell.
2. With the aid of a diagram and chemical equations, describe the construction and operation of a simple cell.
3. State the defects of a simple cell and explain how these defects can be minimized.
4. Describe, with the aid of a diagram, the construction of a typical lead-acid accumulator.
5. Use appropriate diagrams to illustrate how an accumulator may be charged and discharged.
6. Explain how accumulator can be properly maintained.
7. Three cells of e.m.f 1.5 V, 1.6 V, and 1.3 V are connected in series to supply current to an external resistor of 1.6 Ω. The internal resistances of the three cells are 0.26 Ω, 0.35 Ω and 0.42 Ω respectively. Determine the current through external resistor.
8. Four cells each of e.m.f 1.48 V and internal resistance 0.51 Ω are connected in parallel across an external resistor of 2.6 Ω. Determine the current supplied by the battery.

Chapter summary

1. Electromotive force (e.m.f), is the maximum potential difference between two terminals of a battery or a cell when no current is drawn from the battery or cell. In other words, it is the electrical intensity or pressure developed by a source of electrical energy.
2. Electric potential difference (p.d) is the work done per unit charge in moving the electric charge from one point to another point of a conductor.

3. Electric current, I, is defined as the rate at which electric charge, Q, passes a given point by unit of ampere, A, and measured by an instrument called ammeter.
4. Resistance is the opposition to the flow of electric current in a conductor. It is measured in ohms (Ω).
5. The resistance of a conductor is affected by: temperature, its length, cross-sectional area and type of the material of the conductor.
6. When resistors are connected in series, the total resistance in the circuit is equal to the sum of the individual resistance.
7. When resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.
8. When an electric current flows through a conductor of high resistance, the electrical energy gets converted into thermal energy and heats the conductor. This is known as the heating effect of electric current.
9. Every electrical appliance should carry a label stating the potential difference for which it has been designed and the power it can convert when operating at the stated potential difference.
10. Electricity is supplied in consumer's places by low resistance wire made up of either copper or aluminium material insulated with plastic.
11. The wires used in electrical installation are rated according to the maximum current they can carry.
12. Domestic electricity is supplied by two cables; the live cable (L), coloured brown or red, and the neutral cable (N) usually coloured blue or black.
13. The rate at which an appliance dissipates energy is called the rating of that appliance and is usually marked on the body of the appliance.
14. There are two common types of electrochemical cells, namely primary and secondary cells.
15. The total voltage across cells connected in series is equal to the sum of the voltages of the individual cells while in parallel connection, the effective voltage is equal to the voltage of one of the cells.

Revision exercise 9

1. For each of the items (a) - (d), choose the correct answer among the given alternatives and write its letter.

   (a) The amount of charge flowing through a cross-sectional area of a wire per unit time is called:

A. Voltage B. Power
C. Resistance D. Work
E. Current

(b) What is the direction of the conventional current from the battery, through the light bulb in the circuit presented by the Figure 9.64?

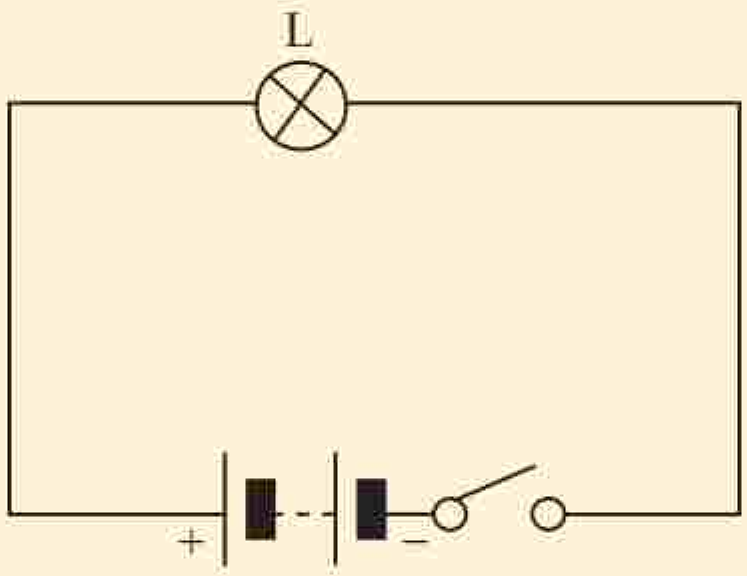


Figure 9.64

A. → B. ← C. ↓ D. ↑ E. Out of the page

(c) The electric current as a function of voltage of a wire is presented by the graph in Figure 9.65. Use this graph for questions (i) and (ii).

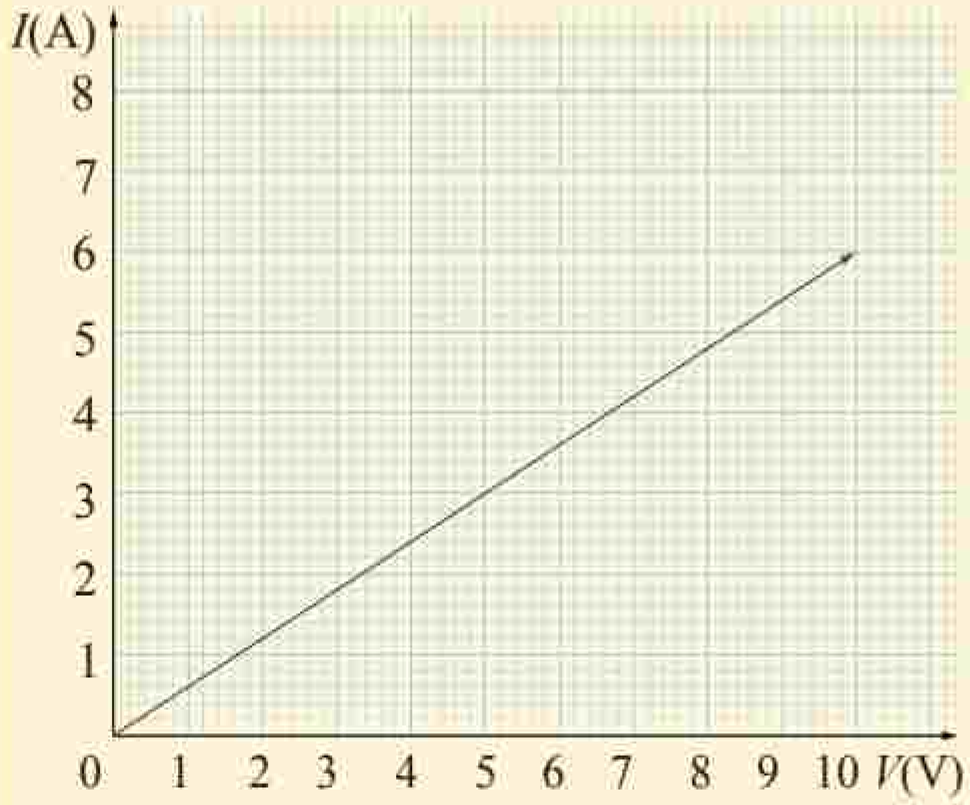


Figure 9.65

(i) What is the resistance of the wire?

A. 1Ω B. 0.8 Ω
C. 1.7 Ω D. 0.4 Ω
E. 0.2 Ω

(ii) What is the power dissipated in the resistor when the applied voltage is 5 V?

A. 5 W B. 10 W
C. 15 W D. 20 W
E. 25 W

(d) Three resistors: $R_1 = 3\ \Omega$, $R_2 = 6\ \Omega$, and $R_3 = 9\ \Omega$ are connected in series and to a 36 V battery as shown in Figure 9.66. What is the ammeter reading when the switch is closed?

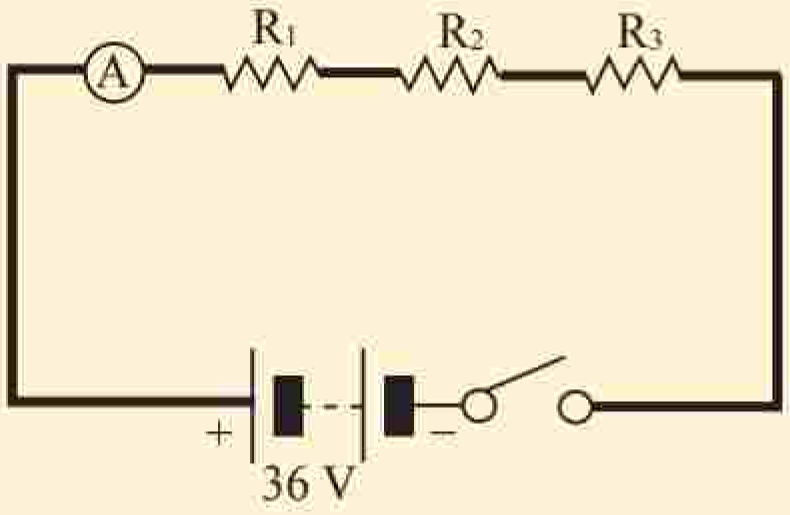


Figure 9.66

A. 6 A B. 5 A
C. 4 A D. 3 A
E. 2 A

2. Differentiate between electromotive force (e.m.f) and potential difference (p.d) of a cell.

3. Calculate the resistivity of a wire of length 2 m and cross-sectional area 0.004 cm^2 if its resistance is 3.0 ohms.

4. When a 6 V battery is connected to a lamp, a current in the circuit is measured to be 0.4 A. What is the resistance of the lamp?

5. The e.m.f of a cell is 9 V. How much work is required to move one electron from the negative terminal to the positive terminal? Ignore the internal resistance of the cell.

6. A bulb with a resistance of 5 Ω is connected to an 8 V battery. If the internal resistance of the battery is neglected;
   (a) How much current will flow through the bulb?
   (b) How much light and heat energy does the lamp produce in one hour?

7. A 12 V battery is connected to two bulbs as shown in Figure 9.67.

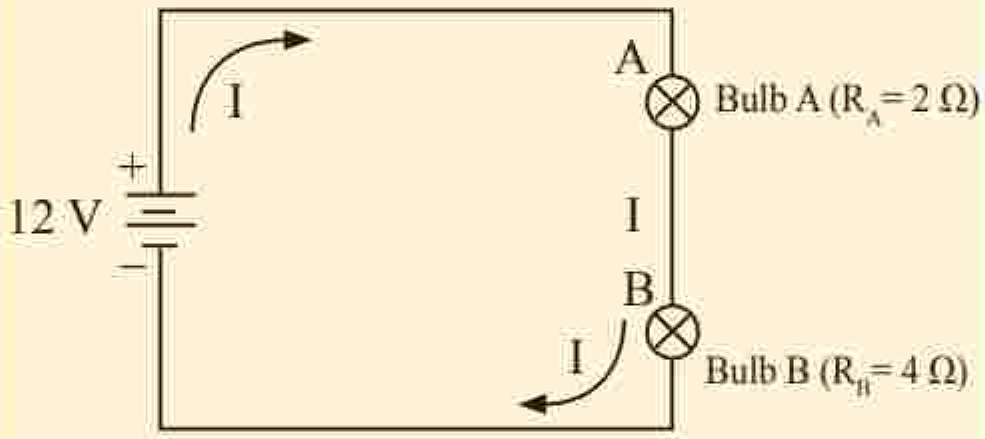


Figure 9.67

   (a) Determine the voltage across:
      (i) bulb A.
      (ii) bulb B.
   (b) How much current will flow in the circuit?

8. A 12 V battery is connected to two bulbs as shown in Figure 9.68.

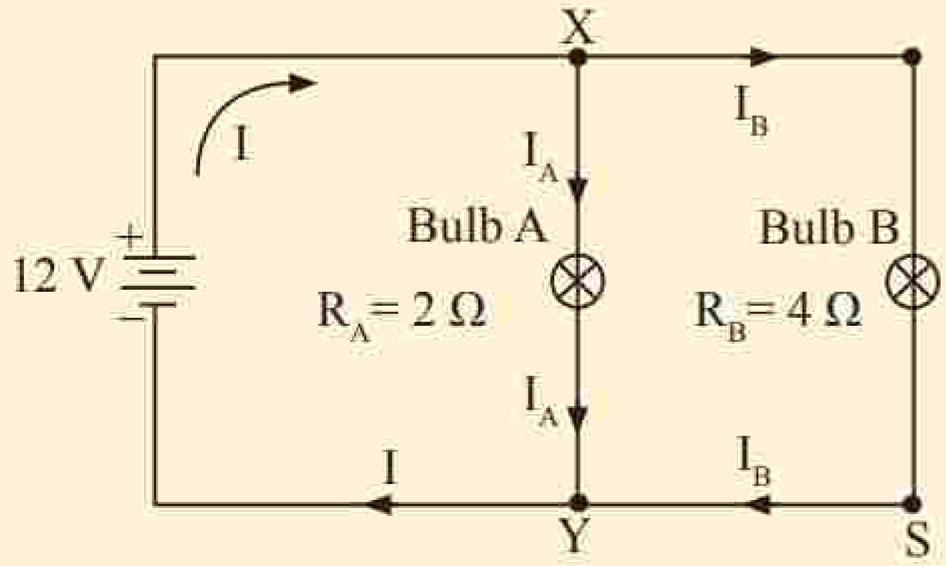


Figure 9.68

   (a) What is the voltage across:
      (i) bulb A?
      (ii) bulb B?
   (b) What is the current flowing through:
      (i) bulb A?
      (ii) bulb B?
   (c) What is the total current, *I*?

9. Explain why resistance cannot be accurately measured using just a voltmeter and an ammeter.

10. A wire of length 28 cm and radius 0.75 mm is wound round a cylindrical tube so as to form a solenoid. If the resistivity of the wire is 5.6×10^{-8} Ωm, calculate its resistance. Determine the material of the wire.

11. Three identical resistors each with a resistance of 10 Ω are connected in series to a 12 V battery with an internal resistance of 2 Ω.
    (a) What is the total resistance of the circuit?

(b) How much power is supplied by the battery?

(c) How much power is dissipated in each of the external resistors?

(d) How much power is dissipated by the battery's internal resistance?

12. The diagram in Figure 9.69 shows an electric circuit containing three resistors.

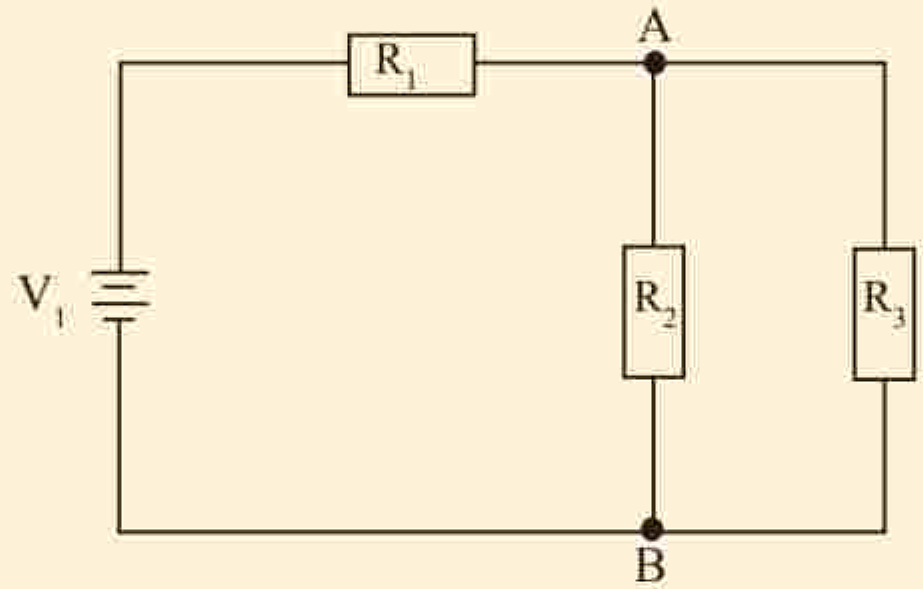


Figure 9.69

Given that, the battery has a voltage of 24 V and a negligible internal resistance and that $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, $R_3 = 10\ \Omega$, determine:

(a) the total resistance in the circuit.

(b) the current that flows through R_1.

(c) the current that flows through R_2.

(d) how much current flows through R_3.

(e) the potential difference between point A and B.

13. Figure 9.70 shows three resistors connected in parallel to a cell with a negligible internal resistance.

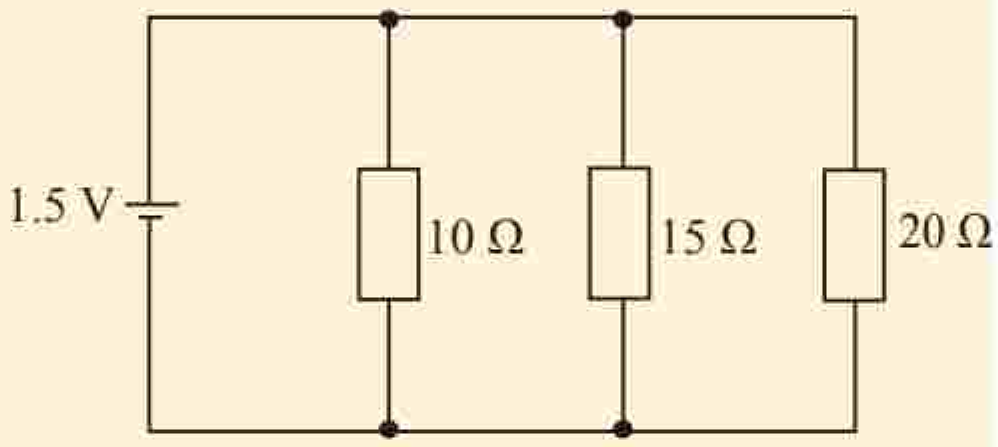


Figure 9.70

(a) What is the total resistance in the circuit?

(b) What is the total current flowing in the circuit?

(c) What current flows in each resistor?

(d) How much power is supplied by the battery?

(e) How much power is dissipated in each resistor?

14. A water boiler is connected to the 240 V power supply. The boiler contains 150 kg of water initially at a temperature of 20 °C. Given that, the resistance of the heating coil in the boiler is 25 Ω:

(a) How long will it take to raise the temperature of water to 40 °C when the heater is turned ON ? (The specific heat capacity of water is 4.18 kJ/kg K).

(b) How many kilowatt-hours of electrical energy will be consumed?

15. An electrical circuit in a domestic house is protected by a 6 A fuse. The circuit is connected to the 240 V mains. The following appliances in Table 9.11 are connected to the circuit.

Table 9.11

Appliance	Bulb 1	Bulb 2	TV	Heater	Iron
Power rating	100 W	75 W	300 W	1500 W	900 W

List several groups of these appliances that could be turned ON at the same time without blowing the fuse. Justify your answer.

Answers to numerical questions

Chapter 1

Exercise 1.1

1. 296 km at an angle of 24.1° from East
2. 29.09 km, 20.1° with 10 km or 69.9° North of East
3. 10 N, 37°
4. 87.18 N, 23.4°
5. 560.8 N, 69.4° South West
6. 21 N, 21.8° with the 15 N force
7. (a) 22.38 N, 30.4° or 14.6° with the 16 N and 8 N force respectively or 8 N force

   (b) 21.17 N, 40.9° with 8 N force

   (c) 13.86 N, 30° with 16 N force

Exercise 1.2

1. Towards the back
2. 39.39 m
3. 1.37 h
4. 332.3 km/h, 285.7° from East
5. (a) −20 km/h (b) 140 km/h
6. (a) $-0.28\ m/s^2$, $0.37\ m/s^2$

   (b) 71.1 s (c) 47.4 m/s

Exercise 1.3

1. (a) 13.7 m at −83.04° or 96.96°

   (b) 10.86 m at −28.4° or 151.6°

3. 113.3 km/h, 92°

Revision exercise 1

1. (a) (i) (b) (iv)
3. (a) 16.52 N (b) 15.93 N
   (c) 10.44 N
4. (a) 120 m/s, East
   (b) 80 m/s, West
   (c) 102 m/s, South
5. (a) 100 N (b) 173.2 N
6. (a) 6.84 m/s, 18.79 m/s

   (b) 14.14 m/s, 14.14 m/s

   (c) 10 m/s, 17.32 m/s

   (d) 1.74 m/s, 19.92 m/s

7. 10 m/s, 18 m/s
8. (a) 11.3°

   (b) 10.2 m/s

9. 398.23 N, −16° or 344°
10. 17.32 N, 34.64 N
11. (b) 15 km/h
12. 2.47 s
13. 13 km/h, 5 km/h

Chapter 2

Exercise 2.2

5. 36 N
6. 0.08
7. 12 N
8. 1.33 cm
9. 850 N

Exercise 2.3

7. (a) 12 N (b) 7 N
9. 0.75
10. 114 N

Revision exercise 2

9. 112.5 m, 450 m
10. (a) 28.8° (b) 2.63 m/s^2
11. (a) (i) 50 N (ii) 50 N (b) 3 m/s^2
12. 0.33, 0.25
13. 40.98 m/s
14. (a) 1.5 m/s^2 (b) 3.5 m/s^2
15. 4.4 m/s^2

Chapter 3

Exercise 3.1

1. 26.3 cm, 3.8 cm, real image
2. (a) 6 cm, 3 cm, behind the mirror
   (b) 0.6, diminished
3. (a) (ii) 20 cm, 0.3, 7.5 cm
4. 12 cm, 8 cm, 2
5. 26.67 cm or 13.33 cm
6. 25 cm
7. 5 cm

Exercise 3.2

1. 47.8 cm
2. 28.9°
3. 2.3 m
4. (a) 1.1 (b) 27°
5. 1.95 m
6. (a) 1.24×10^8 m/s (b) 24.4°

Exercise 3.3

3. 1.25
4. (a) 30° (b) 45° (c) 30°
5. 45°
6. 1.5
7. 1.5
8. (a) 14.17 cm

Exercise 3.5

1. (b) (i) –30 cm, virtual, erect and enlarged
   (ii) at infinity
   (iii) 90 cm, real, inverted and enlarged
2. –10 cm
3. 18 cm, 2, real, inverted and enlarged

4. 25 cm
5. 2 cm
6. 75 cm
7. 4

Revision exercise 3

1. (i) D (ii) D (iii) D (iv) B
   (v) A (vi) B (vii) C (viii) C
4. 60 cm
5. 36 cm, 12 cm
6. –62.33 cm
7. (a) 60 cm (b) 120 cm (c) 41 cm
8. 15 cm
9. 19.4 cm, 5.2 cm
10. 6.67 cm
13. 3.27 cm
14. 35.3°
17. (a) 41.8° (b) 48.8° (c) 62.5°
19. 100 cm, 20 cm
20. (b) 30 cm
21. 20.83 cm
23. 4 cm
24. 77.1°

Chapter 4

Exercise 4.1

1. (a) 6 (b) 25 cm
2. 24 cm, 6
3. 3.6 cm
4. (a) 4.2 cm, 5 cm (b) 6, 5

Exercise 4.2

1. (b) 12 cm
2. 4 cm, 3.3 cm

Exercise 4.3

1. 73.1 cm, 4.9 cm
2. 30.2
3. 35, 144 cm
5. (a) 700
   (b) 12.82 cm

Exercise 4.4

1. 17.2 cm
2. 315 cm

Exercise 4.5

1. (a) 6.7 cm (b) 1.7 cm
2. 2.4 m
3. 10.4 cm

Exercise 4.6

1. (a) 2.1 cm (b) 2.06 cm
   (c) 1.94 cm
2. 33.33 cm, 3 D
3. 2 D
4. (b) (i) 34.62 cm (ii) 2.89 D
   (iii) 3.6

Revision exercise 4

1. (i) C (ii) A (iii) D (iv) A
3. 13.5

5. 15.5 cm, 30
6. (a) 2.6 cm (b) 30
10. (a) 3.75 cm (b) 22.2
11. (a) −48 cm (b) 2.08 D
12. (a) 0.05 m, real
    (b) 0.02, 3.6 cm
13. 13 D

Chapter 5

Exercise 5.2

9. 324.06 m
10. 0.033 m or 3.3 cm
11. $1.7\times10^{-5}\,K^{-1}$
12. −51.9 ºC
13. 178.37 ºC

Exercise 5.3

6. 80 ºC
7. 58.14 ºC
8. 4 ºC

Exercise 5.4

2. 343 K
3. 349.8 K or 76.8 °C
4. (a) 4.39 m^3 (b) 2.88 m^3
5. 29 atm
6. $2.21\times10^{5}\ Nm^{-2}$
7. 2×10^{5} Pa
8. 586 K

Revision exercise 5

1. d
2. c
7. 3.3×10^{5} Pa
8. (b) $1.62\times10^{-5}\ K^{-1}$
9. $3.07\times10^{-6}\ K^{-1}$
10. 0.062 cm
11. 765 ºC
12. 38 cm
13. Too large, 1.2 mm
14. 30.043 cm, 18.026 cm and 10.014 cm
15. 50 cm
18. 7.5 atm
19. (a) 2500 cm^3 (b) 8.33 atm
20. 2.73 m^3
21. 546 K (272.85 ºC)
22. 240.96 ml
24. 1.5%
25. 1.84×10^{5} Pa
26. 13.5 cm^3
27. (a) 1.213×10^{3} Pa
    (b) $1.045\times10^{-3}\ m^3$

Chapter 6

Exercise 6.1

1. (a) (i) (b) (i)
   (c) (iv) (d) (iii)

Revision exercise 6

16. (c)

Chapter 7

Exercise 7.1

3. 40 °C
4. 0.1 kg
5. 420 J/kg°C
6. 21.9 °C
7. 1:2

Exercise 7.2

2. 836 kJ
3. 1670 kJ
4. 0.05 kg
5. 28.72 °C
6. 7410 kJ

Revision exercise 7

1. (i) D (ii) B (iii) B (iv) C
7. 1.3 kg
9. 21.7 °C
10. 27 kJ
11. −10 °C
14. 50 °C
15. 113.56 kJ
16. (a) 387.5 J/kg°C
18. 26.5 W

Chapter 8

Exercise 8.2

4. 29%
5. 0.54
6. 52.6%
7. 75%
8. 30 °C

Revision exercise 8

9. 33.33%
10. 140 m^3
11. (a) 62 °C
    (b) 80 °C
12. 48%

Chapter 9

Exercise 9.2

1. 6. 4 Ω
2. 5.5×10^{-7} Ωm
3. 1.06×10^{-6} Ωm
4. 11.31 m
5. (a) 5.5 Ω (b) 2.18 A (c) 8.73 V
6. (a) 2 Ω (b) 1.5 V
10. $E = 36$ V, $r = 18$ Ω
11. 3.64 V

Exercise 9.3

1. $6\ \Omega$
2. (a) 3.33 A, 4.17 A
   (b) $3.6\ \Omega$, $2.88\ \Omega$
3. (a) 757.58 A
   (b) 8.64×10^{12} J, 2.4×10^{6} kWh
4. (a) 1200 W
   (b) 2.16×10^{7} J, 6 kWh
5. 1.04×10^{7} J, 10.4 MJ
6. (b) 3.56 W
   (c) 0.89 W
7. 1.05 kWh
8. (a) 2.4 A
   (b) 576 W
   (c) 16.59 MW
9. (a) 10.8 MW
   (b) 3 kWh
10. (a) 0.25 A
    (b) $960\ \Omega$

Exercise 9.4

2. (a) 1.6 A

Exercise 9.5

7. 1.67 A
8. 0.543 A

Revision exercise 9

3. $6\times10^{-7}\ \Omega$m
4. $15\ \Omega$
5. 1.44×10^{-18} J
6. (a) 1.6 A
   (b) 46.08 kJ
7. (a) (i) 4 V
   (ii) 8 V (b) 2 A
8. (a) (i) 12 V
   (ii) 12 V
   (b) (i) 6 A
   (ii) 3 A
   (c) 9 A
10. (a) $8.73\times10^{-3}\ \Omega$
11. (a) $32\ \Omega$
    (b) 4.5 W
    (c) 1.41 W
    (d) 0.28 W
12. (a) $10\ \Omega$
    (b) 2.4 A
    (c) 1.2 A
    (d) 1.2 A
    (e) 12 V
13. (a) $4.62\ \Omega$
    (b) 0.33 A
    (c) 0.15 A, 0.1 A, 0.075 A
    (d) 0.49 W
    (e) 0.225 W, 0.15 W, 0.1125 W
14. (a) 1.51 h
    (b) 2.304 kWh

Glossary

Absolute humidity the amount of water vapour present in a unit volume of air, usually is expressed in grams per cubic metre.

Accommodation (of an eye) the adjustment of the focusing power of the eye in order to see objects clearly over a range of distances. Accommodation is achieved by changing the shape of the crystalline lens.

Ambient pressure the pressure of the surrounding medium, such as a gas or liquid which comes into contact with an object.

Angle of deviation the angle through which the direction of a ray of light is changed by a refracting surface of a prism. It is the angle between the incident ray and the refracted (or emergent) ray.

Apical angle an angle included between a refracting surfaces and which is contained in a principal section of a prism.

Apparent expansion the expansion of a liquid with temperature as measured in a graduated container without taking into account the container's expansion.

Boiling point the temperature at which a given substance changes from a liquid to a vapour (gas) at a given pressure. At the boiling point, the vapour pressure of a liquid becomes equal to the local atmospheric pressure surrounding the liquid.

Conductor a material through which electrons will readily flow, thus allowing the material to transfer energy (i.e., heat, electricity, etc.).

Critical angle the angle of incidence in the denser medium for which the angle of refraction in the less dense medium is 90°. It is the smallest angle of incidence at which total internal reflection occurs.

Dew atmospheric moisture that condenses at a rate greater than that at which it can evaporate and appears in the form of droplets on thin, exposed objects in the morning or evening.

Dew point the temperature at which atmospheric air must be cooled (at a constant pressure) to become saturated with water vapour.

Dispersion	(of white light) the splitting of visible white light into its component colours by a dispersive medium. This occurs because each colour has its own frequency of dispersion.
Electrolyte	a substance that conducts an electric current when in solution or molten form.
Electromagnetic wave	a wave that consists of an electric field and a magnetic field oscillating with the same frequency, but perpendicular to each other.
Electromotive force	the work or energy that causes a flow of an electric current through a circuit. It is also the potential difference (voltage) between two electrodes in a cell.
Evaporation	a process by which molecules in a liquid state spontaneously become gaseous. It is the opposite of condensation.
Focal plane	an area in a camera where light is focused.
Frictional force	a force that acts between two surfaces, whether they are moving past each other or there is no relative motion between the two surfaces. Frictional forces oppose relative motion or tendency of motion between the two surfaces.
Frost	solid deposition of water vapour, in form of minute ice crystals, from saturated atmospheric air on objects that are exposed to the air. It is formed when solid surfaces are cooled below the dew point of an adjacent air.
Generator	a device that converts mechanical energy into electrical energy by spinning conductors, usually copper wires, within a magnetic field.
Heat	energy transferred from one body or system to another due to a difference in temperature. It is the energy associated with the random motion of the particles that make up matter.
Heat content	the amount of heat in a quantity of matter at a specific temperature and pressure.
Insulator	a material that does not conduct heat (thermal insulator) or electricity (electrical insulator).
Internal resistance	(of a cell) the resistance to the flow of electric current that occurs inside an electrochemical cell. It is caused by the resistances of the components that make up the cell.

Kinetic friction	the friction between two objects that are moving relative to each other and rub together (like wheels and a road surface).
Latent heat	the energy released or absorbed by a chemical substance during a change of state that occurs without a change in temperature.
Limiting friction	the friction force that just balances a moving force applied to a solid body resting on a solid surface when the body fails to move.
Magnification	the apparent enlargement of an object by an optical instrument.
Melting point	the temperature at which a solid substance becomes a liquid.
Mirage	a naturally occurring optical phenomenon in which light rays are bent to produce a displaced image of distant objects or the sky.
Monochromatic light	a light consisting of a single wavelength or frequency, which appears as a single colour.
Near point	the nearest point at which an object can be seen clearly by un aided human eye. For a normal human eye, this distance is 25 cm.
Nitrocellulose	a highly flammable compound formed by treating nitrating cellulose with a solution and nitric acid mixture.
Normal force	the component force that is perpendicular to the surface that an object contacts.
Optical density	a measure of the extent to which a substance transmits light or electromagnetic radiation.
Polarization	(in a cell) a chemical reaction that occurs in a cell while a current is flowing causing hydrogen bubbles to form on the surface of the electrode. The bubbles form a barrier that increases internal resistance resulting to poor perfomance of the cell.
Polychromatic light	light consisting of multiple wavelengths which appear as different colours.
Prism	a 3-dimensional transparent body, usually made of optical glass, with at least two polished plane faces inclined towards each other from which light is reflected or through which light is refracted.

Rainbow	an arc of coloured light in the sky, produced by reflection, refraction and dispersion of light within droplets of rain in the air. This results in splitting up the white light of the sun into its component colours.
Real image	an image produced in an optical instrument, such as a lens or mirror, and can be projected onto a screen. Real images are formed by actual intersection of light rays.
Regelation	the phenomenon of melting under pressure and freezing again when the pressure is reduced.
Relative humidity	the ratio of the actual vapour pressure to the saturation vapour pressure. It is the amount of water vapour present in the atmosphere expressed as a percentage of the amount needed to achieve saturation at the same temperature.
Relative velocity	a measurement of velocity between two objects moving in different frames of reference.
Resistivity	a physical property of a material to resist or oppose the movement of electric charge through it.
Resolution	(of vectors) the splitting of a vector quantity into two or more components which are usually at right angles to each other.
Scalar	a physical quantity that has a magnitude but no direction.
Secondary colour	a colour that is obtained by mixing two primary colours.
Static friction	the friction between two solid objects that are not moving relative to each other.
Temperature	the degree of hotness or coldness of a body. It is a measure of the average kinetic energy of the individual particles (atoms or molecules) that make up the body.
Thermal equilibrium	the condition under which two substances in physical contact with each other exchange no heat energy, because they have equal temperature.
Thermal expansion	the increase in the dimensions of a material as a result of temperature change.
Thermostat	a device that opens or closes a circuit in response to changes in temperature.

Vapour pressure	a measure of the pressure exerted by a vapour of an evaporating liquid at a given temperature.
Vector	a physical quantity that is described by both magnitude and direction.
Virtual image	an image produced in an optical instrument, such as a lens or mirror, and cannot be projected onto a screen. Virtual images are formed by an apparent intersection of light rays.
Wavelength	the distance between two successive or adjacent points on a wave. Is the distance that the wave travels on one complete cycle.

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Index

Physics for Secondary Schools Student's Book Form Three